Quantum cryptanalysis

Daniel J. Bernstein

Main question in quantum cryptanalysis: What is the most efficient quantum algorithm to attack this cryptosystem?

(For comparison, main question in non-quantum cryptanalysis: What is the most efficient non-quantum algorithm to attack this cryptosystem?)

"Quantum algorithm" means an algorithm that a quantum computer can run.

i.e. a sequence of instructions, where each instruction is in a quantum computer's supported instruction set.

How do we know which instructions a quantum computer will support?

(Something to think about: Do we really know the answer for non-quantum computers?)

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Data stored in 4 qubits: a list of 16 numbers, not all zero. e.g.:
$[3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3]$.

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Data stored in 4 qubits: a list of 16 numbers, not all zero. e.g.:
$[3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3]$.

Data stored in 64 qubits:
a list of 2^{64} numbers, not all zero.

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a list of 8 numbers, not all zero.

e.g.: $[3, 1, 4, 1, 5, 9, 2, 6]$.

e.g.: $[-2, 7, -1, 8, 1, -8, -2, 8]$.

e.g.: $[0, 0, 0, 0, 0, 1, 0, 0]$.

Data stored in 4 qubits: a list of 16 numbers, not all zero. e.g.:
$[3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3]$.

Data stored in 64 qubits:

a list of 2^{64} numbers, not all zero.

Data stored in 1000 qubits: a list of 2^{1000} numbers, not all zero.

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state") stored in 3 bits:

3 elements of $\{0, 1\}$.

$0, 0)$.

$1, 1)$.

$1, 1)$.

ored in 64 bits:

64 elements of $\{0, 1\}$.

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$, 0, 0, 1, 1, 0, 0, 0,$

$, 1, 0, 0, 0, 0, 0, 1,$

$, 0, 0, 1, 0, 0, 0, 1,$

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NOT gates

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$100 = 4$ with probability 0;
$101 = 5$ with probability 1;
$110 = 6$ with probability 0;
$111 = 7$ with probability 0.

5 is guaranteed outcome.

NOT gates

NOT_0 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto [1, 3, 1, 4, 9, 5, 6, 2]$.

NOT_0 gate on 4 qubits:
$[3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3] \mapsto [1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9]$.

e.g.: Say 3 qubits have state
$[0, 0, 0, 0, 0, 1, 0, 0]$.

Measurement produces
$000 = 0$ with probability 0;
$001 = 1$ with probability 0;
$010 = 2$ with probability 0;
$011 = 3$ with probability 0;
$100 = 4$ with probability 0;
$101 = 5$ with probability 1;
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$[1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9]$.

NOT_1 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[4, 1, 3, 1, 2, 6, 5, 9]$.

e.g.: Say 3 qubits have state $[0, 0, 0, 0, 0, 1, 0, 0]$.

Measurement produces
$000 = 0$ with probability 0;
$001 = 1$ with probability 0;
$010 = 2$ with probability 0;
$011 = 3$ with probability 0;
$100 = 4$ with probability 0;
$101 = 5$ with probability 1;
$110 = 6$ with probability 0;
$111 = 7$ with probability 0.

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NOT gates

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NOT_0 gate on 4 qubits:
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NOT_1 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto [4, 1, 3, 1, 2, 6, 5, 9]$.

NOT_2 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto [5, 9, 2, 6, 3, 1, 4, 1]$.

y 3 qubits have state
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ement produces
with probability 0;
with probability 0;
with probability 0;
with probability 0;
with probability 0;
with probability 1;
with probability 0;
with probability 0.

ranteed outcome.

NOT gates

NOT_0 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[1, 3, 1, 4, 9, 5, 6, 2]$.

NOT_0 gate on 4 qubits:
$[3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3] \mapsto$
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NOT_1 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
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NOT_2 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[5, 9, 2, 6, 3, 1, 4, 1]$.

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NOT gates

NOT_0 gate on 3 qubits:

$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$

$[1, 3, 1, 4, 9, 5, 6, 2]$.

NOT_0 gate on 4 qubits:

$[3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3] \mapsto$

$[1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9]$.

NOT_1 gate on 3 qubits:

$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$

$[4, 1, 3, 1, 2, 6, 5, 9]$.

NOT_2 gate on 3 qubits:

$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$

$[5, 9, 2, 6, 3, 1, 4, 1]$.

state

[1, 0, 0, 0, 0, 0, 0, 0

[0, 1, 0, 0, 0, 0, 0, 0

[0, 0, 1, 0, 0, 0, 0, 0

[0, 0, 0, 1, 0, 0, 0, 0

[0, 0, 0, 0, 1, 0, 0, 0

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NOT gates

NOT_0 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[1, 3, 1, 4, 9, 5, 6, 2]$.

NOT_0 gate on 4 qubits:
$[3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3] \mapsto$
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NOT_1 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
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NOT_2 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[5, 9, 2, 6, 3, 1, 4, 1]$.

state	measure
$[1, 0, 0, 0, 0, 0, 0, 0]$	000
$[0, 1, 0, 0, 0, 0, 0, 0]$	001
$[0, 0, 1, 0, 0, 0, 0, 0]$	010
$[0, 0, 0, 1, 0, 0, 0, 0]$	011
$[0, 0, 0, 0, 1, 0, 0, 0]$	100
$[0, 0, 0, 0, 0, 1, 0, 0]$	101
$[0, 0, 0, 0, 0, 0, 1, 0]$	110
$[0, 0, 0, 0, 0, 0, 0, 1]$	111

Operation on quantum state
NOT_0, swapping pairs.
Operation after measuremen
flipping bit 0 of result.
Flip: output is not input.

NOT gates

NOT_0 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[1, 3, 1, 4, 9, 5, 6, 2]$.

NOT_0 gate on 4 qubits:
$[3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3] \mapsto$
$[1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9]$.

NOT_1 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
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NOT_2 gate on 3 qubits:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[5, 9, 2, 6, 3, 1, 4, 1]$.

state	measurement
$[1, 0, 0, 0, 0, 0, 0, 0]$	000
$[0, 1, 0, 0, 0, 0, 0, 0]$	001
$[0, 0, 1, 0, 0, 0, 0, 0]$	010
$[0, 0, 0, 1, 0, 0, 0, 0]$	011
$[0, 0, 0, 0, 1, 0, 0, 0]$	100
$[0, 0, 0, 0, 0, 1, 0, 0]$	101
$[0, 0, 0, 0, 0, 0, 1, 0]$	110
$[0, 0, 0, 0, 0, 0, 0, 1]$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

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ate on 3 qubits:

., $5, 9, 2, 6] \mapsto$

4, 9, 5, 6, 2].

ate on 4 qubits:

5,9,2,6,5,3,5,8,9,7,9,3] $\mapsto$

9,5,6,2,3,5,8,5,7,9,3,9].

ate on 3 qubits:

., $5, 9, 2, 6] \mapsto$

., 2, 6, 5, 9].

ate on 3 qubits:

., $5, 9, 2, 6] \mapsto$

5, 3, 1, 4, 1].

state	measurement
$[1, 0, 0, 0, 0, 0, 0, 0]$	000
$[0, 1, 0, 0, 0, 0, 0, 0]$	001
$[0, 0, 1, 0, 0, 0, 0, 0]$	010
$[0, 0, 0, 1, 0, 0, 0, 0]$	011
$[0, 0, 0, 0, 1, 0, 0, 0]$	100
$[0, 0, 0, 0, 0, 1, 0, 0]$	101
$[0, 0, 0, 0, 0, 0, 1, 0]$	110
$[0, 0, 0, 0, 0, 0, 0, 1]$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controll

e.g. C_1N

[3, 1, 4, 1

[3, 1, 1, 4

ubits:

] $\mapsto$

].

ubits:

3,5,8,9,7,9,3] $\mapsto$

5,8,5,7,9,3,9].

ubits:

] $\mapsto$

].

ubits:

] $\mapsto$

].

state	measurement
$[1, 0, 0, 0, 0, 0, 0, 0]$	000
$[0, 1, 0, 0, 0, 0, 0, 0]$	001
$[0, 0, 1, 0, 0, 0, 0, 0]$	010
$[0, 0, 0, 1, 0, 0, 0, 0]$	011
$[0, 0, 0, 0, 1, 0, 0, 0]$	100
$[0, 0, 0, 0, 0, 1, 0, 0]$	101
$[0, 0, 0, 0, 0, 0, 1, 0]$	110
$[0, 0, 0, 0, 0, 0, 0, 1]$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (C

e.g. C_1NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6]$
$[3, 1, 1, 4, 5, 9, 6, 2]$

$,3] \mapsto$
$,9].$

state	measurement
$[1, 0, 0, 0, 0, 0, 0, 0]$	000
$[0, 1, 0, 0, 0, 0, 0, 0]$	001
$[0, 0, 1, 0, 0, 0, 0, 0]$	010
$[0, 0, 0, 1, 0, 0, 0, 0]$	011
$[0, 0, 0, 0, 1, 0, 0, 0]$	100
$[0, 0, 0, 0, 0, 1, 0, 0]$	101
$[0, 0, 0, 0, 0, 0, 1, 0]$	110
$[0, 0, 0, 0, 0, 0, 0, 1]$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (CNOT) ga

e.g. C_1NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 1, 4, 5, 9, 6, 2].$

state	measurement
$[1, 0, 0, 0, 0, 0, 0, 0]$	000
$[0, 1, 0, 0, 0, 0, 0, 0]$	001
$[0, 0, 1, 0, 0, 0, 0, 0]$	010
$[0, 0, 0, 1, 0, 0, 0, 0]$	011
$[0, 0, 0, 0, 1, 0, 0, 0]$	100
$[0, 0, 0, 0, 0, 1, 0, 0]$	101
$[0, 0, 0, 0, 0, 0, 1, 0]$	110
$[0, 0, 0, 0, 0, 0, 0, 1]$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 1, 4, 5, 9, 6, 2]$.

state	measurement
$[1, 0, 0, 0, 0, 0, 0, 0]$	000
$[0, 1, 0, 0, 0, 0, 0, 0]$	001
$[0, 0, 1, 0, 0, 0, 0, 0]$	010
$[0, 0, 0, 1, 0, 0, 0, 0]$	011
$[0, 0, 0, 0, 1, 0, 0, 0]$	100
$[0, 0, 0, 0, 0, 1, 0, 0]$	101
$[0, 0, 0, 0, 0, 0, 1, 0]$	110
$[0, 0, 0, 0, 0, 0, 0, 1]$	111

Operation on quantum state:
NOT_0, swapping pairs.

Operation after measurement:
flipping bit 0 of result.

Flip: output is not input.

Controlled-NOT (CNOT) gates

e.g. $\mathrm{C}_1\mathrm{NOT}_0$:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 1, 4, 5, 9, 6, 2]$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

state	measurement
$[1, 0, 0, 0, 0, 0, 0, 0]$	000
$[0, 1, 0, 0, 0, 0, 0, 0]$	001
$[0, 0, 1, 0, 0, 0, 0, 0]$	010
$[0, 0, 0, 1, 0, 0, 0, 0]$	011
$[0, 0, 0, 0, 1, 0, 0, 0]$	100
$[0, 0, 0, 0, 0, 1, 0, 0]$	101
$[0, 0, 0, 0, 0, 0, 1, 0]$	110
$[0, 0, 0, 0, 0, 0, 0, 1]$	111

Operation on quantum state:
NOT_0, swapping pairs.

Operation after measurement:
flipping bit 0 of result.

Flip: output is not input.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 1, 4, 5, 9, 6, 2]$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 1, 9, 5, 6, 2]$.

state	measurement
$[1,0,0,0,0,0,0,0]$	000
$[0,1,0,0,0,0,0,0]$	001
$[0,0,1,0,0,0,0,0]$	010
$[0,0,0,1,0,0,0,0]$	011
$[0,0,0,0,1,0,0,0]$	100
$[0,0,0,0,0,1,0,0]$	101
$[0,0,0,0,0,0,1,0]$	110
$[0,0,0,0,0,0,0,1]$	111

Operation on quantum state:
NOT_0, swapping pairs.

Operation after measurement:
flipping bit 0 of result.

Flip: output is not input.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$[3,1,4,1,5,9,2,6] \mapsto$
$[3,1,1,4,5,9,6,2]$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$[3,1,4,1,5,9,2,6] \mapsto$
$[3,1,4,1,9,5,6,2]$.

e.g. C_0NOT_2:
$[3,1,4,1,5,9,2,6] \mapsto$
$[3,9,4,6,5,1,2,1]$.

state	measurement
0, 0, 0, 0, 0]	000
0, 0, 0, 0, 0]	001
0, 0, 0, 0, 0]	010
1, 0, 0, 0, 0]	011
0, 1, 0, 0, 0]	100
0, 0, 1, 0, 0]	101
0, 0, 0, 1, 0]	110
0, 0, 0, 0, 1]	111

on on quantum state:
swapping pairs.

on after measurement:
bit 0 of result.

tput is not input.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 1, 4, 5, 9, 6, 2]$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 1, 9, 5, 6, 2]$.

e.g. C_0NOT_2:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 9, 4, 6, 5, 1, 2, 1]$.

Toffoli g

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e.g. C_2C
$[3, 1, 4, 1$
$[3, 1, 4, 1$

measurement

0] 000

0] 001

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0] 011

0] 100

0] 101

0] 110

1] 111

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Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 1, 4, 5, 9, 6, 2]$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 1, 9, 5, 6, 2]$.

e.g. C_0NOT_2:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 9, 4, 6, 5, 1, 2, 1]$.

Toffoli gates

Also known as CC
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e.g. $C_2C_1NOT_0$:
$[3, 1, 4, 1, 5, 9, 2, 6]$
$[3, 1, 4, 1, 5, 9, 6, 2]$

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Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 1, 4, 5, 9, 6, 2]$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 1, 9, 5, 6, 2]$.

e.g. C_0NOT_2:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 9, 4, 6, 5, 1, 2, 1]$.

Toffoli gates

Also known as CCNOT gate
controlled-controlled-NOT g

e.g. $C_2C_1NOT_0$:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 1, 5, 9, 6, 2]$.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$[3,1,4,1,5,9,2,6] \mapsto$
$[3,1,1,4,5,9,6,2]$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$[3,1,4,1,5,9,2,6] \mapsto$
$[3,1,4,1,9,5,6,2]$.

e.g. C_0NOT_2:
$[3,1,4,1,5,9,2,6] \mapsto$
$[3,9,4,6,5,1,2,1]$.

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:
$[3,1,4,1,5,9,2,6] \mapsto$
$[3,1,4,1,5,9,6,2]$.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 1, 4, 5, 9, 6, 2]$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 1, 9, 5, 6, 2]$.

e.g. C_0NOT_2:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 9, 4, 6, 5, 1, 2, 1]$.

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 1, 5, 9, 6, 2]$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

Controlled-NOT (CNOT) gates

e.g. $\mathrm{C}_1\mathrm{NOT}_0$:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 1, 4, 5, 9, 6, 2]$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\mathrm{C}_2\mathrm{NOT}_0$:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 1, 9, 5, 6, 2]$.

e.g. $\mathrm{C}_0\mathrm{NOT}_2$:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 9, 4, 6, 5, 1, 2, 1]$.

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $\mathrm{C}_2\mathrm{C}_1\mathrm{NOT}_0$:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 1, 5, 9, 6, 2]$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $\mathrm{C}_0\mathrm{C}_1\mathrm{NOT}_2$:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 6, 5, 9, 2, 1]$.

ed-NOT (CNOT) gates

IOT$_0$:

$., 5, 9, 2, 6] \mapsto$

$4, 5, 9, 6, 2]$.

on after measurement:

bit 0 *if* bit 1 is set; i.e.,

$q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

IOT$_0$:

$., 5, 9, 2, 6] \mapsto$

$., 9, 5, 6, 2]$.

IOT$_2$:

$., 5, 9, 2, 6] \mapsto$

$5, 5, 1, 2, 1]$.

Toffoli gates

Also known as CCNOT gates: controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:

$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$

$[3, 1, 4, 1, 5, 9, 6, 2]$.

Operation after measurement:

$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $C_0C_1NOT_2$:

$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$

$[3, 1, 4, 6, 5, 9, 2, 1]$.

More sh

Combine

to build

CNOT) gates

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easurement:

t 1 is set; i.e.,

$, q_1, q_0 \oplus q_1)$.

] $\mapsto$

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] $\mapsto$

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Toffoli gates

Also known as CCNOT gates: controlled-controlled-NOT gates.

e.g. $\mathrm{C}_2\mathrm{C}_1\mathrm{NOT}_0$:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 1, 5, 9, 6, 2]$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $\mathrm{C}_0\mathrm{C}_1\mathrm{NOT}_2$:
$[3, 1, 4, 1, 5, 9, 2, 6] \mapsto$
$[3, 1, 4, 6, 5, 9, 2, 1]$.

More shuffling

Combine NOT, CN
to build other per

ates

nt:
i.e.,
q_1).

Toffoli gates

Also known as CCNOT gates: controlled-controlled-NOT gates.

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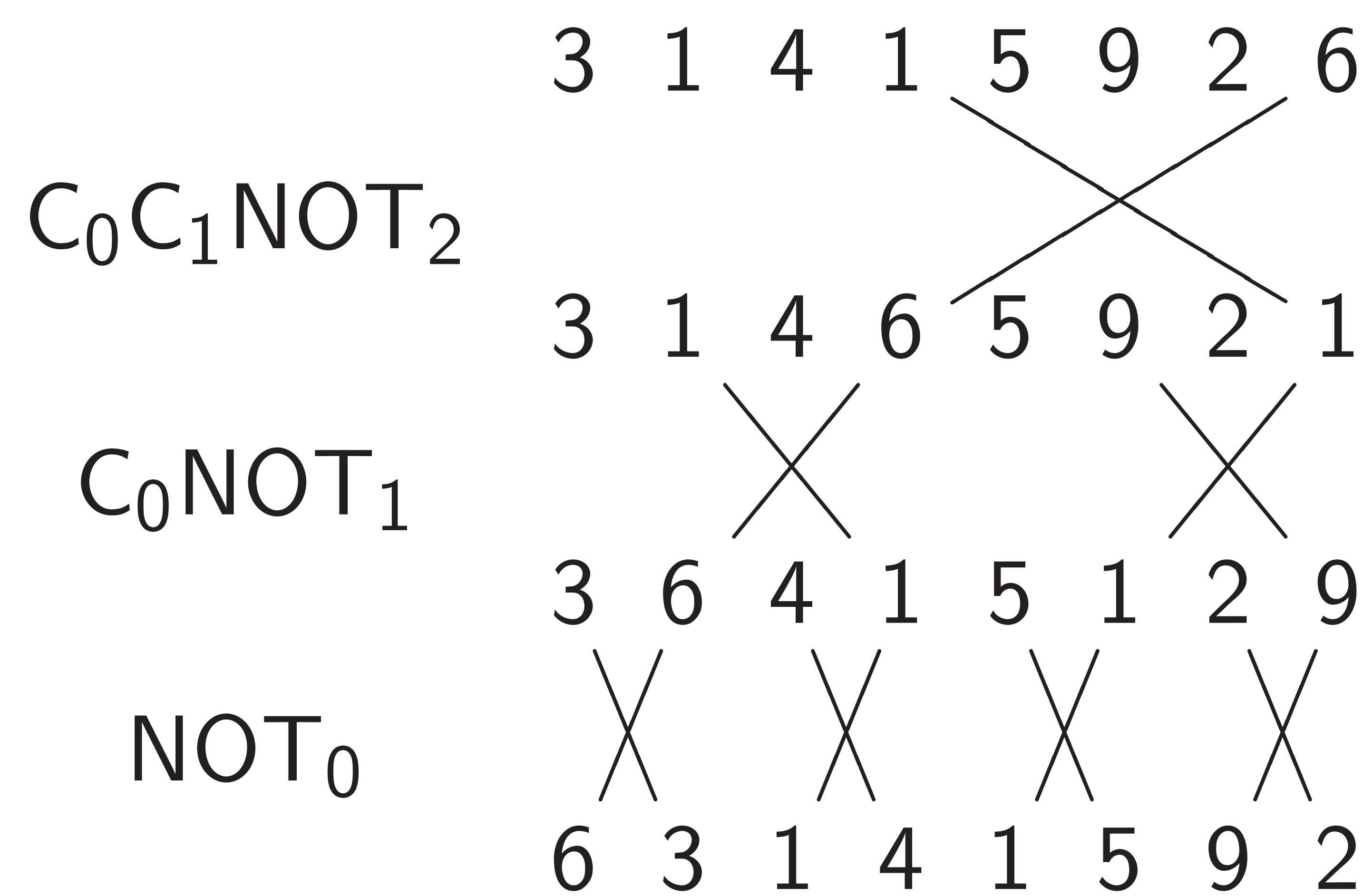
e.g. series of gates to rotate 8 positions by distance 1:



ates

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C_1NOT_0:
$., 5, 9, 2, 6] \mapsto$
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C_1NOT_2:
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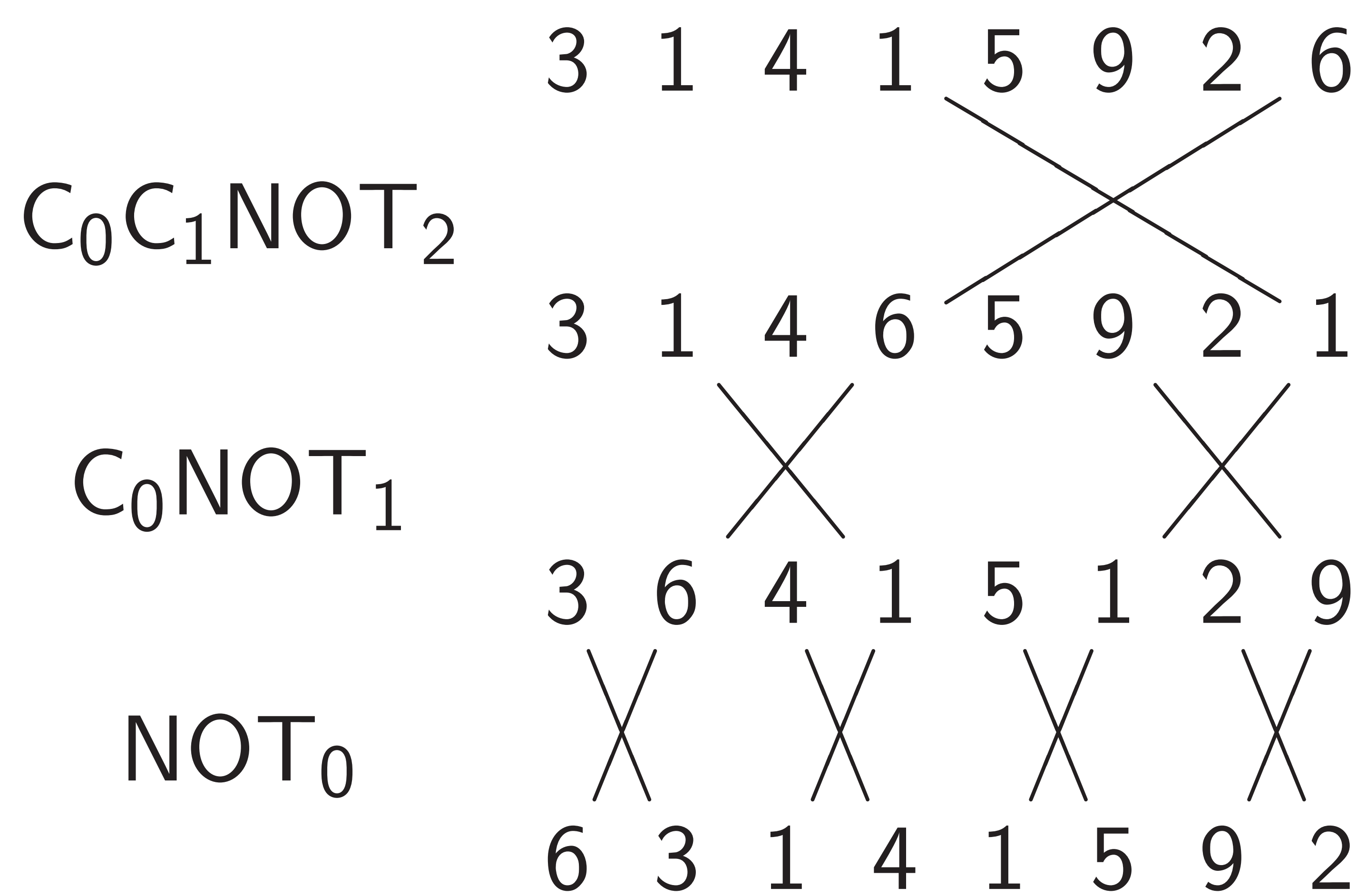
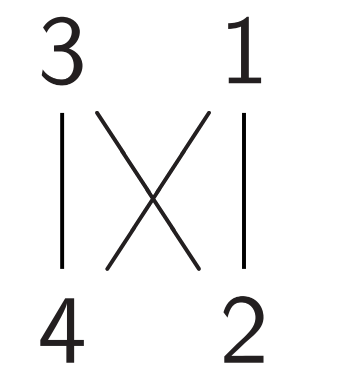


Hadama

Hadama

$[a, b] \mapsto$



NOT gates:
ed-NOT gates.

| $\mapsto$
|.

easurement:
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| $\mapsto$
|.

More shuffling

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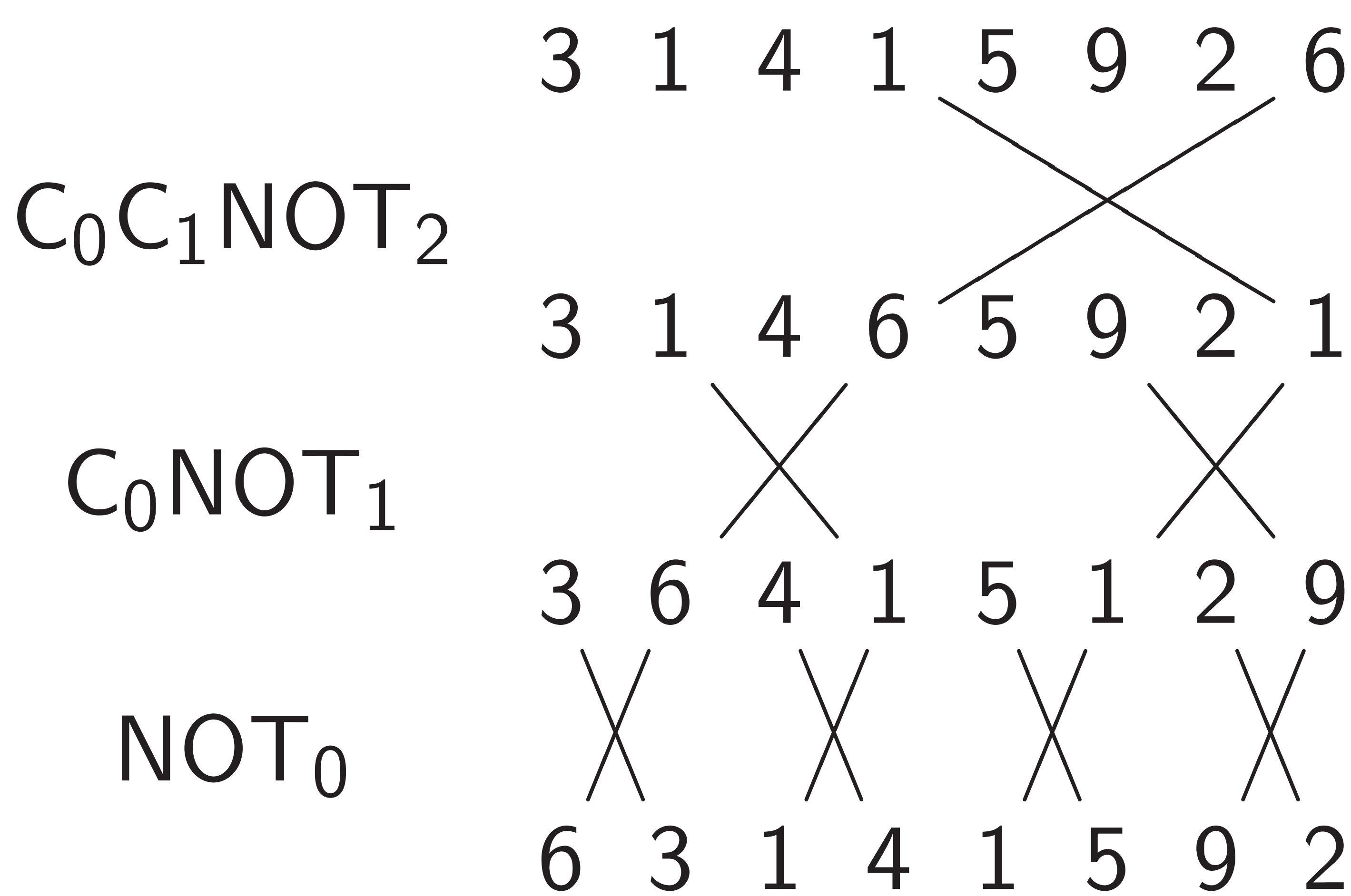
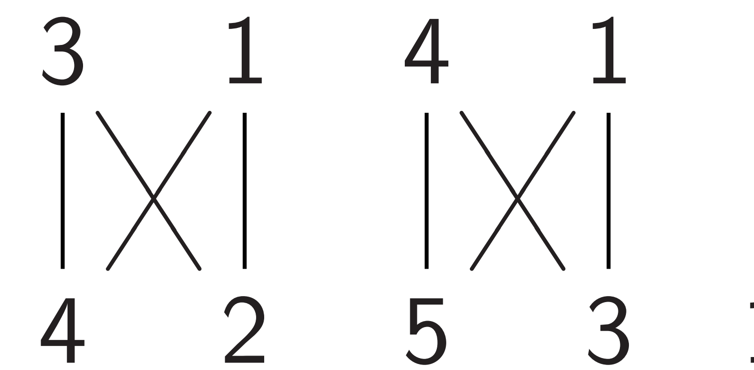


Hadamard gates

Hadamard_0:

$[a, b] \mapsto [a + b, a$ -



es:

gates.

nt:

$q_1 q_2$).

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

e.g. series of gates to rotate 8 positions by distance 1:

3 1 4 1 5 9 2 6

$C_0C_1NOT_2$

3 1 4 6 5 9 2 1

C_0NOT_1

3 6 4 1 5 1 2 9

NOT_0

6 3 1 4 1 5 9 2

Hadamard gates

Hadamard_0:

$[a, b] \mapsto [a + b, a - b]$.

3 1   4 1   5 9   2

4 2   5 3   14 −4   8

More shuffling

Combine NOT, CNOT, Toffoli
to build other permutations.

e.g. series of gates to
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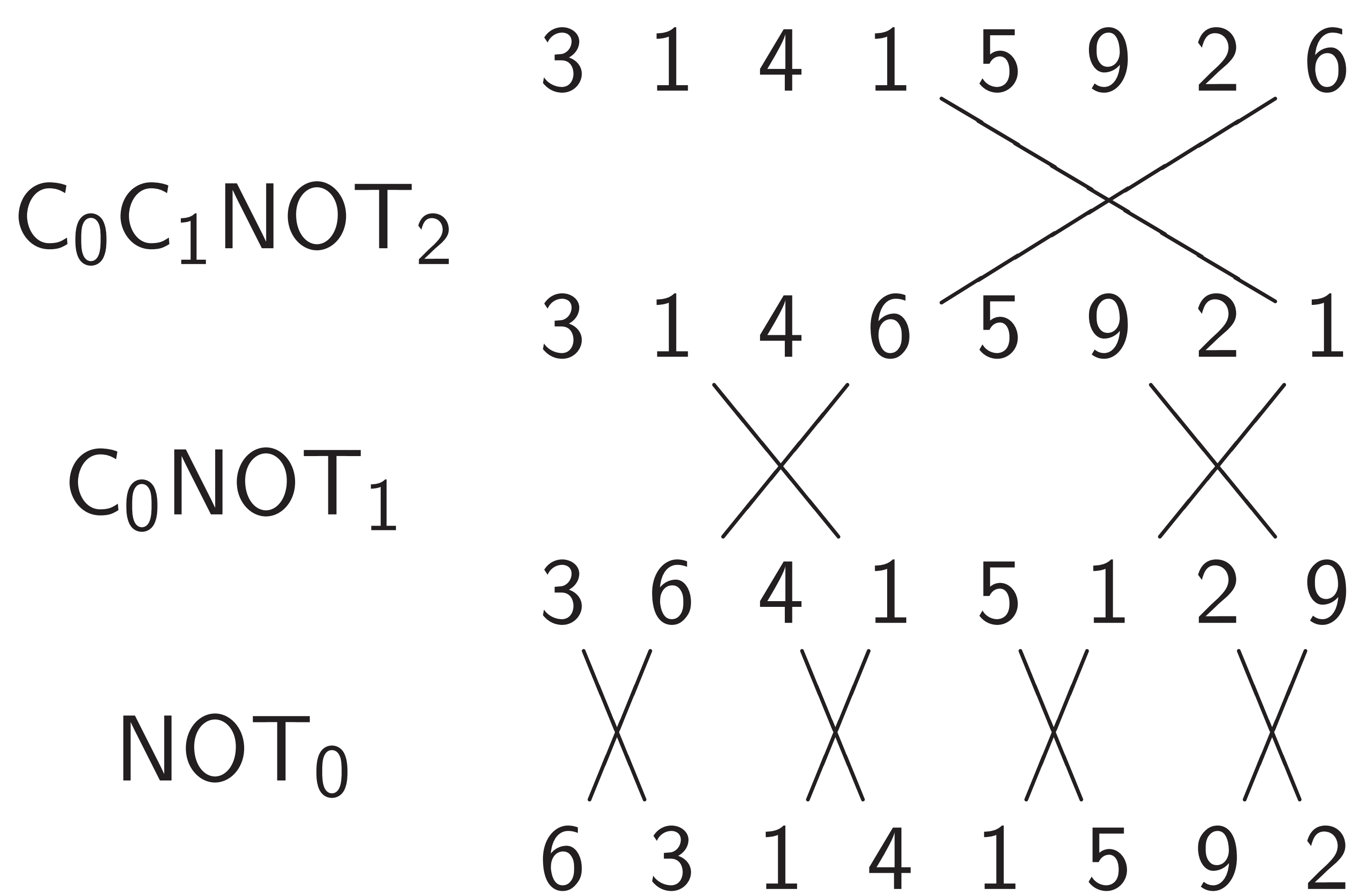

Hadamard gates

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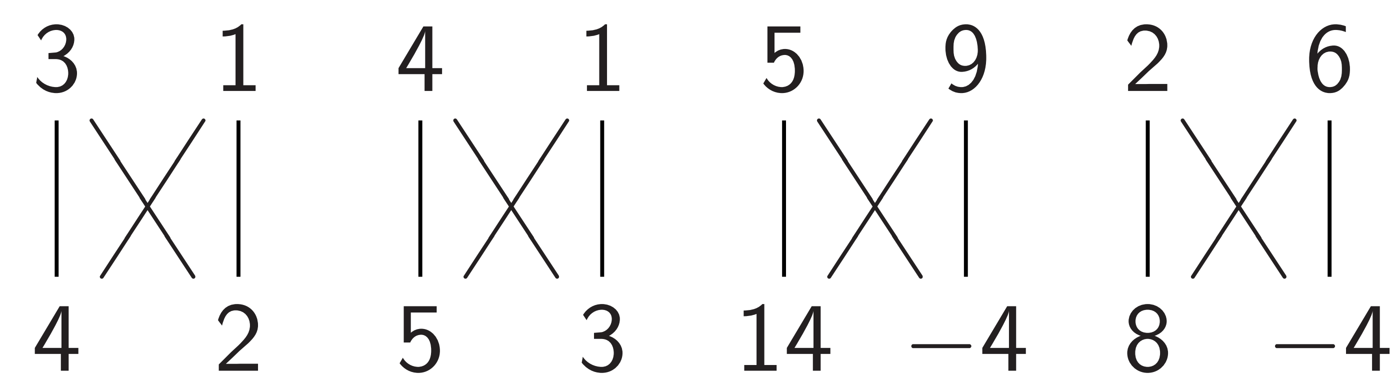

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

e.g. series of gates to rotate 8 positions by distance 1:

3 1 4 1 5 9 2 6

$C_0C_1NOT_2$

3 1 4 6 5 9 2 1

C_0NOT_1

3 6 4 1 5 1 2 9

NOT_0

6 3 1 4 1 5 9 2

Hadamard gates

Hadamard_0:

$[a, b] \mapsto [a + b, a - b]$.

3 1 4 1 5 9 2 6

4 2 5 3 14 −4 8 −4

Hadamard_1:

$[a, b, c, d] \mapsto$
$[a + c, b + d, a - c, b - d]$.

3 1 4 1 5 9 2 6

7 2 −1 0 7 15 3 3

uffling

e NOT, CNOT, Toffoli
other permutations.

es of gates to
positions by distance 1:

3 1 4 1 5 9 2 6

OT_2

3 1 4 6 5 9 2 1

T_1

3 6 4 1 5 1 2 9

$_0$

6 3 1 4 1 5 9 2

Hadamard gates

$\mathrm{Hadamard}_0$:

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3 1   4 1   5 9   2 6

4 2   5 3   14 −4   8 −4

$\mathrm{Hadamard}_1$:

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3 1 4 1   5 9 2 6

7 2 −1 0   7 15 3 3

Some us

Hadama

3 1

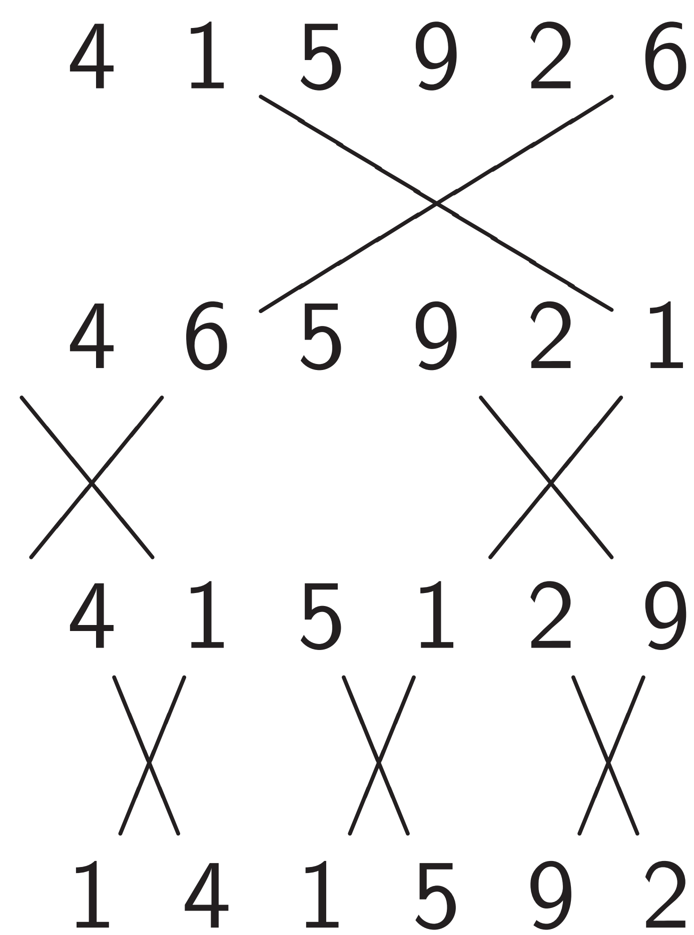
4 2

2 4

6 −2

NOT, Toffoli
mutations.

s to
by distance 1:



Hadamard gates

Hadamard$_0$:

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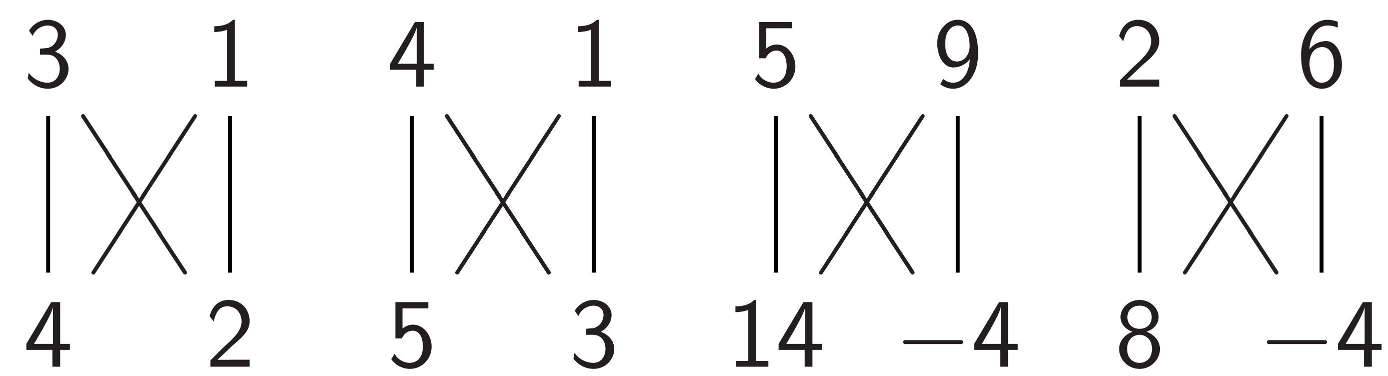

Hadamard$_1$:

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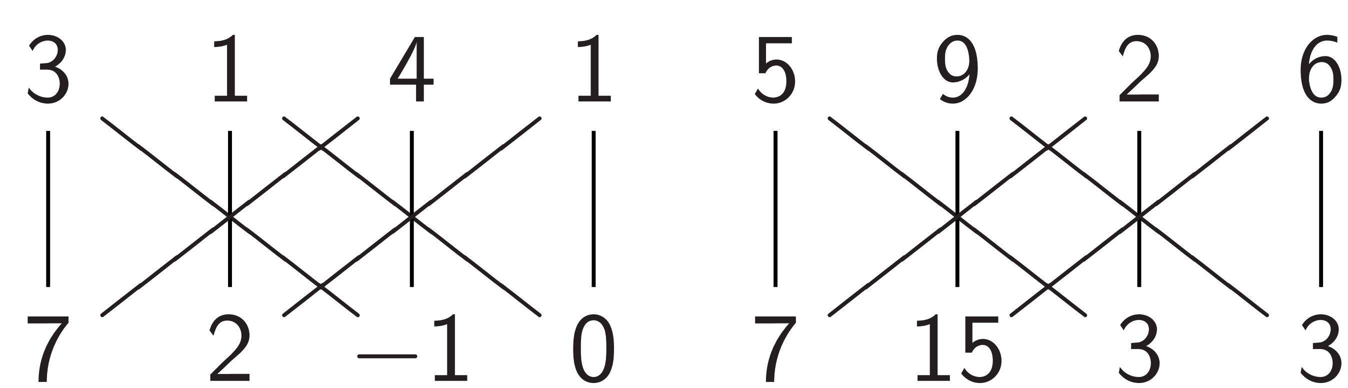
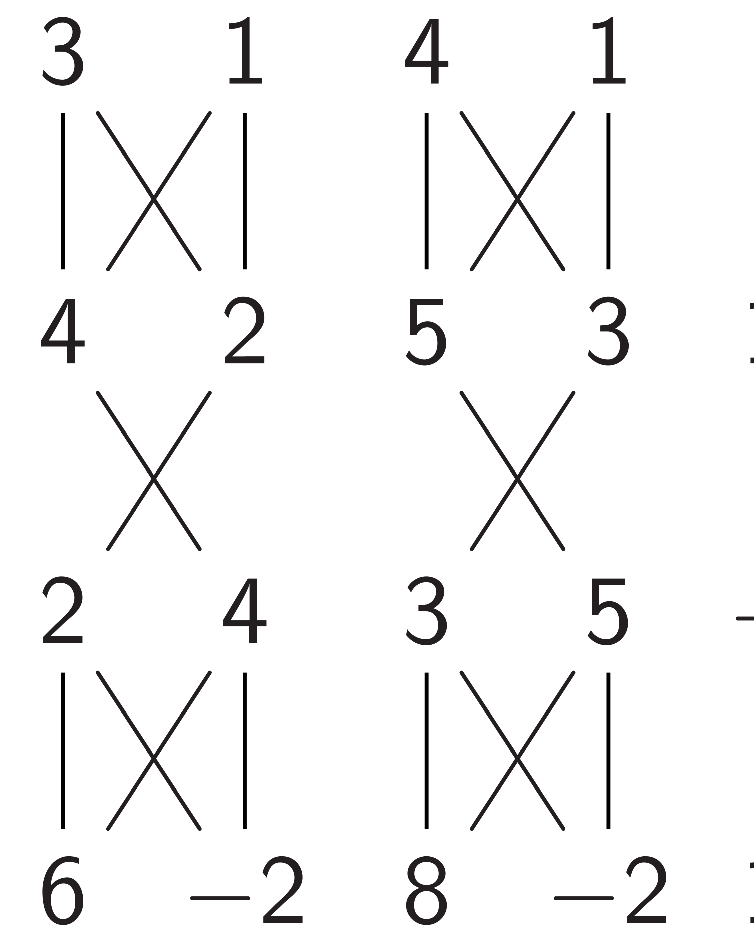


Some uses of Had

Hadamard$_0$, NOT$_($



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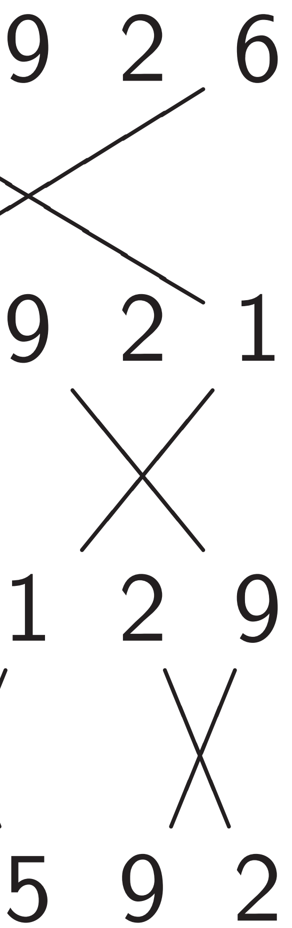
ce 1:



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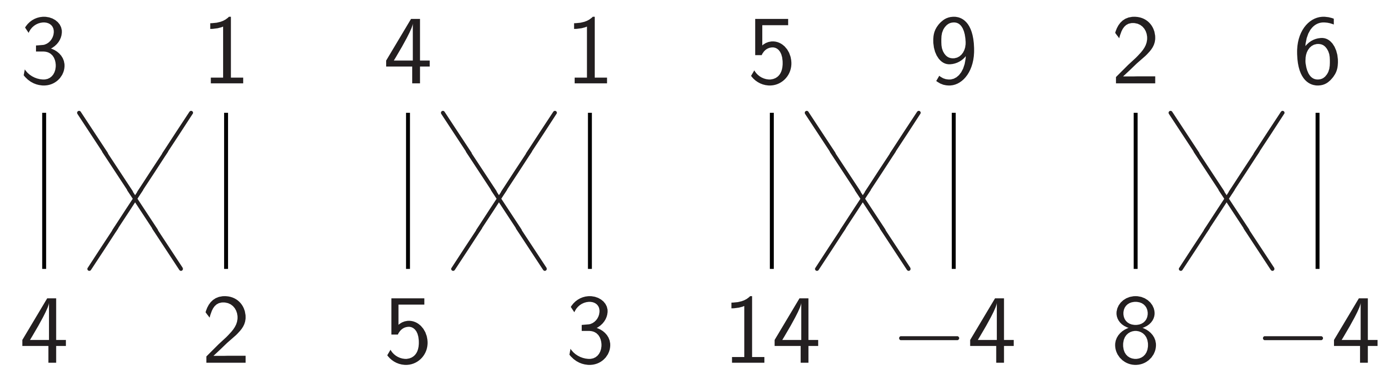

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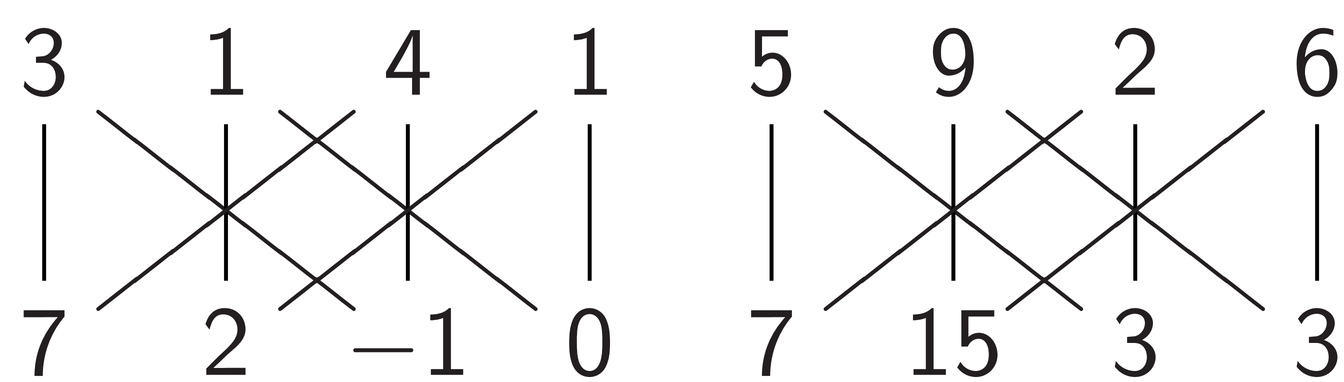
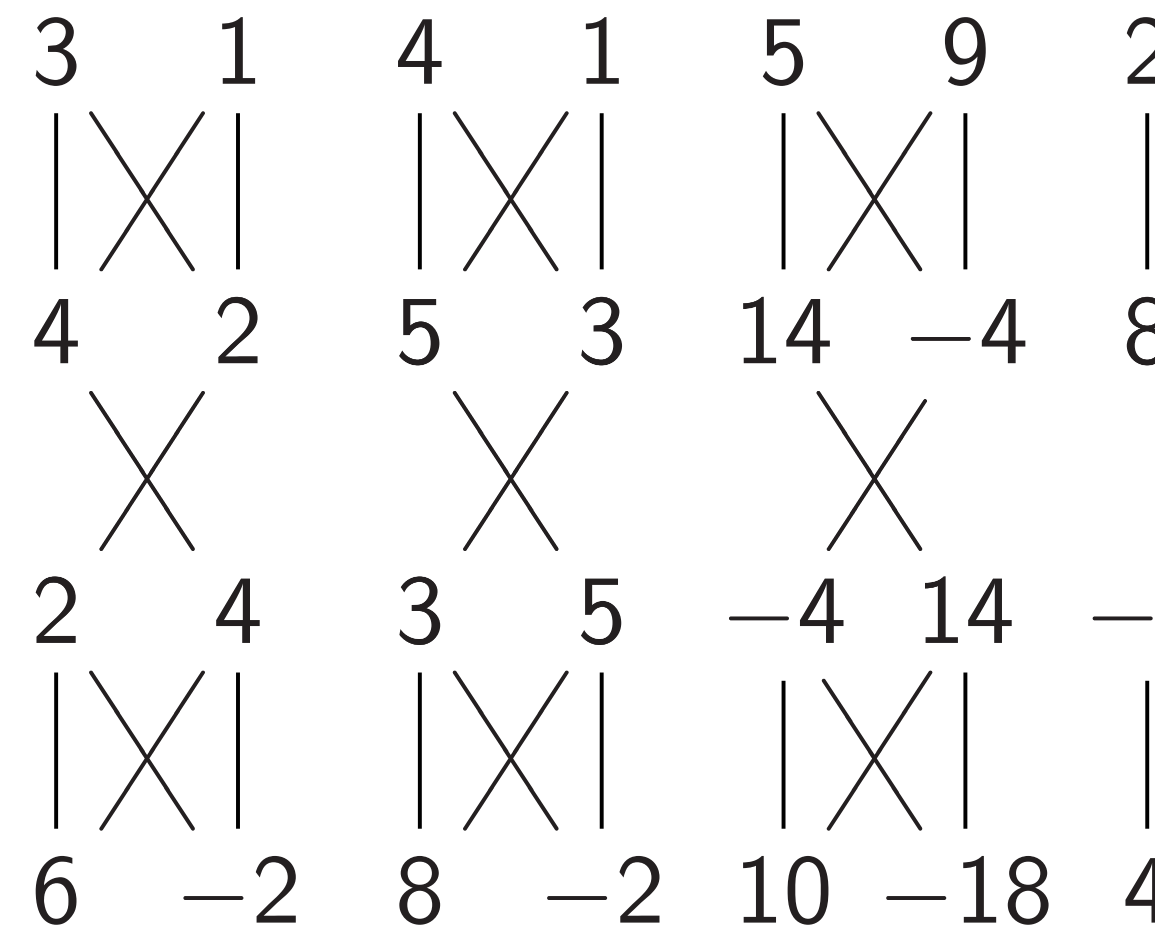


Some uses of Hadamard gat

Hadamard$_0$, NOT$_0$, Hadama



Hadamard gates

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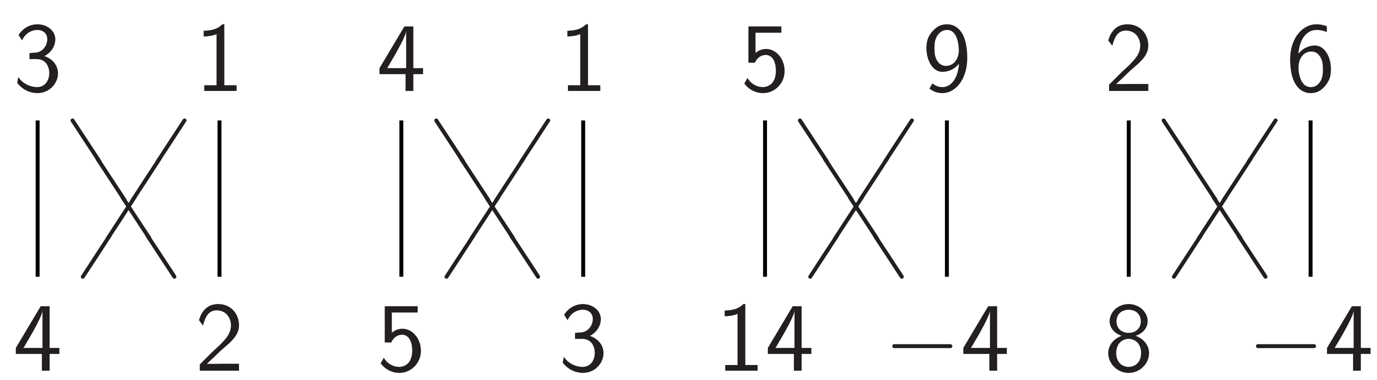

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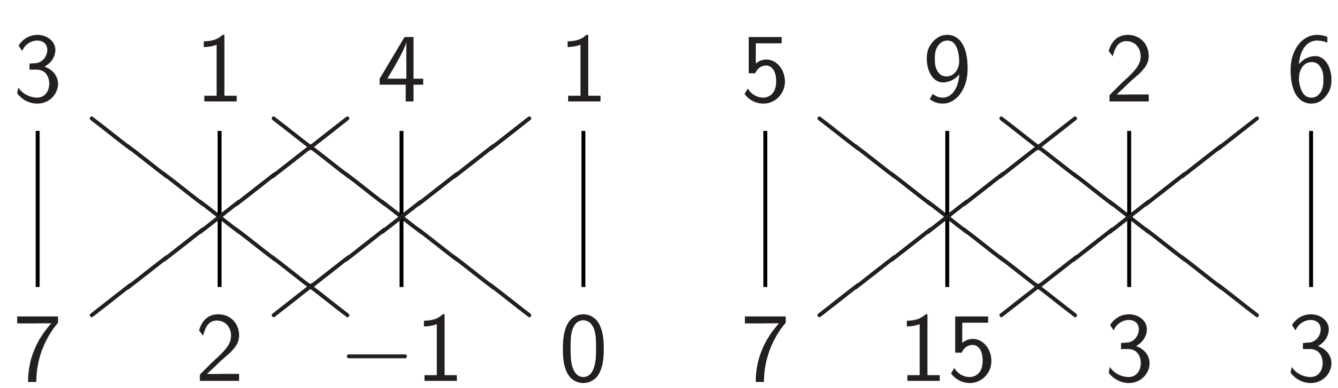

Some uses of Hadamard gates

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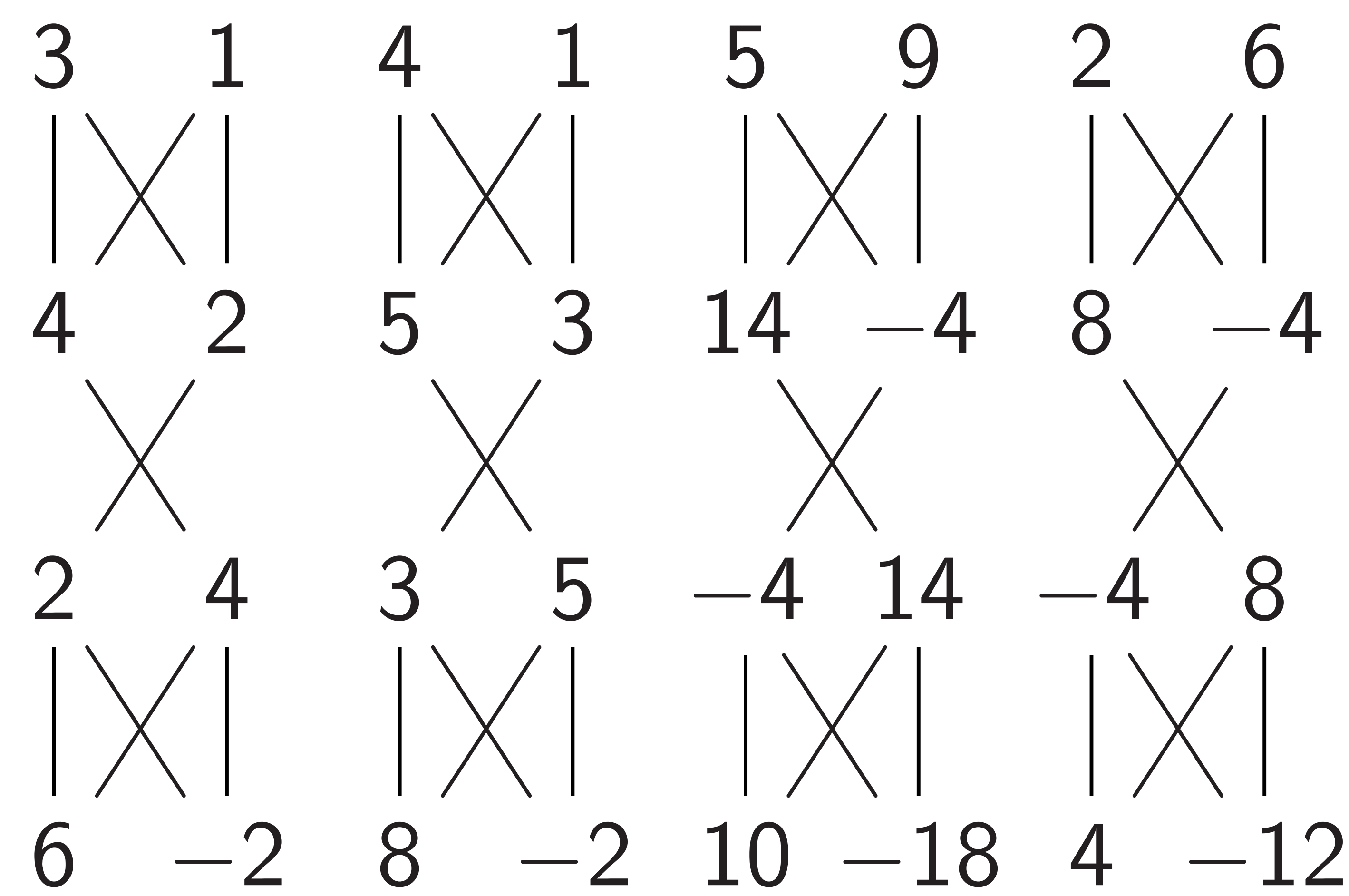

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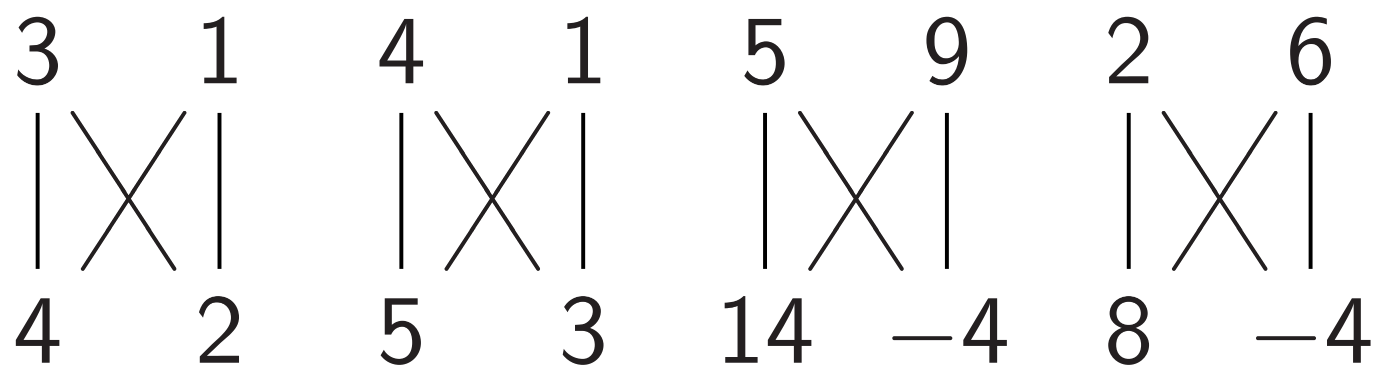

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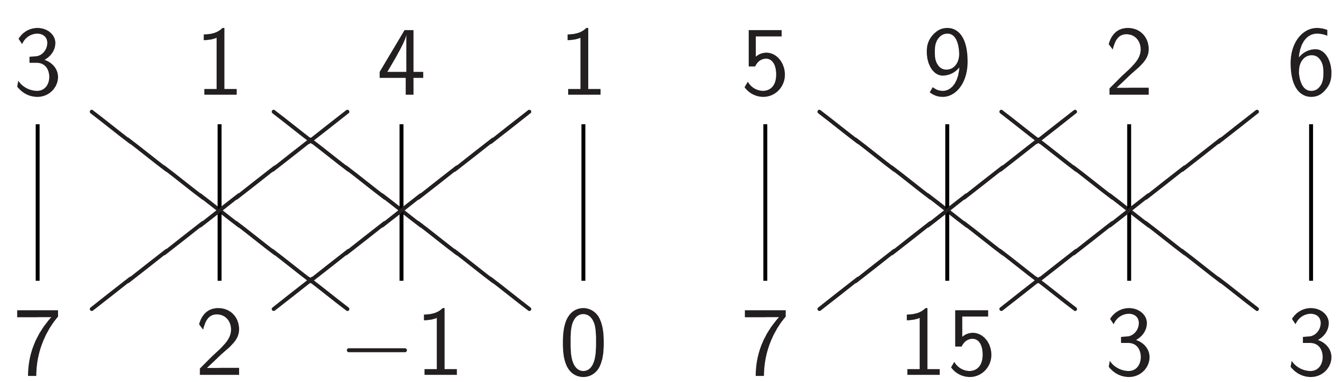

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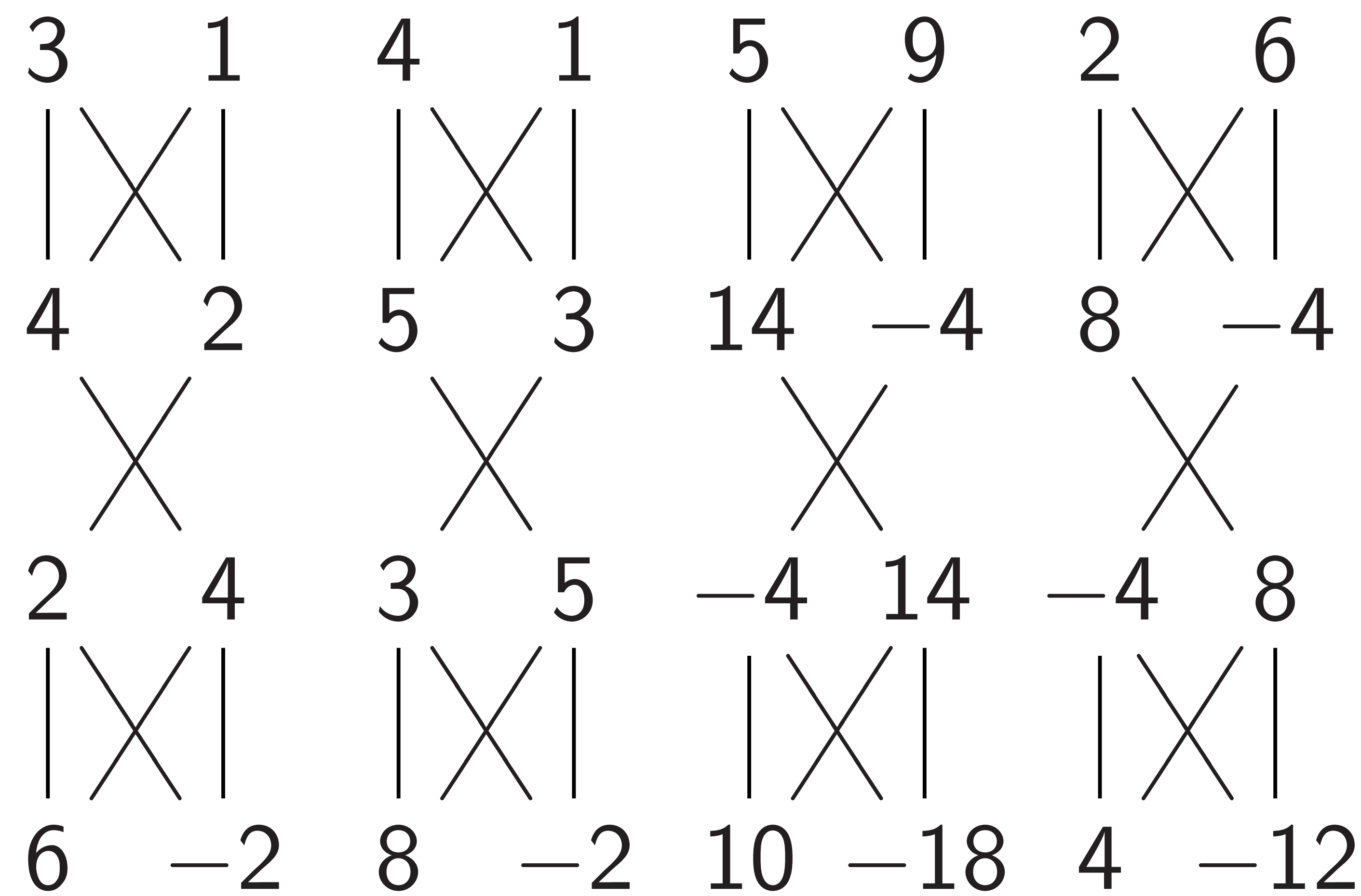

“Multiplied each amplitude by 2.”

This is not physically observable.

Hadamard gates

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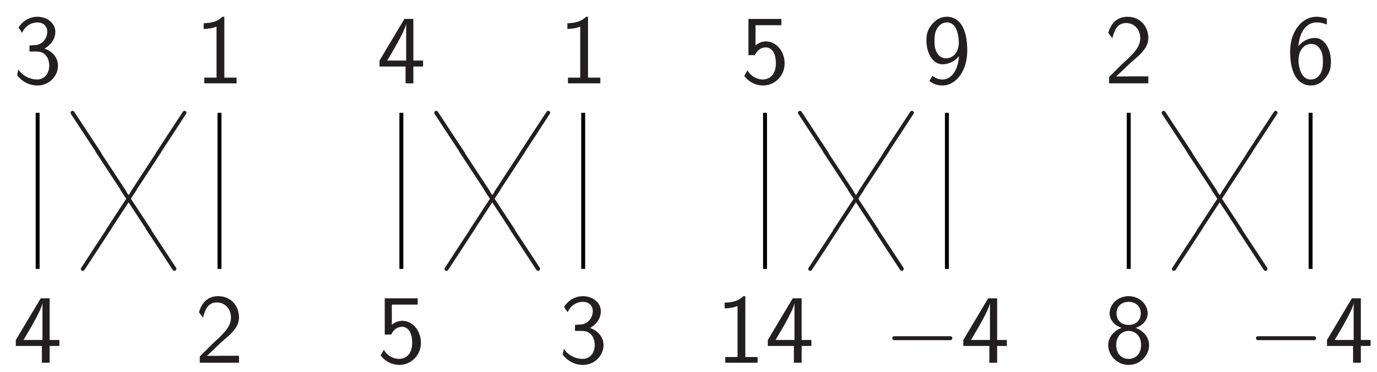

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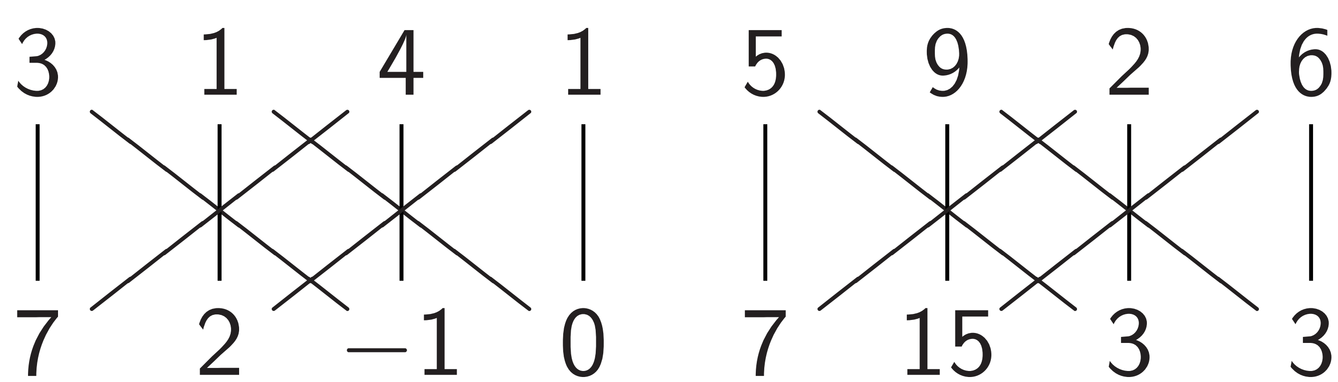

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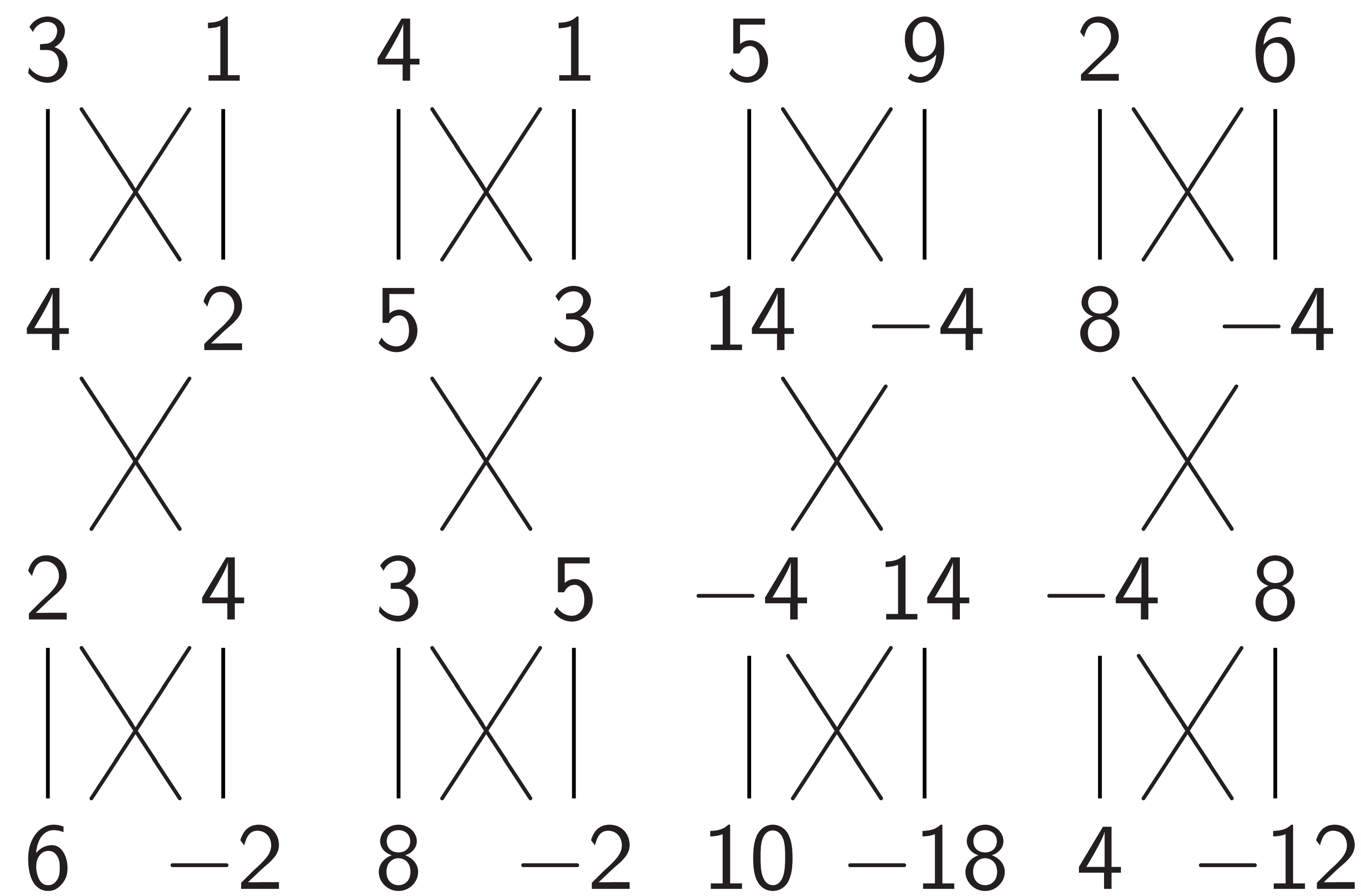

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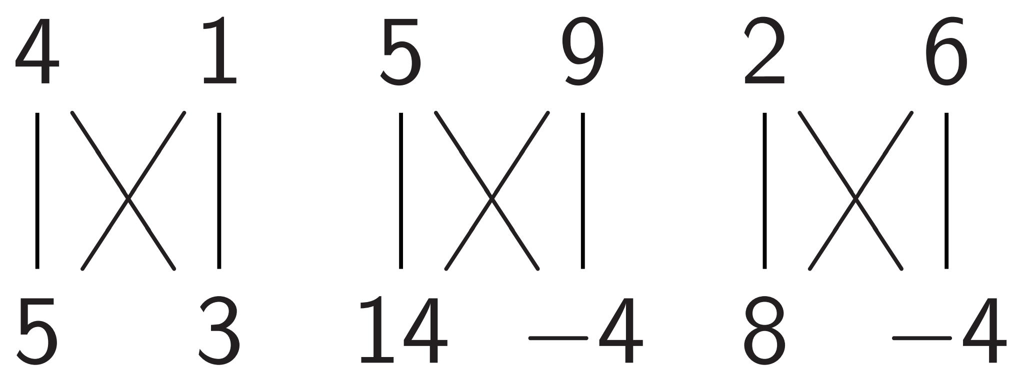
"Negated amplitude if q_0 is set."
No effect on measuring *now*.

rd gates

rd$_0$:

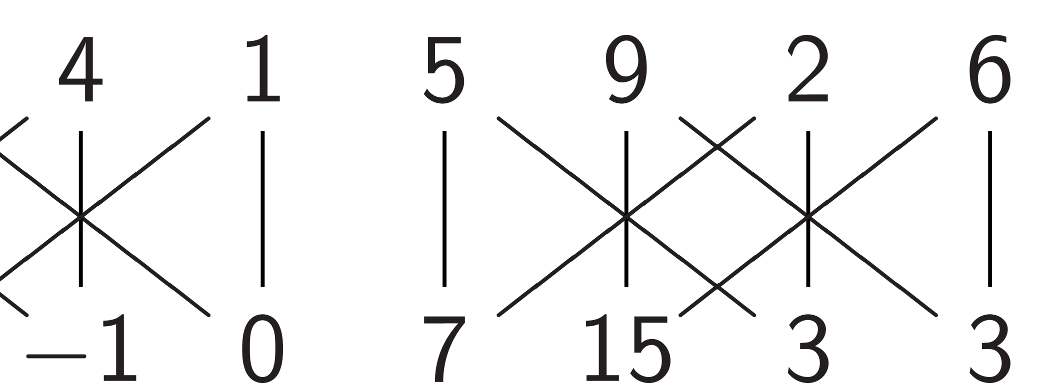
$[a + b, a - b]$.



rd$_1$:

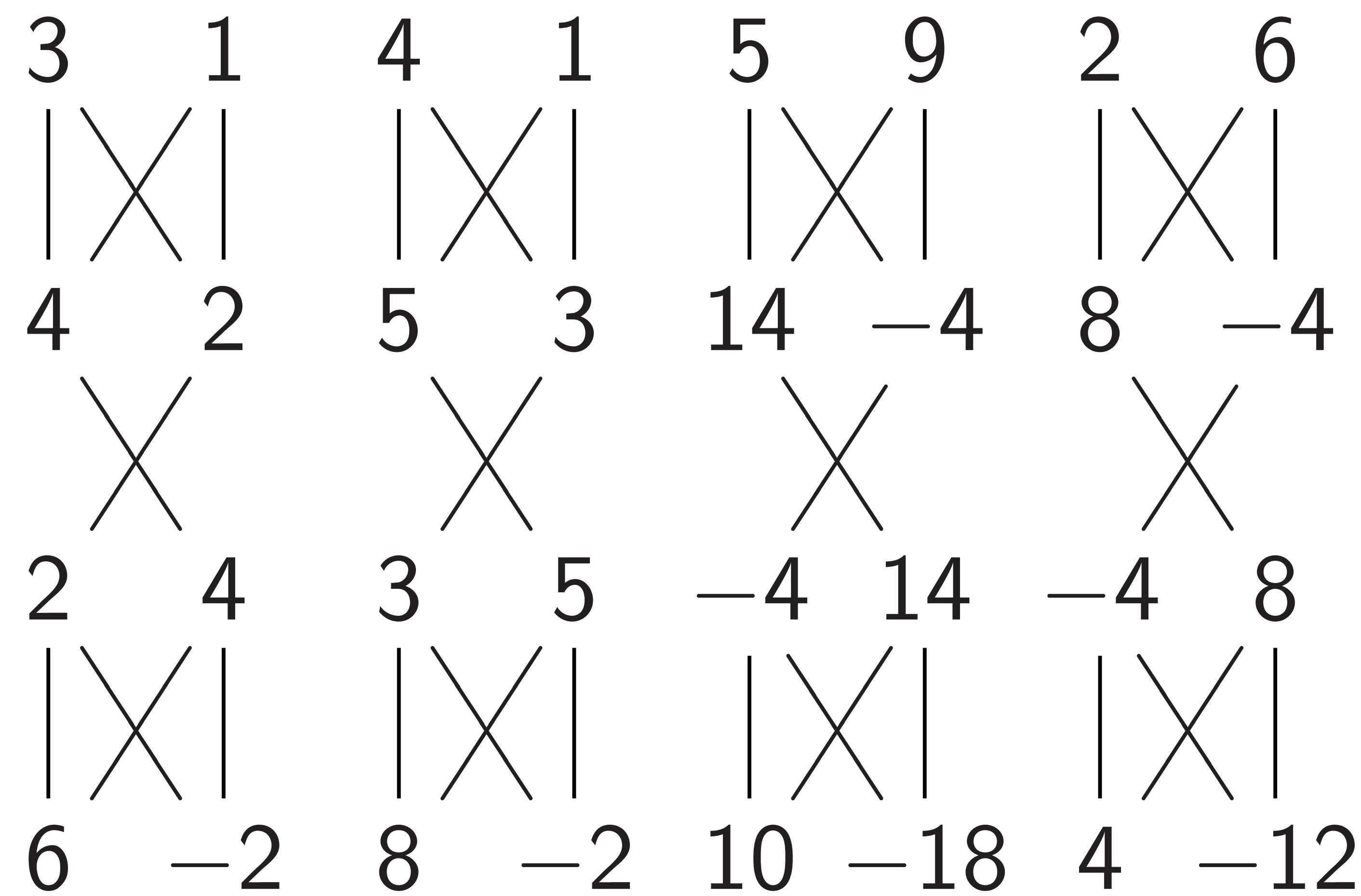
$d] \mapsto$

$+ d, a - c, b - d]$.



Some uses of Hadamard gates

Hadamard$_0$, NOT$_0$, Hadamard$_0$:



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Fancier

“Negate

Assumes

C_0C_1N

Hadama

NOT

Hadama

C_0C_1N

$- b]$.

5 9 2 6

14 −4 8 −4

$c, b - d]$.

5 9 2 6

7 15 3 3

Some uses of Hadamard gates

Hadamard$_0$, NOT$_0$, Hadamard$_0$:

3 1 4 1 5 9 2 6

4 2 5 3 14 −4 8 −4

2 4 3 5 −4 14 −4 8

6 −2 8 −2 10 −18 4 −12

"Multiplied each amplitude by 2."
This is not physically observable.

"Negated amplitude if q_0 is set."
No effect on measuring *now*.

Fancier example:

"Negate amplitud

Assumes $q_2 = 0$:

3 1

$C_0C_1NOT_2$

3 1

Hadamard$_2$

3 1

NOT$_2$

3 1

Hadamard$_2$

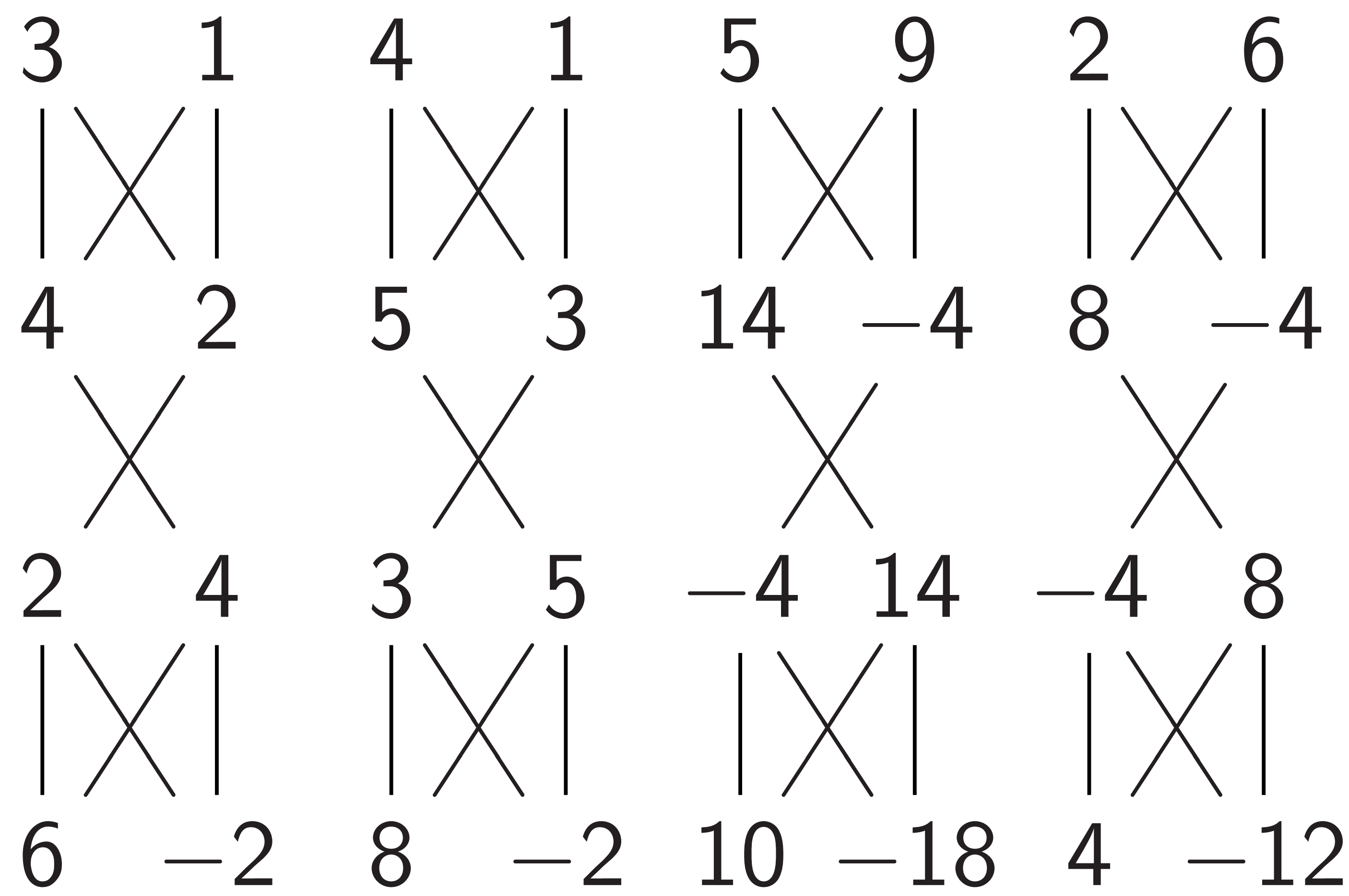
6 2

$C_0C_1NOT_2$

6 2

Some uses of Hadamard gates

Hadamard$_0$, NOT$_0$, Hadamard$_0$:



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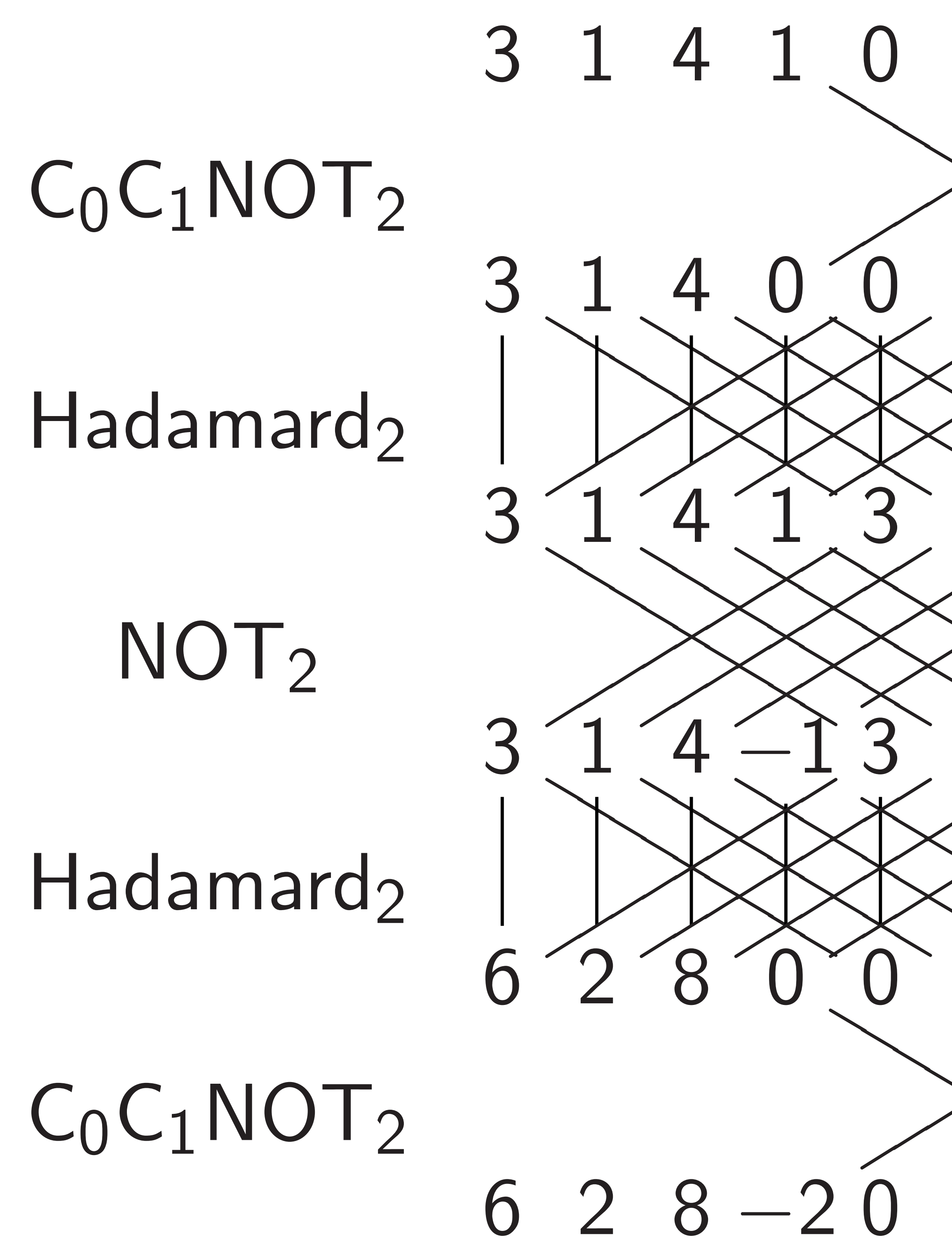
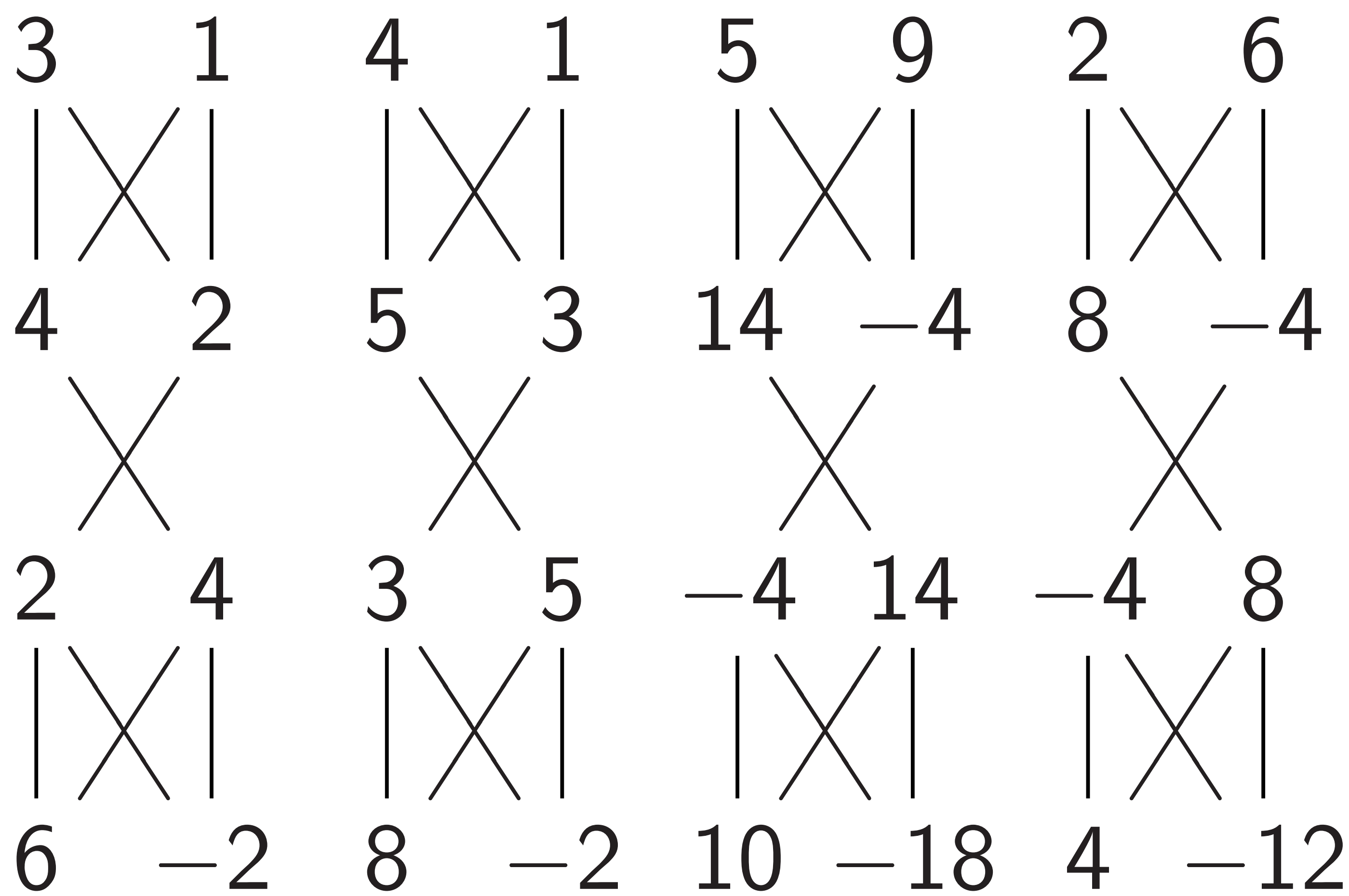
"Negate amplitude if $q_0 q_1$ is

Assumes $q_2 = 0$: "ancilla" q



Some uses of Hadamard gates

Hadamard_0, NOT_0, Hadamard_0:



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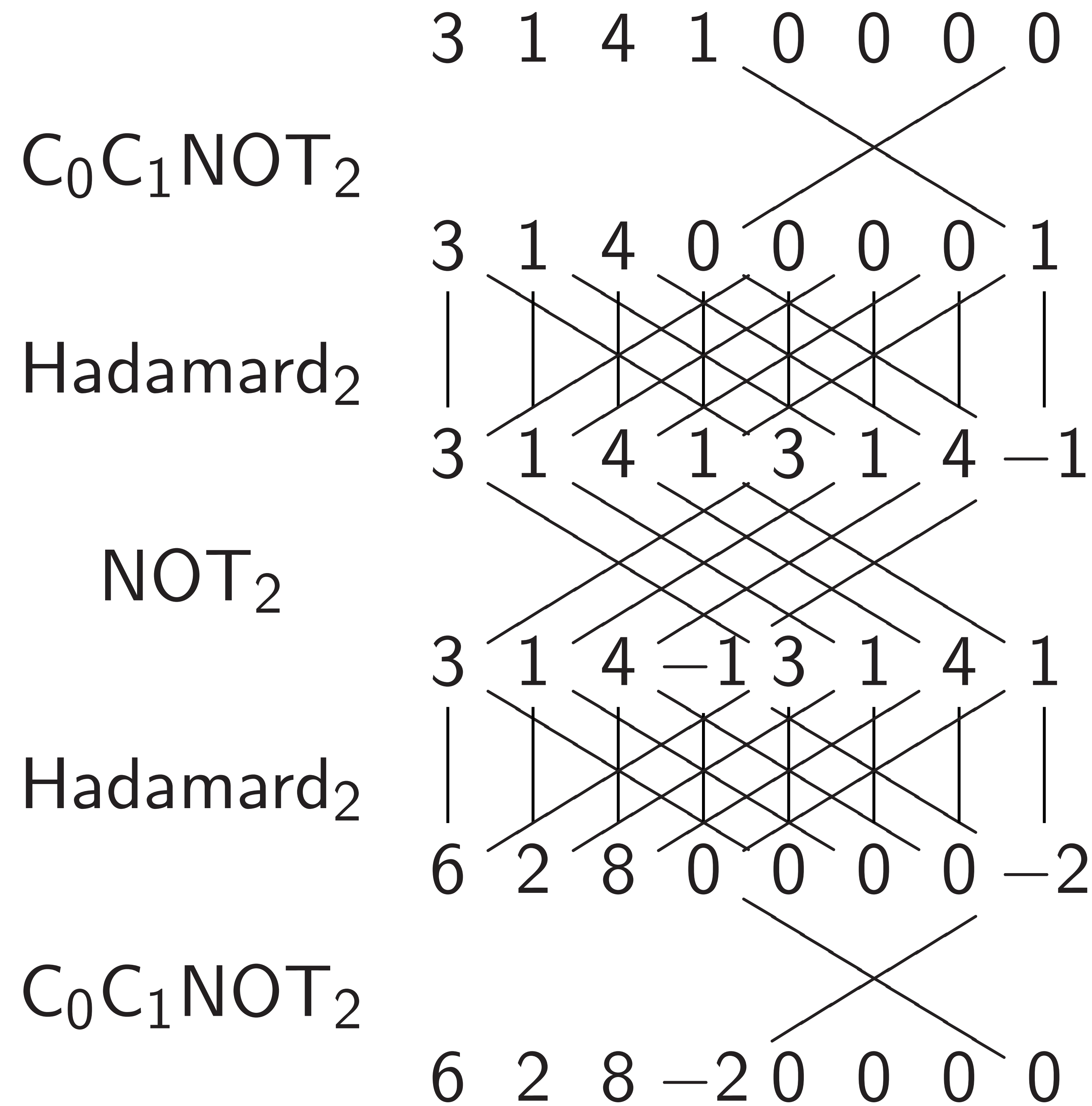

ses of Hadamard gates

rd$_0$, NOT$_0$, Hadamard$_0$:

4	1	5	9	2	6
5	3	14	−4	8	−4
3	5	−4	14	−4	8
8	−2	10	−18	4	−12

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not physically observable.

d amplitude if q_0 is set.”
t on measuring *now*.

Fancier example:
“Negate amplitude if $q_0 q_1$ is set.”
Assumes $q_2 = 0$: “ancilla” qubit.

	3	1	4	1	0	0	0	0
C$_0$C$_1$NOT$_2$								
	3	1	4	0	0	0	0	1
Hadamard$_2$								
	3	1	4	1	3	1	4	−1
NOT$_2$								
	3	1	4	−1	3	1	4	1
Hadamard$_2$								
	6	2	8	0	0	0	0	−2
C$_0$C$_1$NOT$_2$								
	6	2	8	−2	0	0	0	0

Affects r
amplitud
[3, 1, 4, 1

amard gates

$_0$, Hadamard$_0$:

5 9 2 6

14 -4 8 -4

-4 14 -4 8

10 -18 4 -12

mplitude by 2."

ally observable.

de if q_0 is set."

uring *now*.

Fancier example:

"Negate amplitude if $q_0 q_1$ is set."

Assumes $q_2 = 0$: "ancilla" qubit.

3 1 4 1 0 0 0 0

$\mathrm{C}_0\mathrm{C}_1\mathrm{NOT}_2$

3 1 4 0 0 0 0 1

Hadamard$_2$

3 1 4 1 3 1 4 -1

NOT_2

3 1 4 -1 3 1 4 1

Hadamard$_2$

6 2 8 0 0 0 0 -2

$\mathrm{C}_0\mathrm{C}_1\mathrm{NOT}_2$

6 2 8 -2 0 0 0 0

Affects measureme

amplitude around

$[3, 1, 4, 1] \mapsto [1.5,$

es

ard$_0$:

2 6

3 -4

4 8

4 -12

by 2.”

able.

set.”

.

Fancier example:

“Negate amplitude if $q_0 q_1$ is set.”

Assumes $q_2 = 0$: “ancilla” qubit.

$$
\begin{array}{lcccccccc}
 & 3 & 1 & 4 & 1 & 0 & 0 & 0 & 0 \\
\mathrm{C_0C_1NOT_2} & & & & & & & & \\
 & 3 & 1 & 4 & 0 & 0 & 0 & 0 & 1 \\
\mathrm{Hadamard_2} & & & & & & & & \\
 & 3 & 1 & 4 & 1 & 3 & 1 & 4 & -1 \\
\mathrm{NOT_2} & & & & & & & & \\
 & 3 & 1 & 4 & -1 & 3 & 1 & 4 & 1 \\
\mathrm{Hadamard_2} & & & & & & & & \\
 & 6 & 2 & 8 & 0 & 0 & 0 & 0 & -2 \\
\mathrm{C_0C_1NOT_2} & & & & & & & & \\
 & 6 & 2 & 8 & -2 & 0 & 0 & 0 & 0
\end{array}
$$

Affects measurements: “Neg

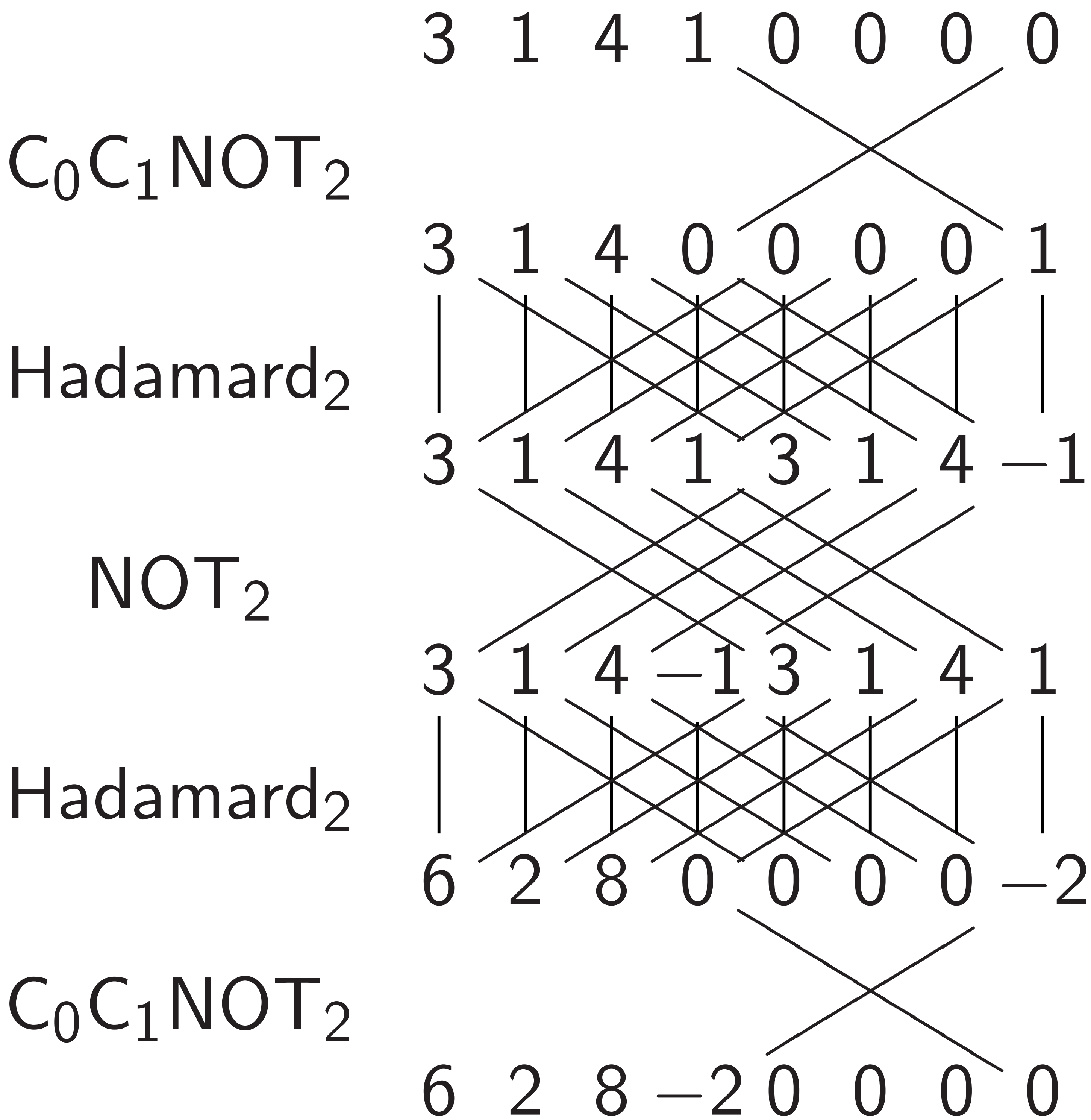
amplitude around its averag

$[3, 1, 4, 1] \mapsto [1.5, 3.5, 0.5, 3.$

Fancier example:

"Negate amplitude if $q_0 q_1$ is set."

Assumes $q_2 = 0$: "ancilla" qubit.



Affects measurements: "Negate amplitude around its average."

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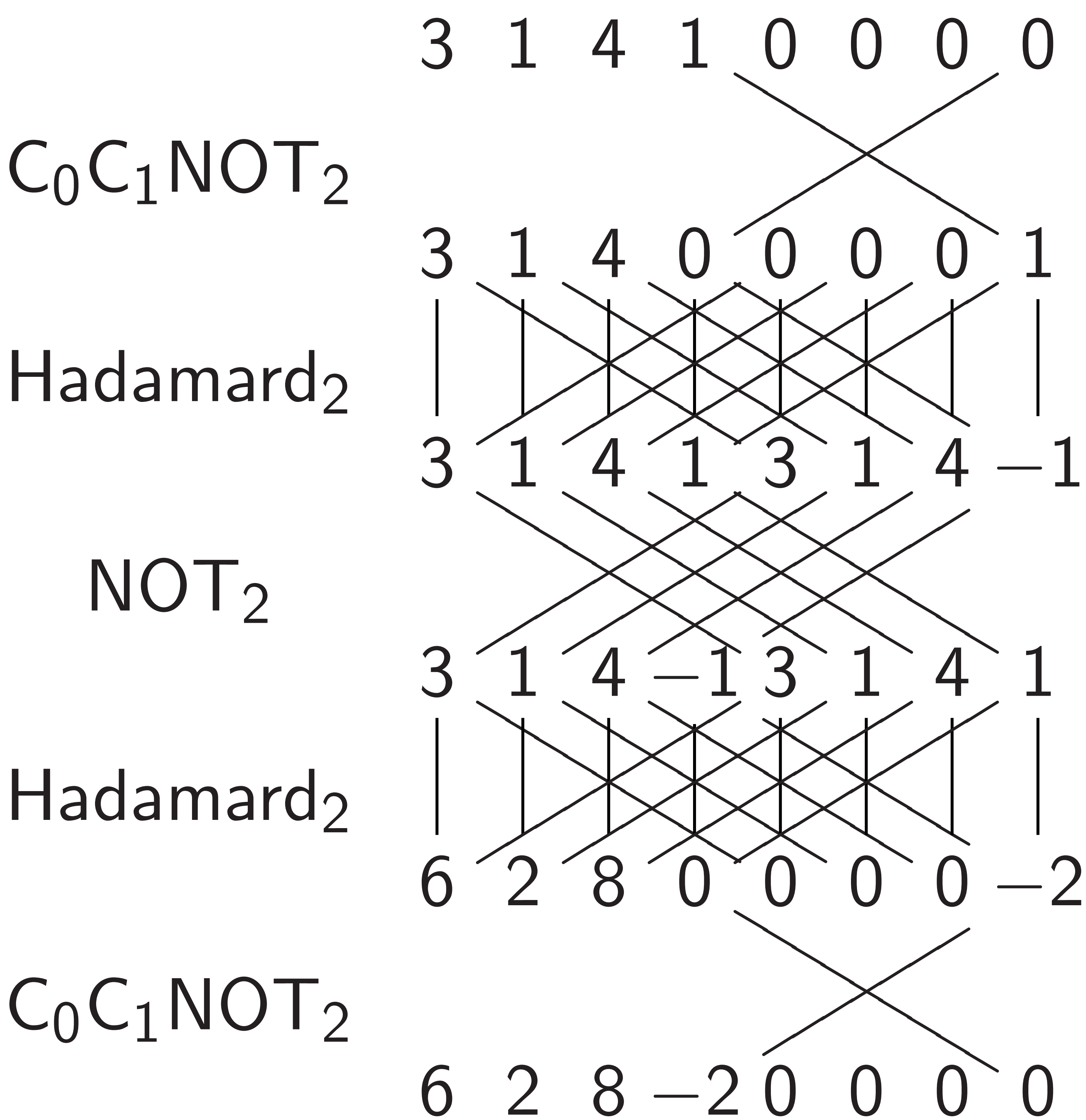

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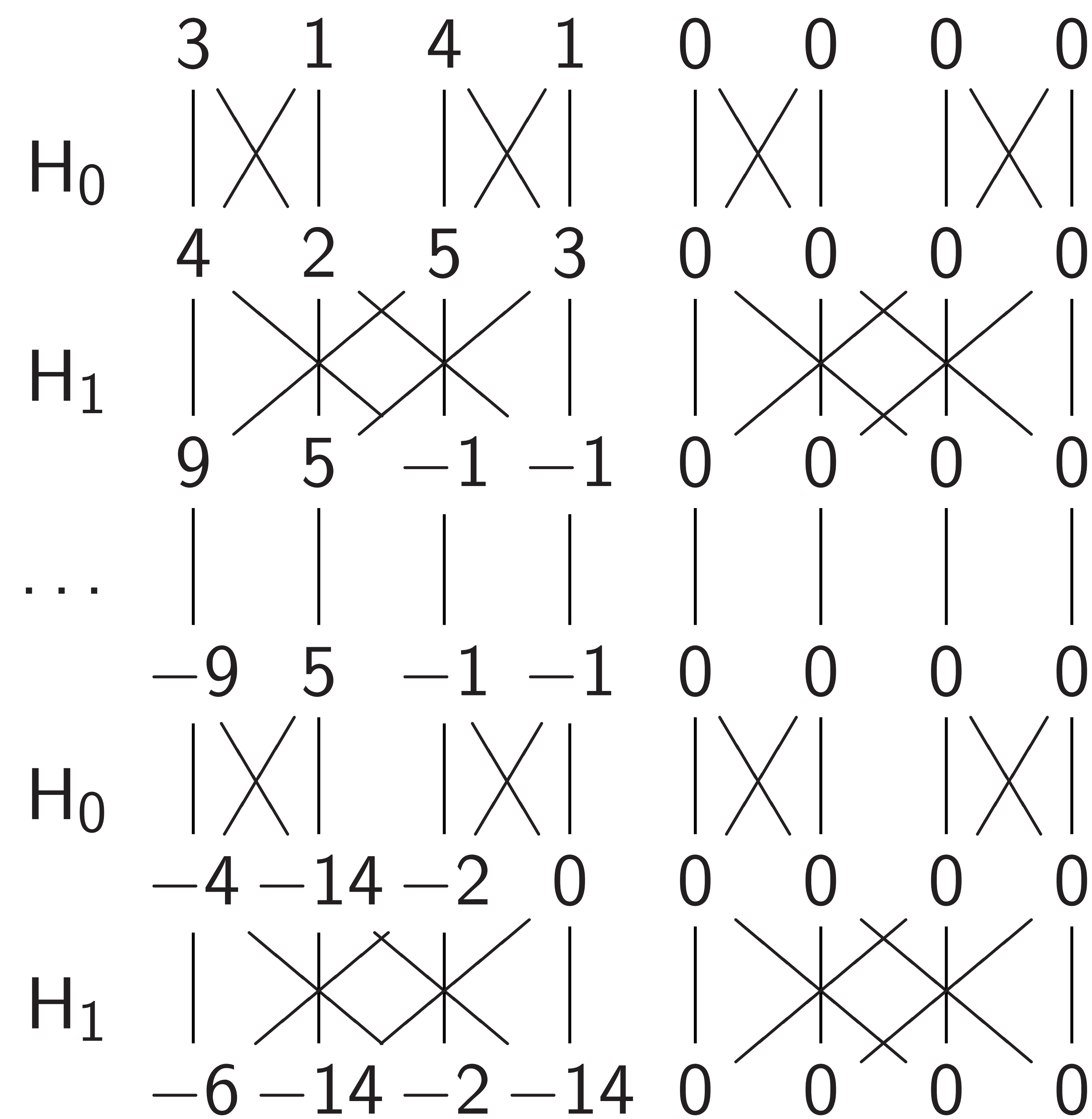


example:

amplitude if $q_0 q_1$ is set."

s $q_2 = 0$: "ancilla" qubit.

3 1 4 1 0 0 0 0

OT$_2$

3 1 4 0 0 0 0 1

ard$_2$

3 1 4 1 3 1 4 −1

$_2$

3 1 4 −1 3 1 4 1

ard$_2$

6 2 8 0 0 0 0 −2

OT$_2$

6 2 8 −2 0 0 0 0

Affects measurements: "Negate amplitude around its average."

$[3, 1, 4, 1] \mapsto [1.5, 3.5, 0.5, 3.5]$.

3 1 4 1 0 0 0 0

H_0

4 2 5 3 0 0 0 0

H_1

9 5 −1 −1 0 0 0 0

. . .

−9 5 −1 −1 0 0 0 0

H_0

−4 −14 −2 0 0 0 0 0

H_1

−6 −14 −2 −14 0 0 0 0

Simon's

Assumpt

- Given
  can ef
- Nonze
- $f(u) =$
- f has

Goal: Fi

e if $q_0 q_1$ is set.”
“ancilla” qubit.

4	1	0	0	0	0
4	0	0	0	0	1
4	1	3	1	4	-1
4	-1	3	1	4	1
8	0	0	0	0	-2
8	-2	0	0	0	0

Affects measurements: “Negate amplitude around its average.”
$[3, 1, 4, 1] \mapsto [1.5, 3.5, 0.5, 3.5]$.

	3	1	4	1	0	0	0	0
H_0								
	4	2	5	3	0	0	0	0
H_1								
	9	5	-1	-1	0	0	0	0
...								
	-9	5	-1	-1	0	0	0	0
H_0								
	-4	-14	-2	0	0	0	0	0
H_1								
	-6	-14	-2	-14	0	0	0	0

Simon’s algorithm

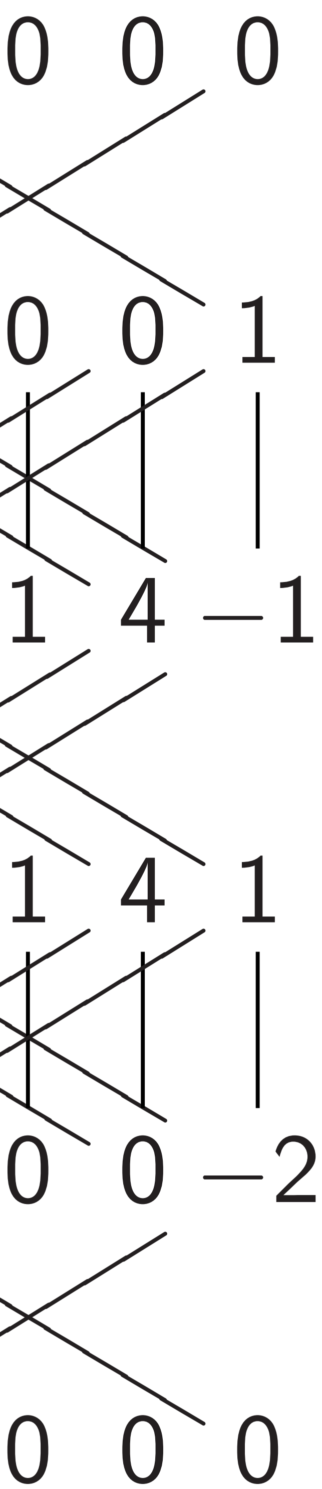
Assumptions:

- Given any $u \in \{$
  can efficiently co
- Nonzero $s \in \{0,$
- $f(u) = f(u \oplus s)$
- f has no other c

Goal: Figure out s

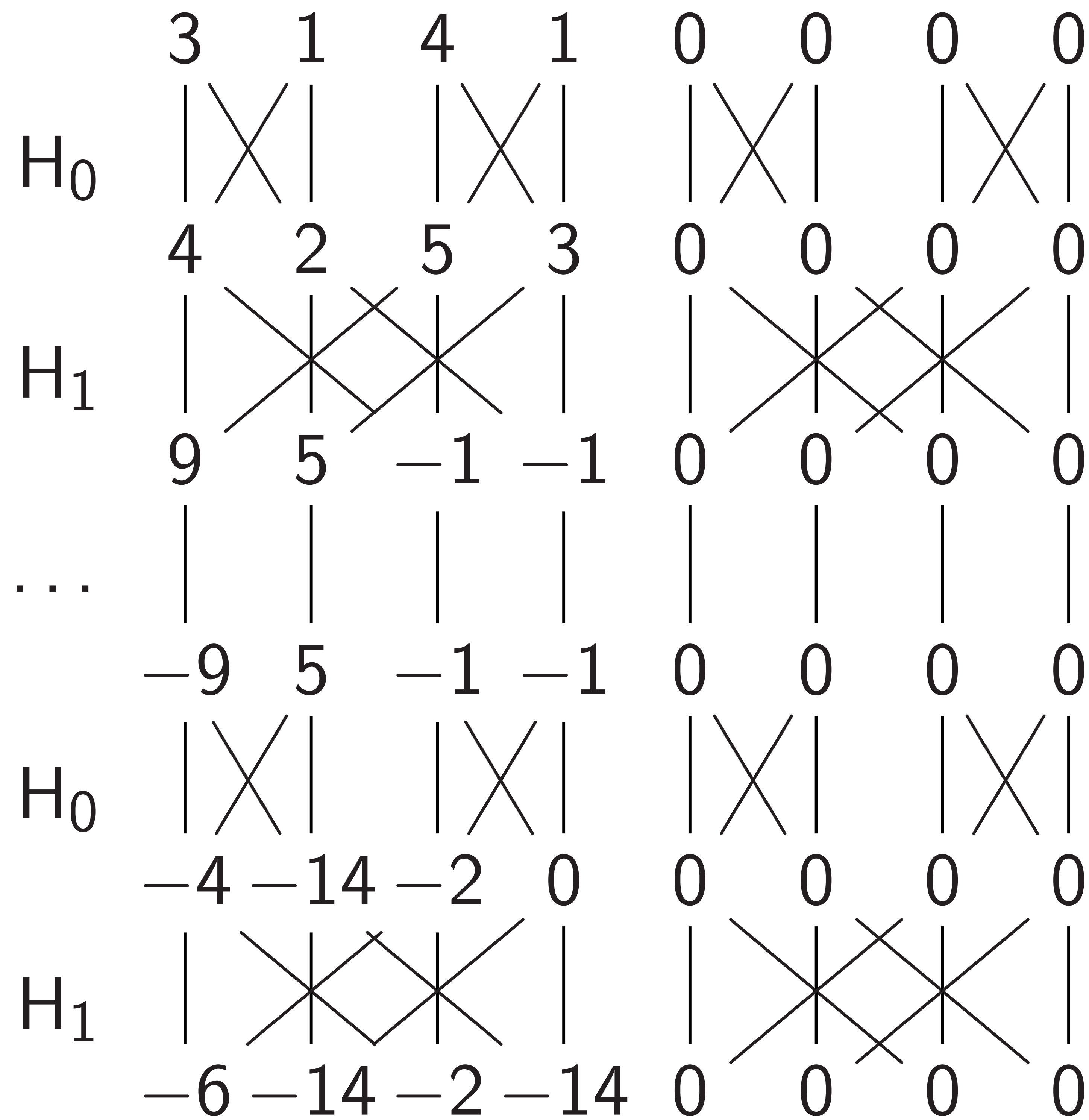
s set.”

qubit.



Affects measurements: “Negate amplitude around its average.”

$[3, 1, 4, 1] \mapsto [1.5, 3.5, 0.5, 3.5]$.



Simon’s algorithm

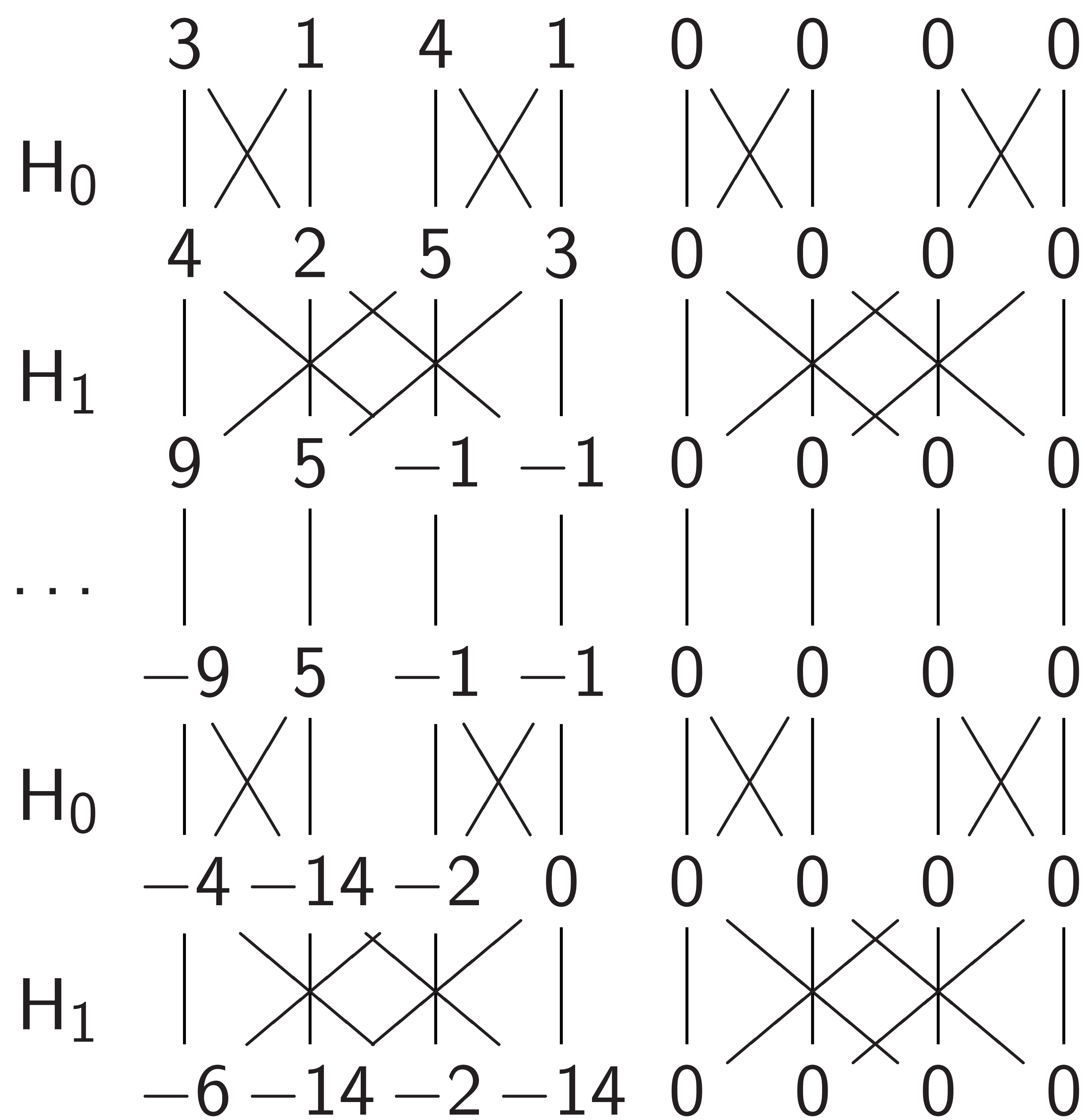
Assumptions:

- Given any $u \in \{0, 1\}^n$, can efficiently compute $f($
- Nonzero $s \in \{0, 1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Affects measurements: "Negate amplitude around its average."

$[3, 1, 4, 1] \mapsto [1.5, 3.5, 0.5, 3.5]$.



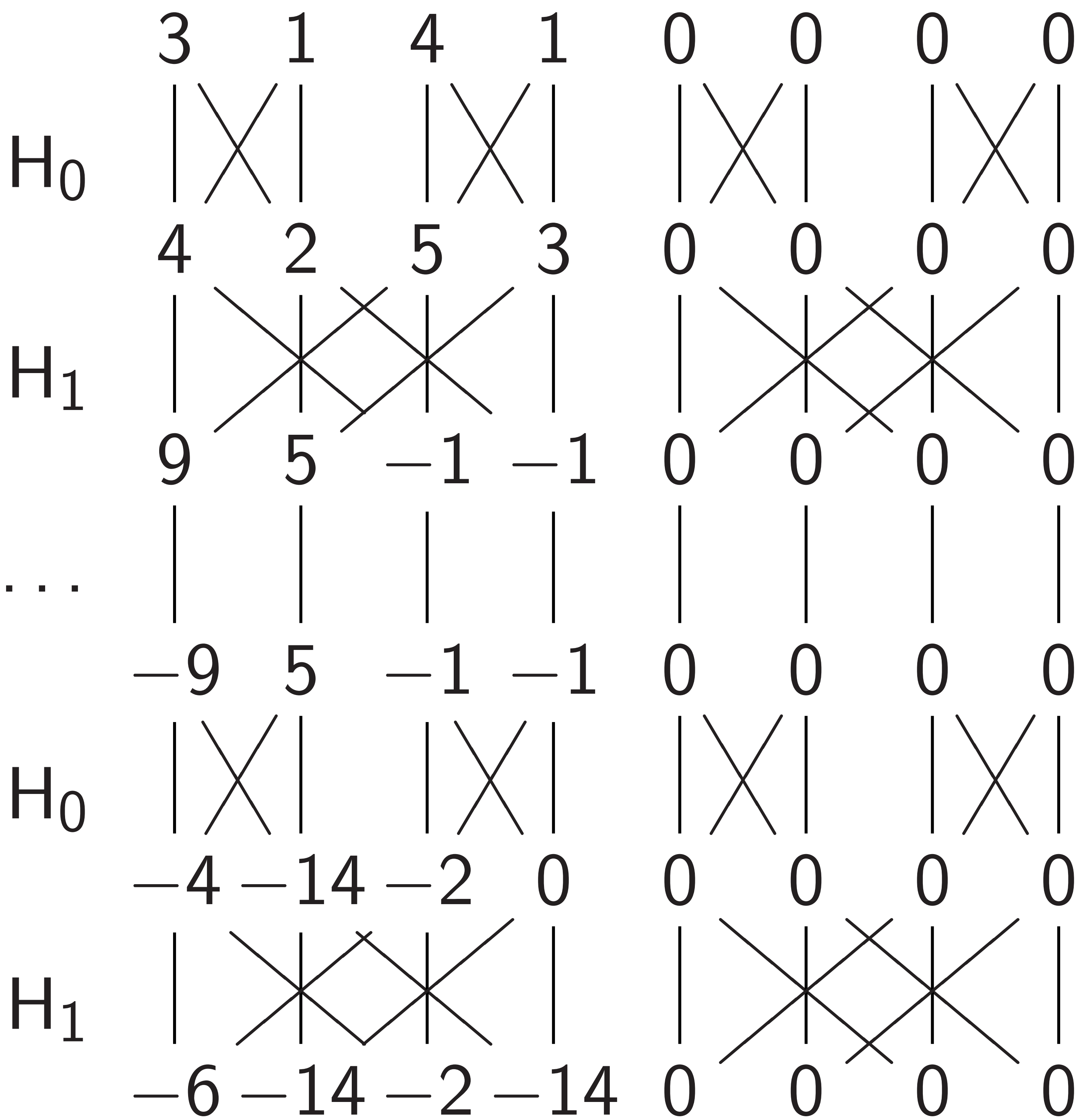
Simon's algorithm

Assumptions:

- Given any $u \in \{0, 1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0, 1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Affects measurements: "Negate amplitude around its average."
$[3,1,4,1] \mapsto [1.5,3.5,0.5,3.5]$.



Simon's algorithm

Assumptions:

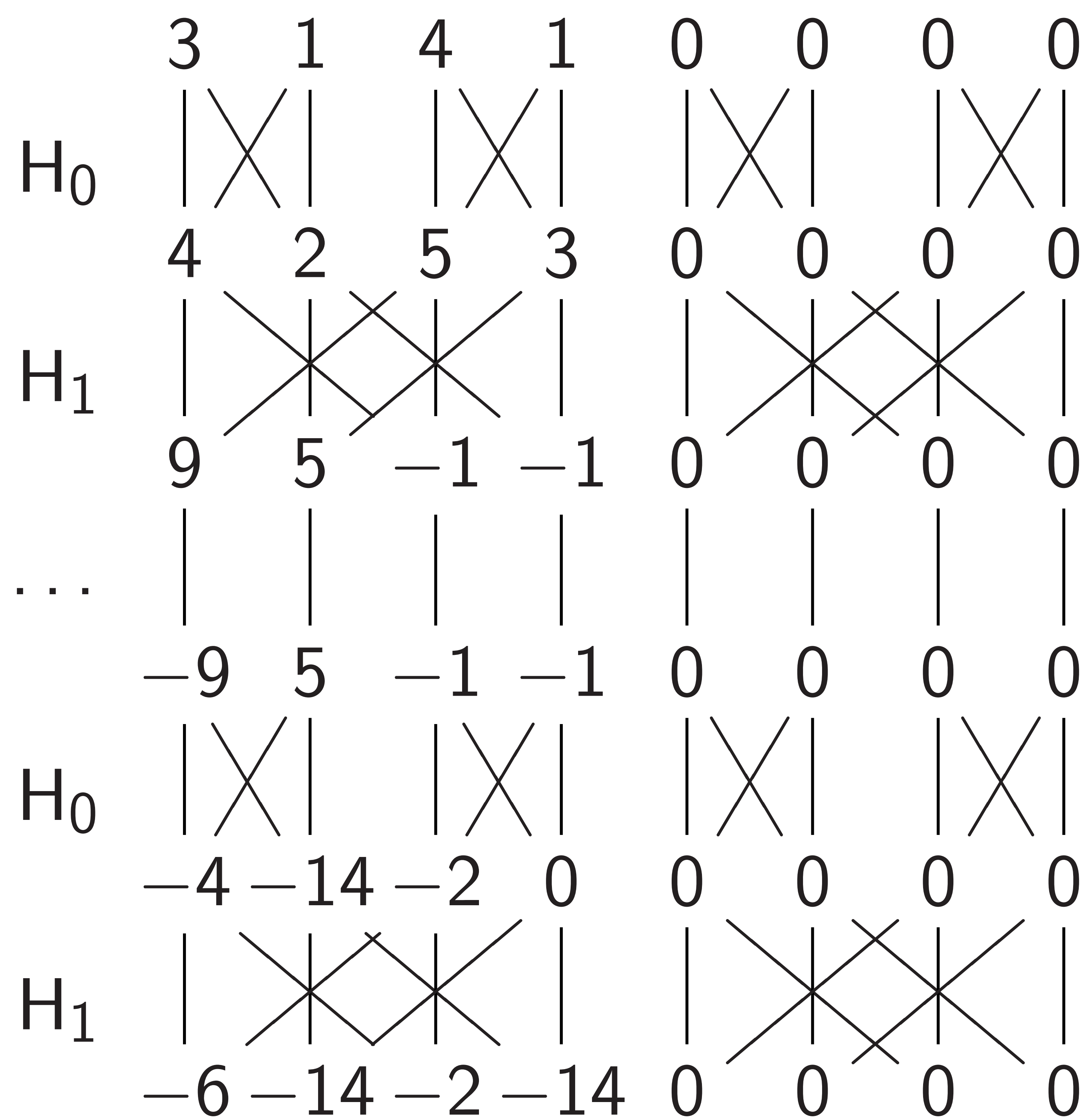
- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Affects measurements: "Negate amplitude around its average."

$[3,1,4,1] \mapsto [1.5,3.5,0.5,3.5]$.



Simon's algorithm

Assumptions:

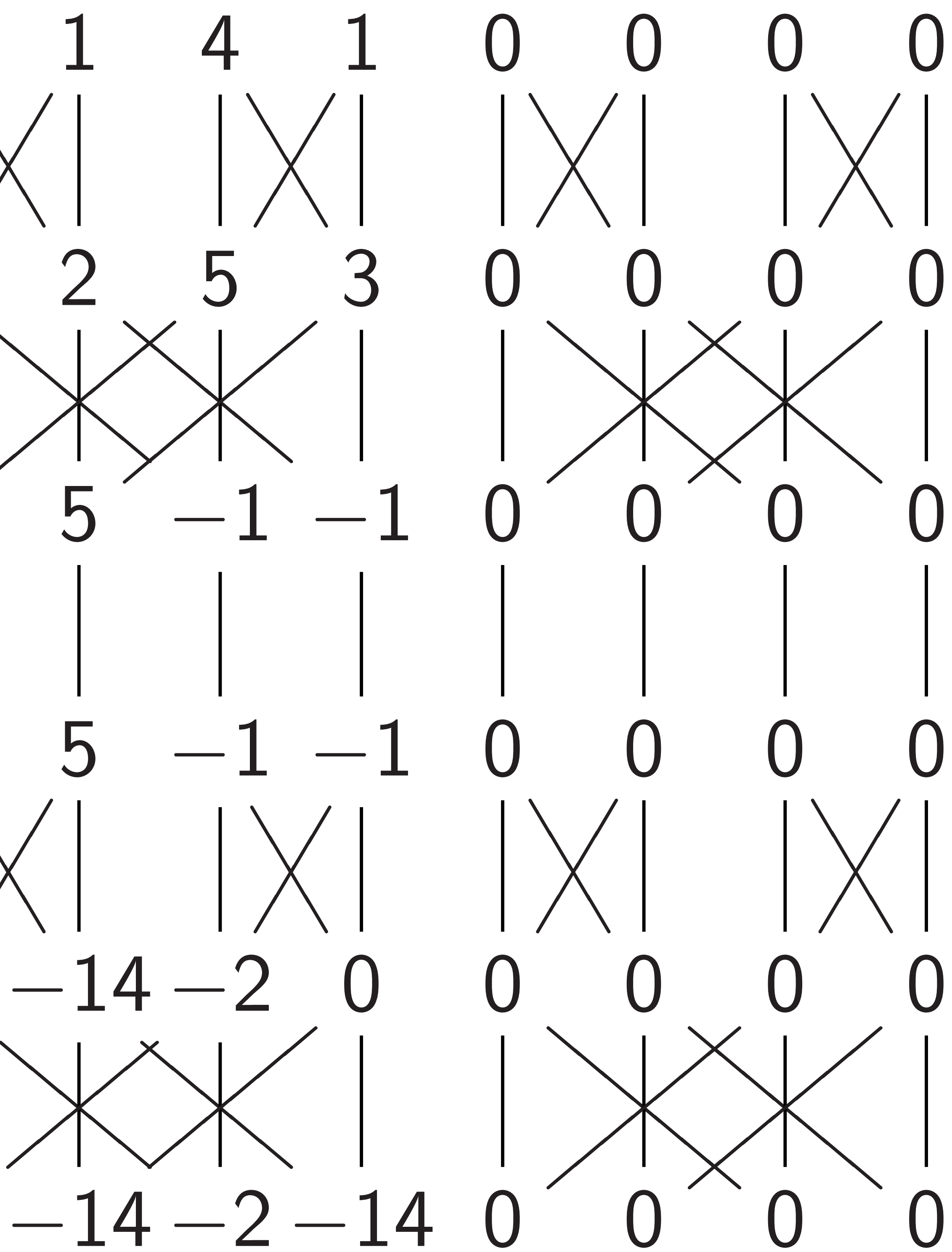
- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

measurements: “Negate
de around its average.”
] $\mapsto$ [1.5, 3.5, 0.5, 3.5].



Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

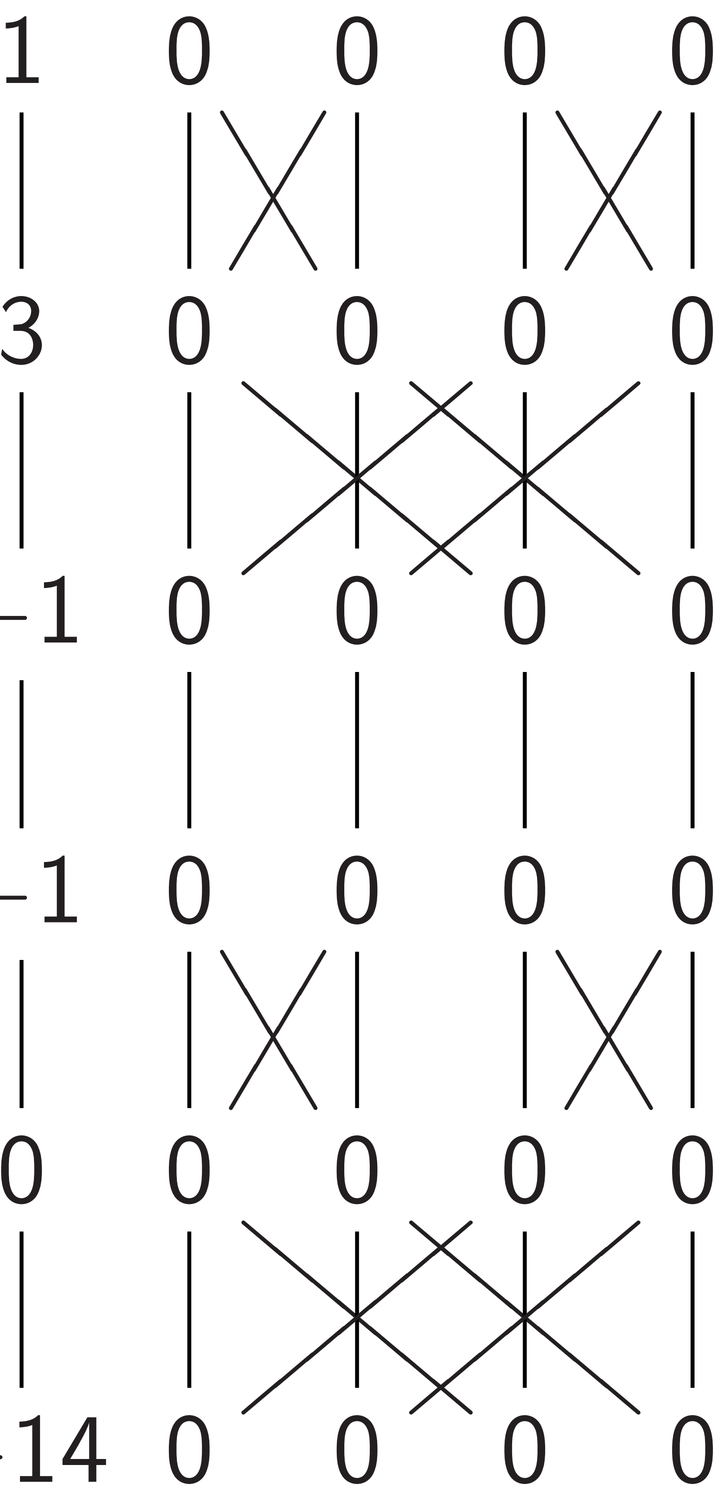
Example

Step 1.

1, 0, 0,
0, 0, 0,
0, 0, 0,
0, 0, 0,
0, 0, 0,
0, 0, 0,
0, 0, 0,
0, 0, 0,

This exa
with 3-b

ents: “Negate
its average.”
$3.5, 0.5, 3.5]$.



Simon’s algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon’s algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon

Step 1. Set up pu

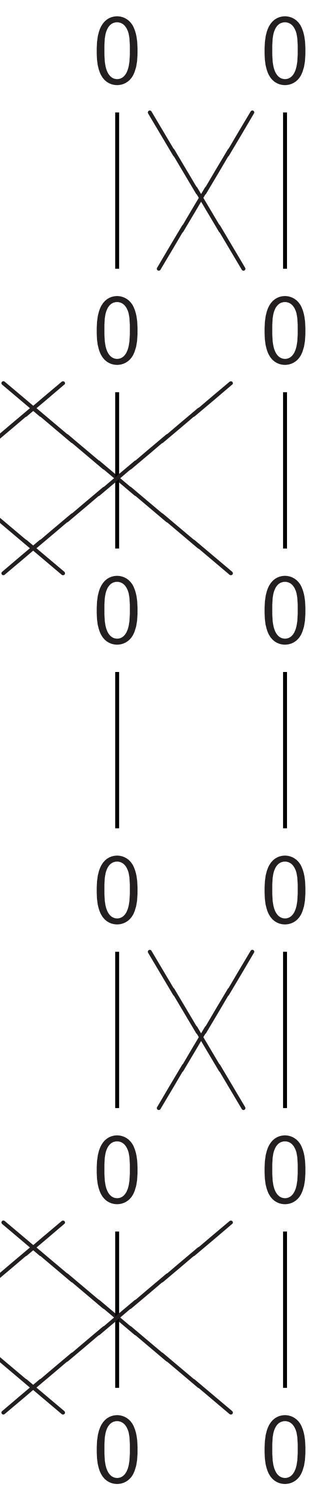
1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0,

This example is fo
with 3-bit input ar

gate

e.”

5].



Simon’s algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon’s algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon’s algorith

Step 1. Set up pure zero sta

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

This example is for a functio

with 3-bit input and 3-bit ou

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 1. Set up pure zero state:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

This example is for a function f with 3-bit input and 3-bit output.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 2.0. Hadamard$_0$:

1, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Simon's algorithm

Assumptions:

- Given any $u \in \{0, 1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0, 1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 2.1. Hadamard$_1$:

1, 1, 1, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 2.2. Hadamard$_2$:

1, 1, 1, 1, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe. Step 3 will apply the function f (a specific function in this example), computing $f(u)$ in universe u.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3a. C_0NOT_3:

1, 0, 1, 0, 1, 0, 1, 0,
0, 1, 0, 1, 0, 1, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3b. More entry shuffling:

1, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3c. More entry shuffling:

1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 1.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3d. More entry shuffling:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3e. More entry shuffling:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3f. More entry shuffling:

0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3g. More entry shuffling:

0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3h. More entry shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3i. More entry shuffling:

0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 1, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3j. Final entry shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 3j. Final entry shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

Each column is a parallel universe performing its own computations.

Surprise: u and $u \oplus 101$ match.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 4.0. Hadamard$_0$:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, $\overline{1}$, 0, 0, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 1, 0, 0, 1, $\overline{1}$,
1, $\overline{1}$, 0, 0, 1, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 1, 0, 0, 1, $\overline{1}$, 0, 0.

Notation: $\overline{1}$ means -1.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 4.1. Hadamard$_1$:

$0, 0, 0, 0, 0, 0, 0, 0,$

$1, \overline{1}, \overline{1}, 1, 1, 1, \overline{1}, \overline{1},$

$0, 0, 0, 0, 0, 0, 0, 0,$

$1, 1, \overline{1}, \overline{1}, 1, \overline{1}, \overline{1}, 1,$

$1, \overline{1}, 1, \overline{1}, 1, 1, 1, 1,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$1, 1, 1, 1, 1, \overline{1}, 1, \overline{1}.$

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 4.2. Hadamard$_2$:

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, 2, 0, \overline{2},$

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$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, 2, 0, 0, 2, 0, 2.$

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Non-quantum algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum evaluations of f.

Example of Simon's algorithm

Step 4.2. Hadamard$_2$:

0, 0, 0, 0, 0, 0, 0, 0,
2, 0, $\overline{2}$, 0, 0, $\overline{2}$, 0, 2,
0, 0, 0, 0, 0, 0, 0, 0,
2, 0, $\overline{2}$, 0, 0, 2, 0, $\overline{2}$,
2, 0, 2, 0, 0, $\overline{2}$, 0, $\overline{2}$,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
2, 0, 2, 0, 0, 2, 0, 2.

Step 5: Measure. Obtain some information about the surprise: a random vector orthogonal to 101.

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2, 0, 2, 0, 0, $\overline{2}$, 0, $\overline{2}$,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
2, 0, 2, 0, 0, 2, 0, 2.

Step 5: Measure. Obtain some information about the surprise: a random vector orthogonal to 101.

Repeat to figure out 101.

Example of Simon's algorithm

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$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$

$2,\ 0,\ \overline{2},\ 0,\ 0,\ \overline{2},\ 0,\ 2,$

$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$

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Generalize Step 3 to any function $u \mapsto f(u)$ with $f(u) = f(u \oplus s)$. "Usually" algorithm figures out s.

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Many spectacular applications.

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Assume: unique s
has $f(s) = 0$.

Goal: Figure out s

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Grover's algorithm takes only $2^{n/2}$ quantum evaluations of f. e.g. 2^{64} instead of 2^{128}.

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Start from uniform superposition over n-bit strings u: each $a_u = 1$.

Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
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This is fast if f is fast.

Grover's algorithm

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Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

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Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

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Measure the n qubits.
With high probability this finds s.

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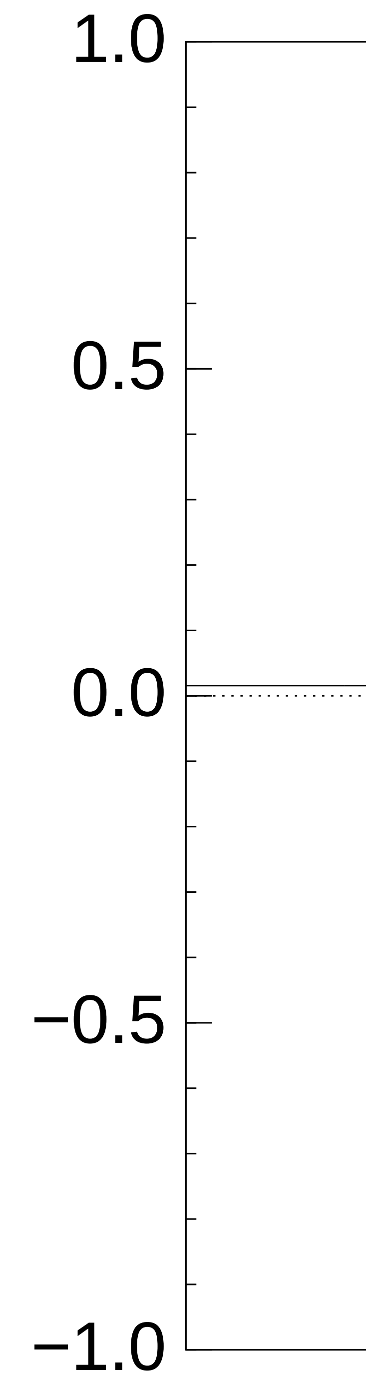

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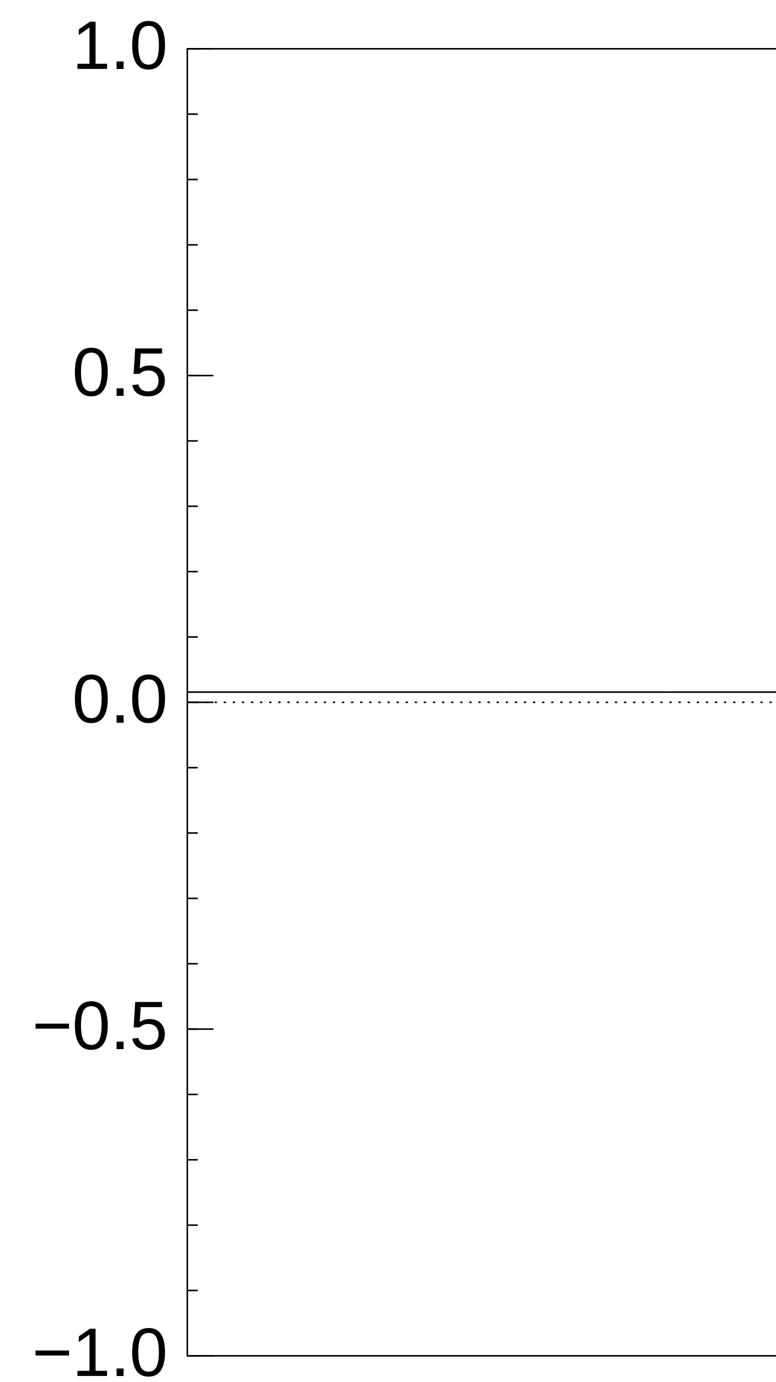
Normalized graph
for an example wit
after 0 steps:



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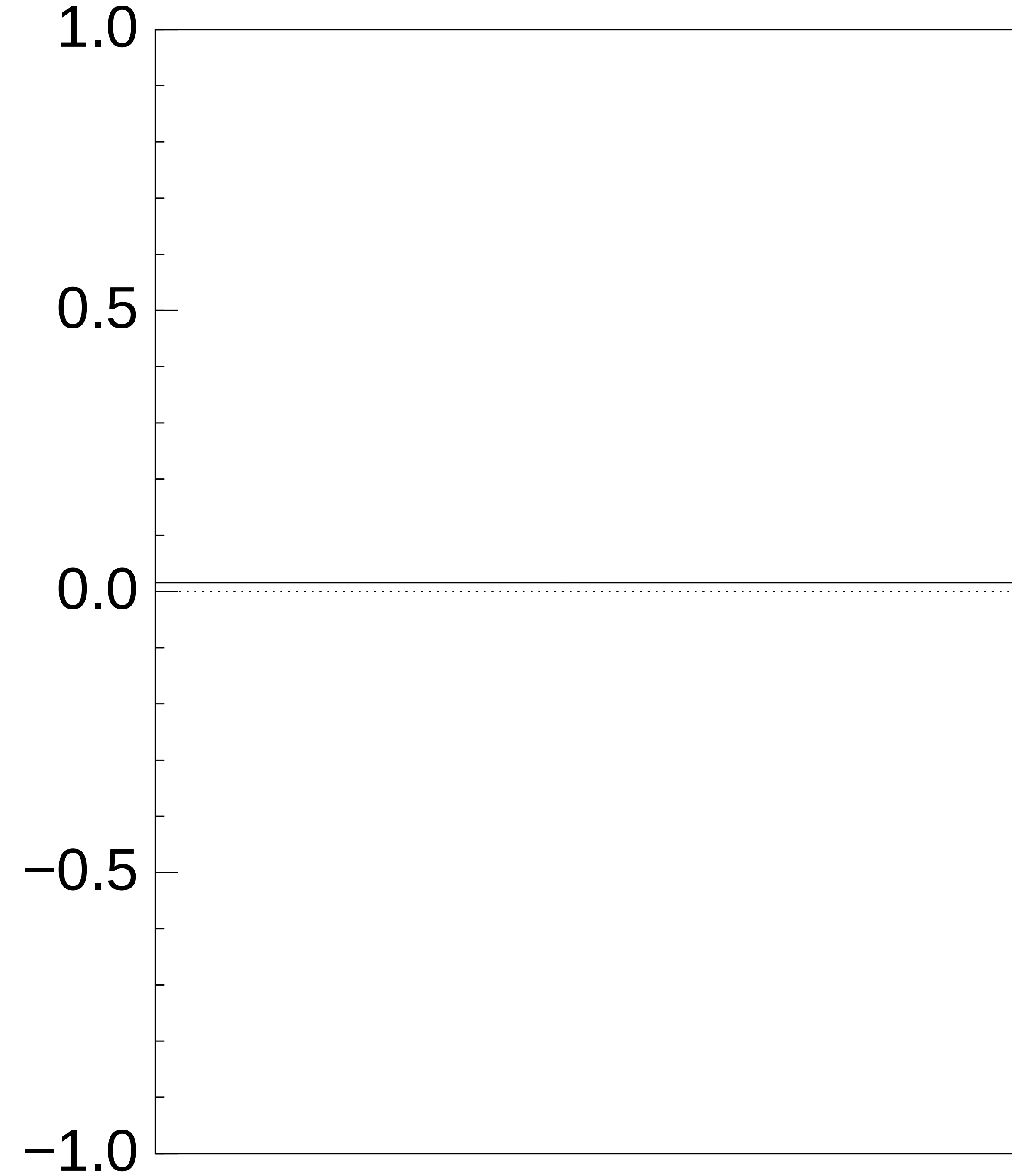
Normalized graph of $u \mapsto a_u$
for an example with $n = 12$
after 0 steps:



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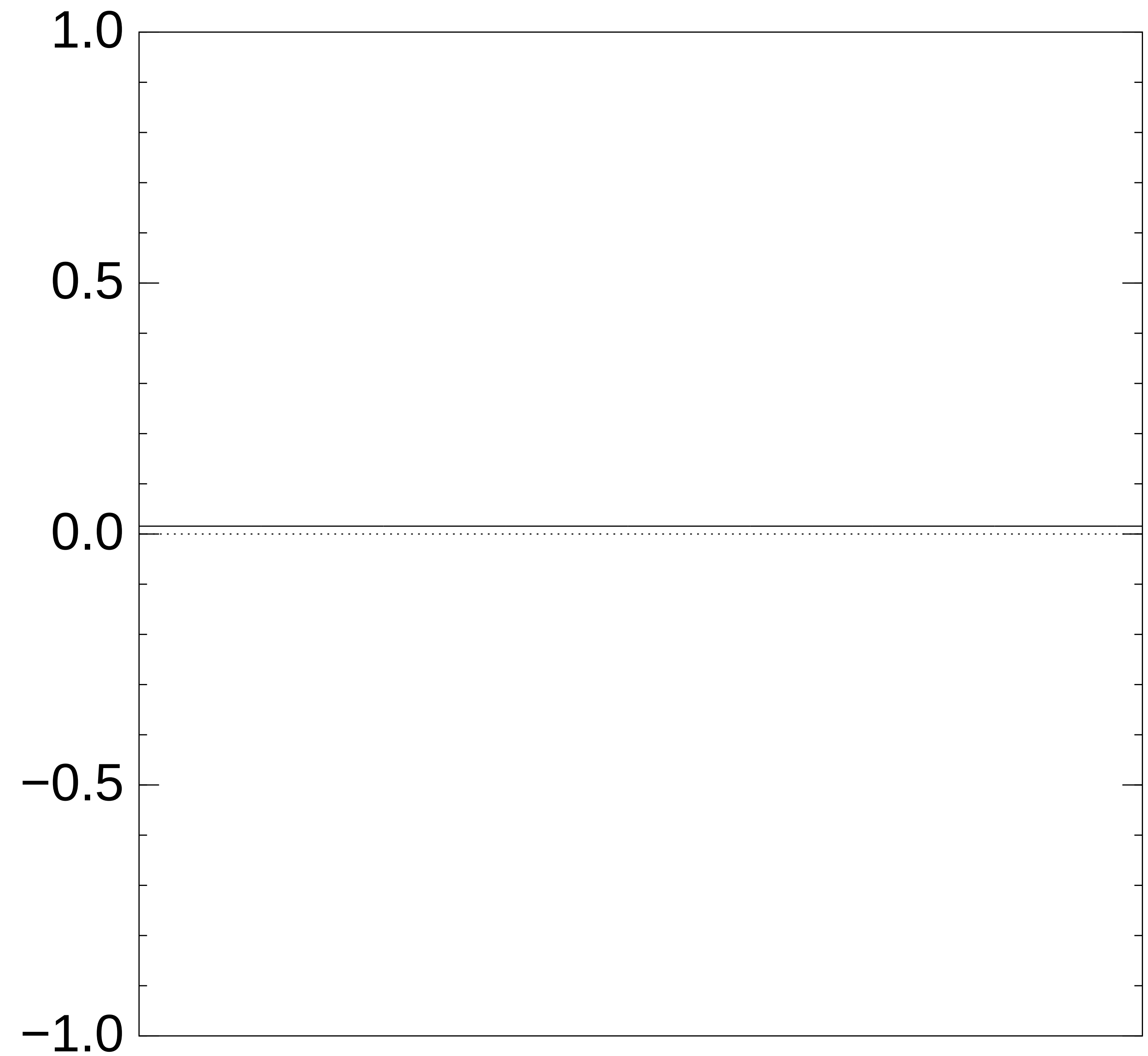

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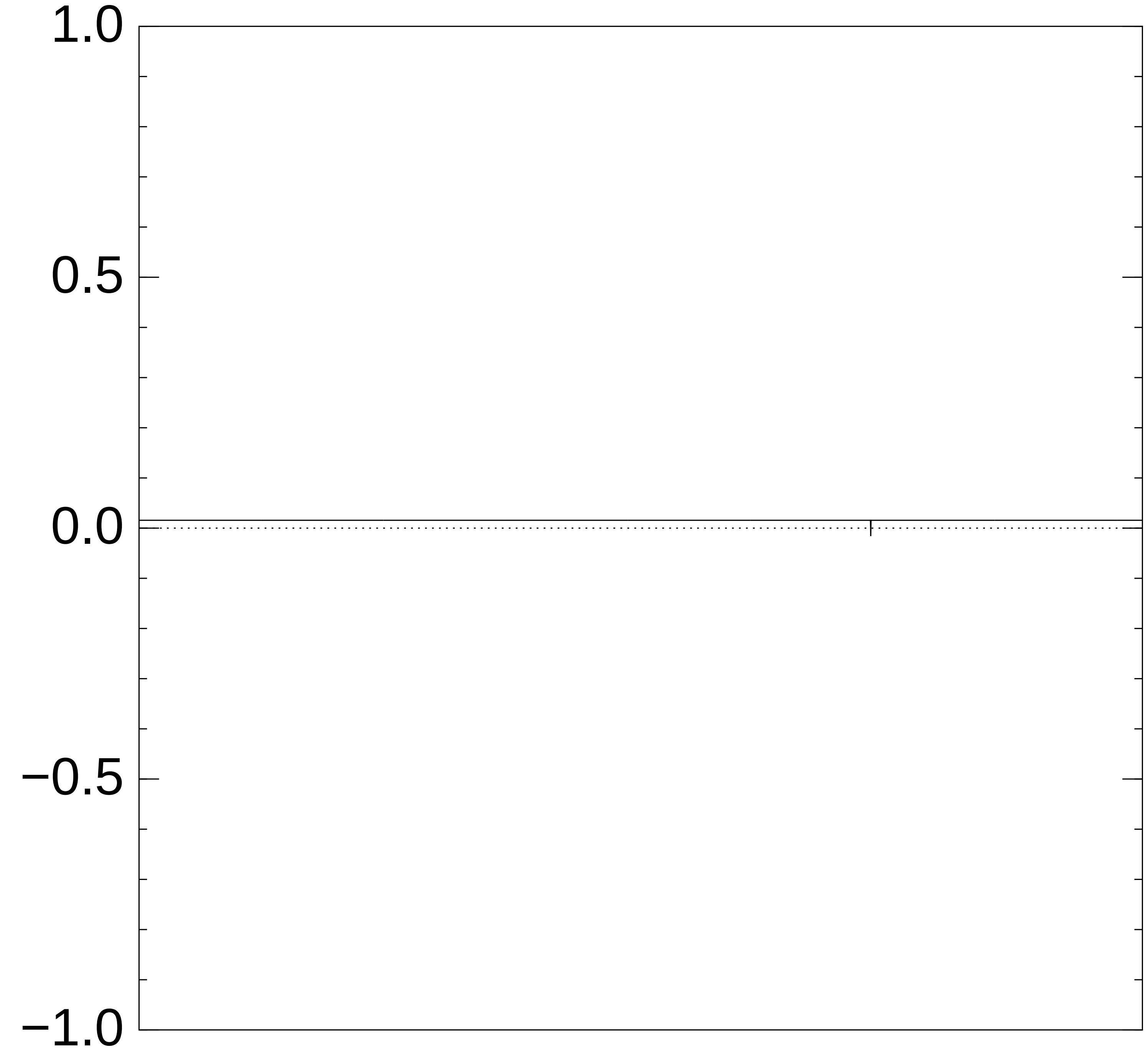
Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after Step 1:



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Step 1: Set $a \leftarrow b$ where
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Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

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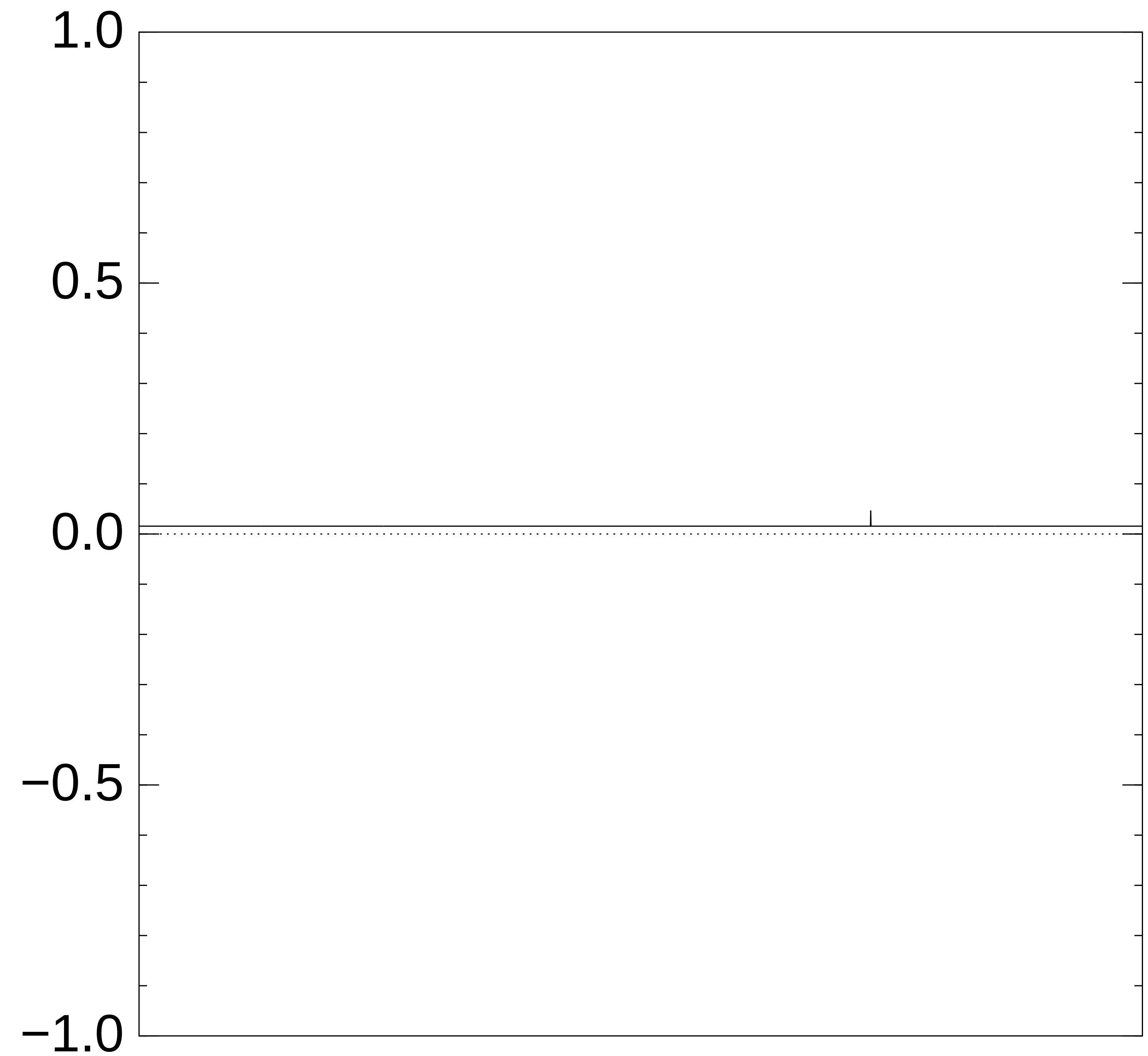

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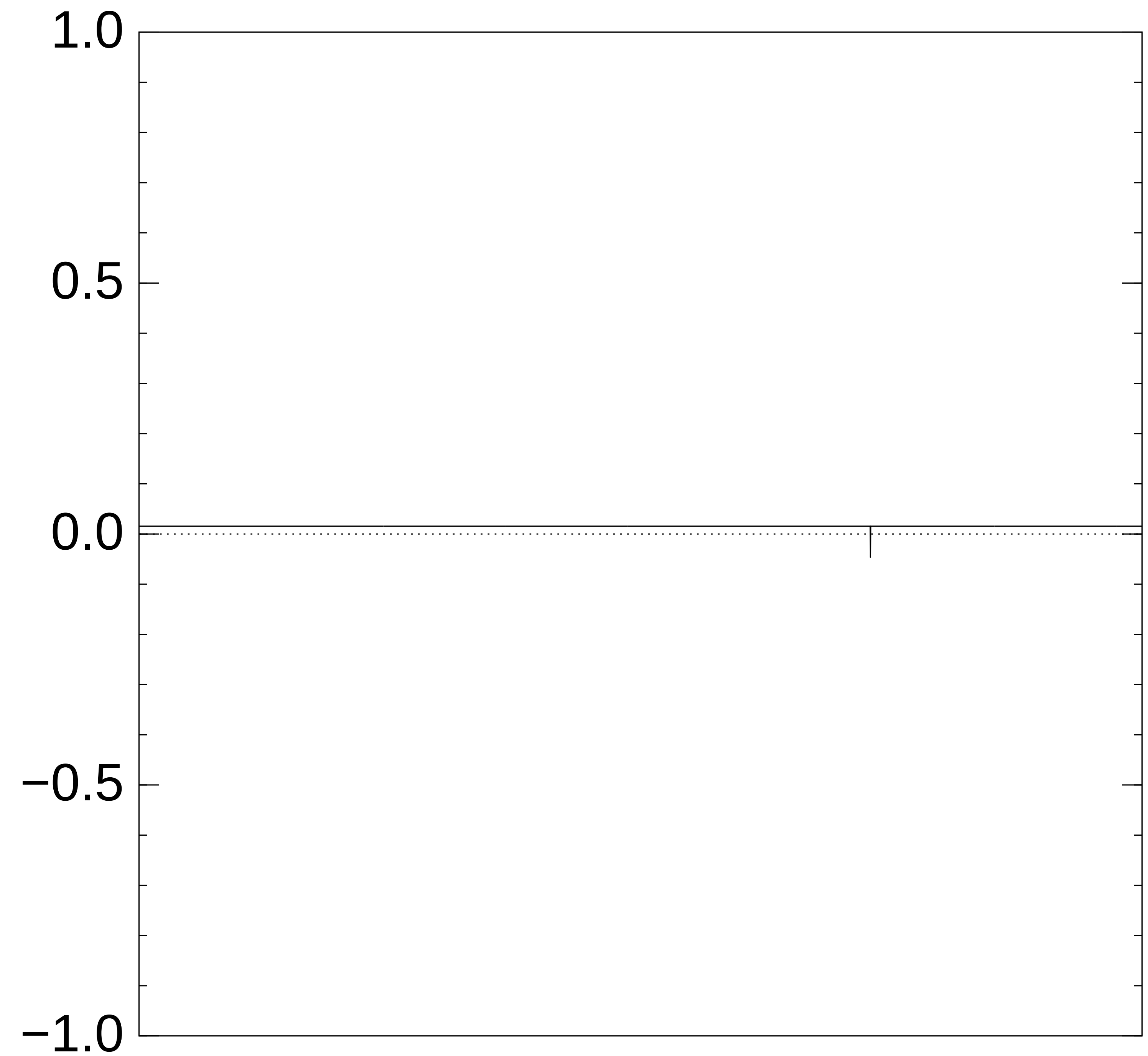

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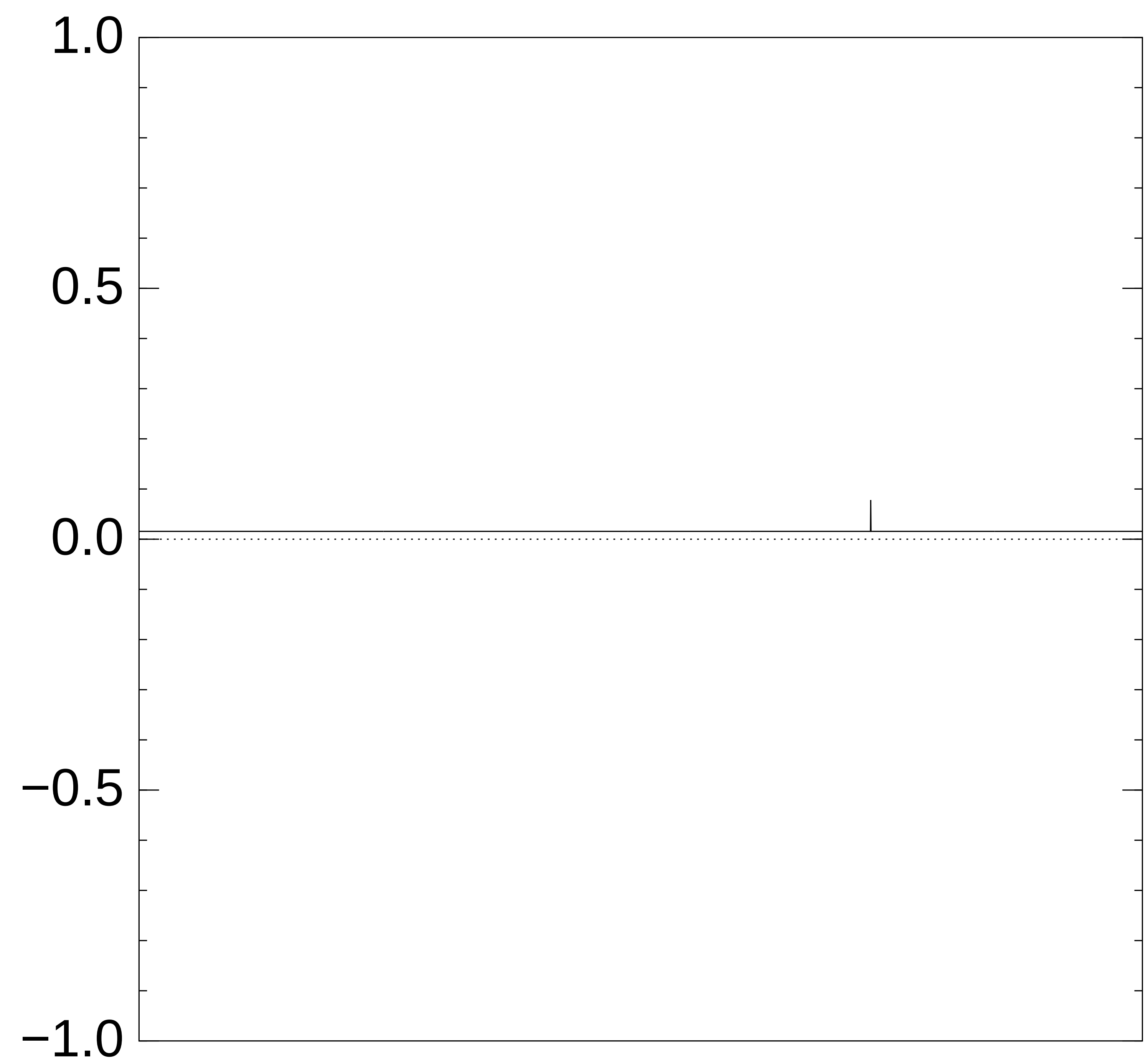
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $2 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

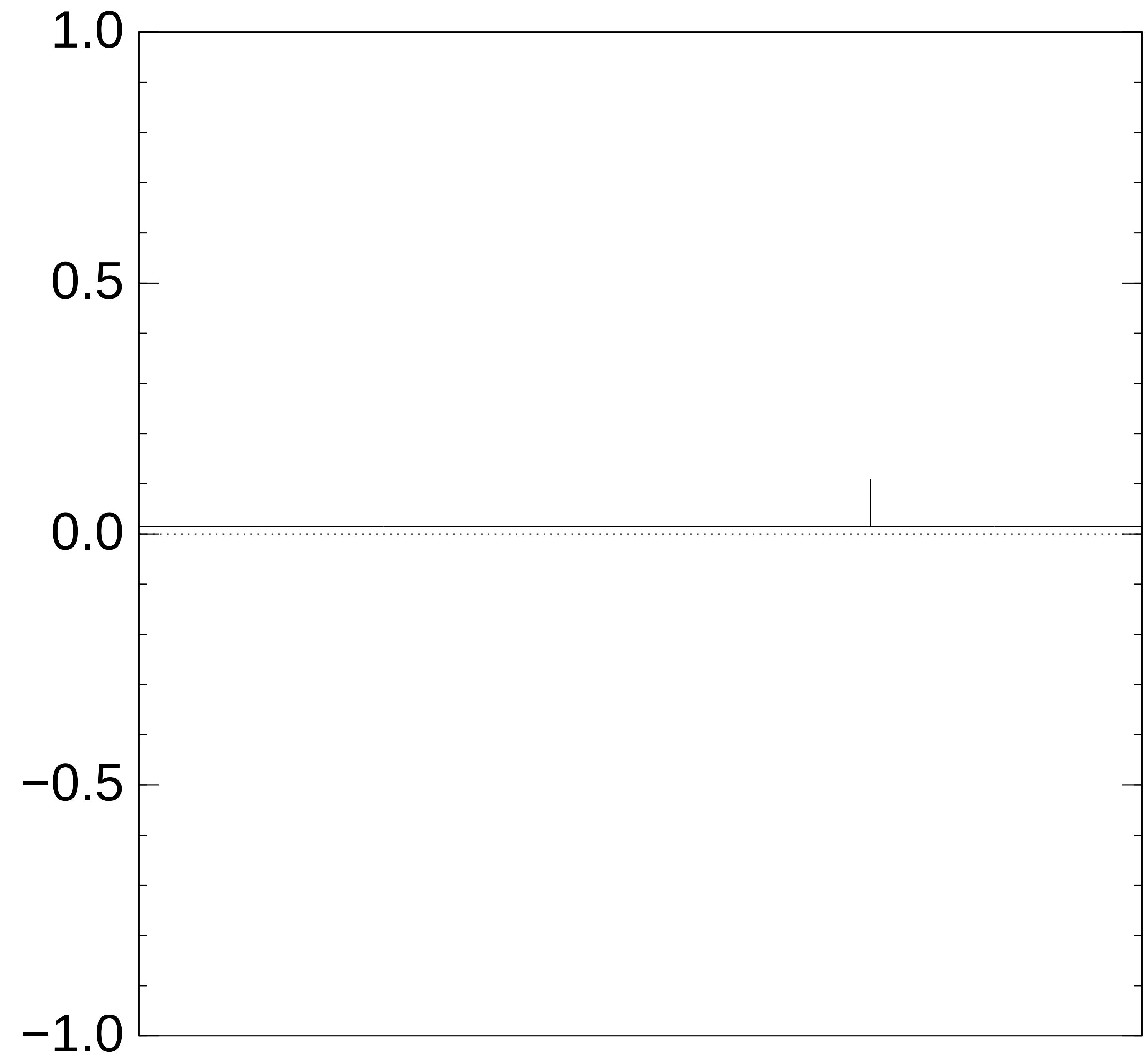
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $3 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

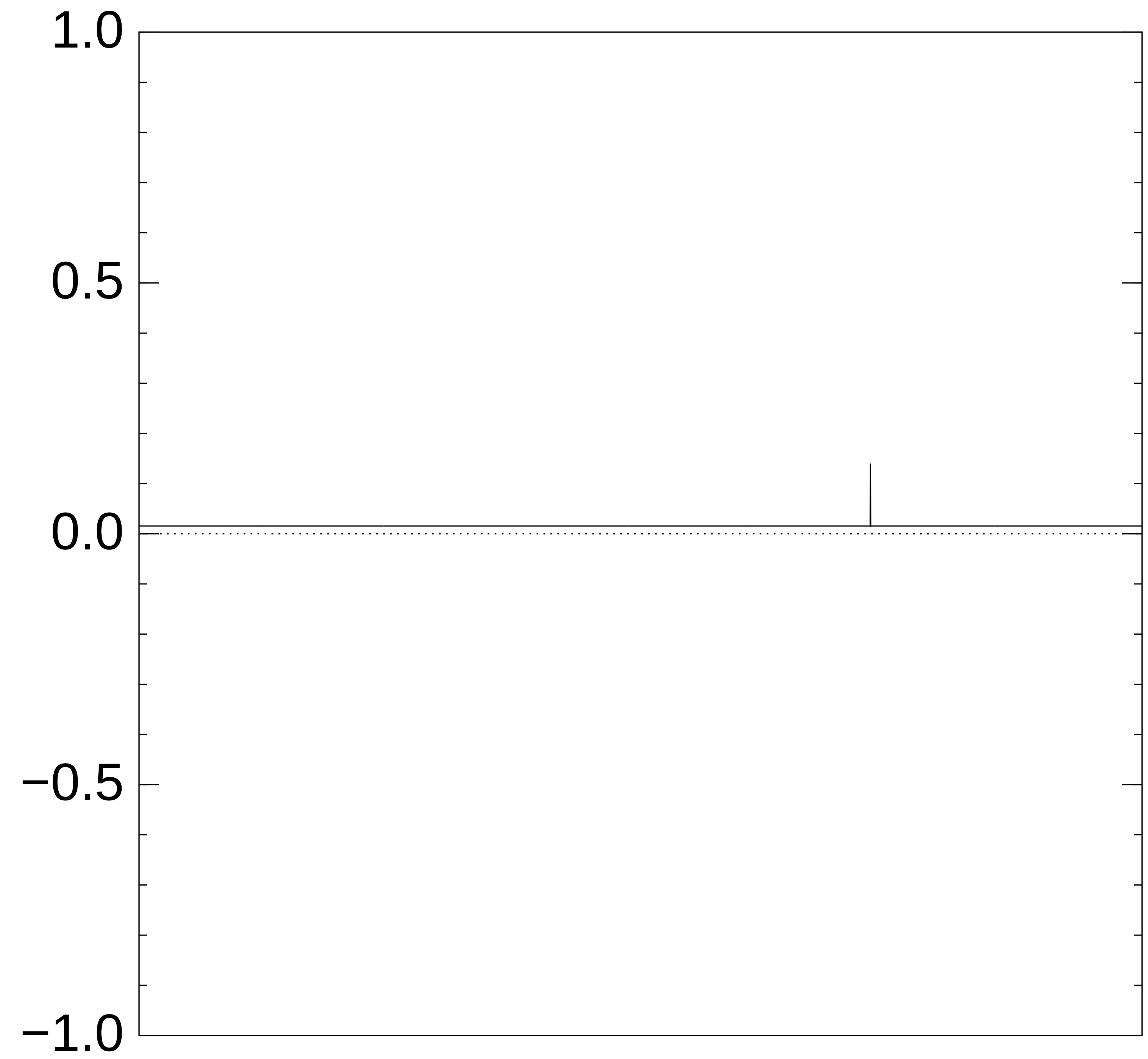
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $4 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

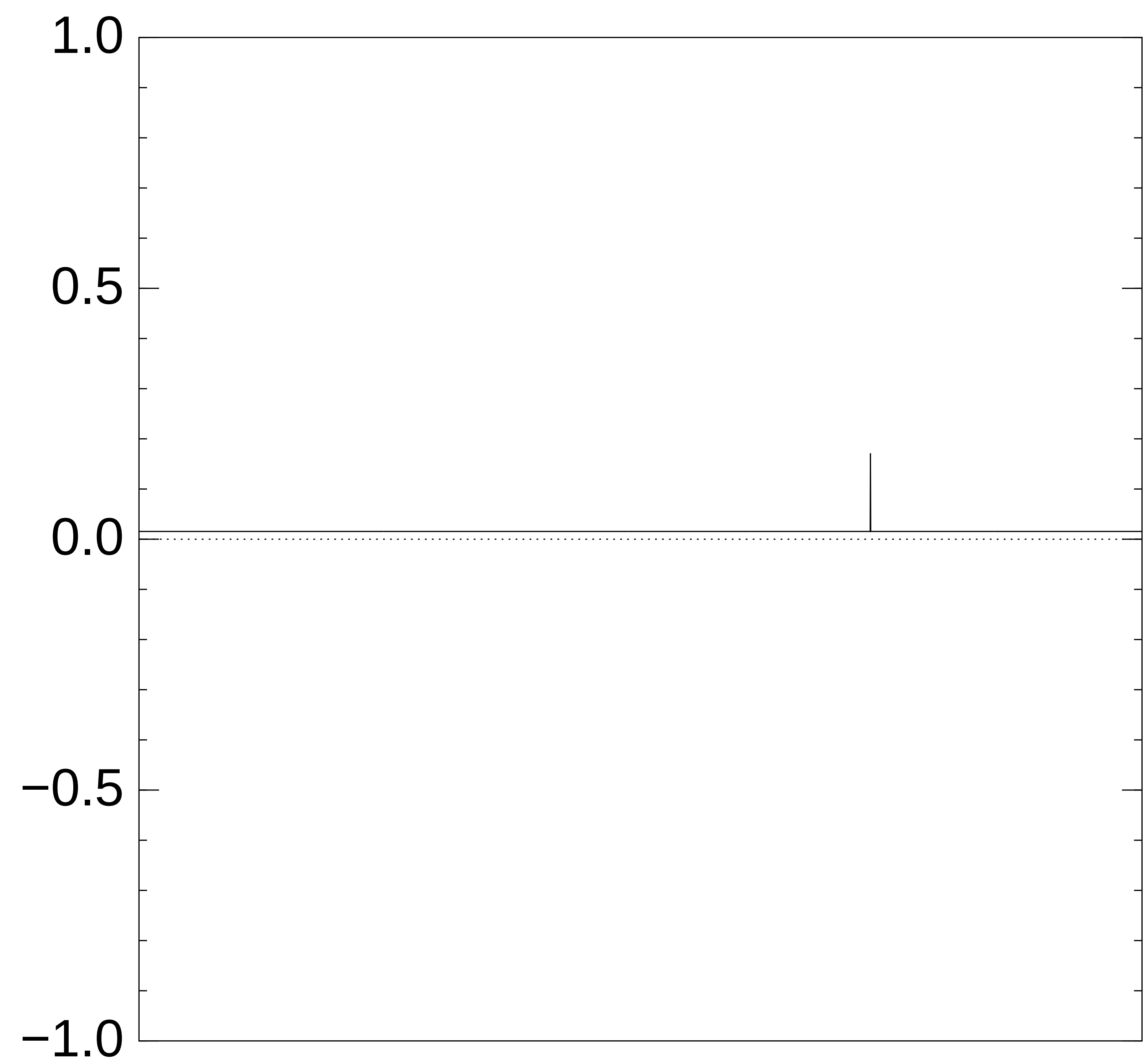
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $5 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

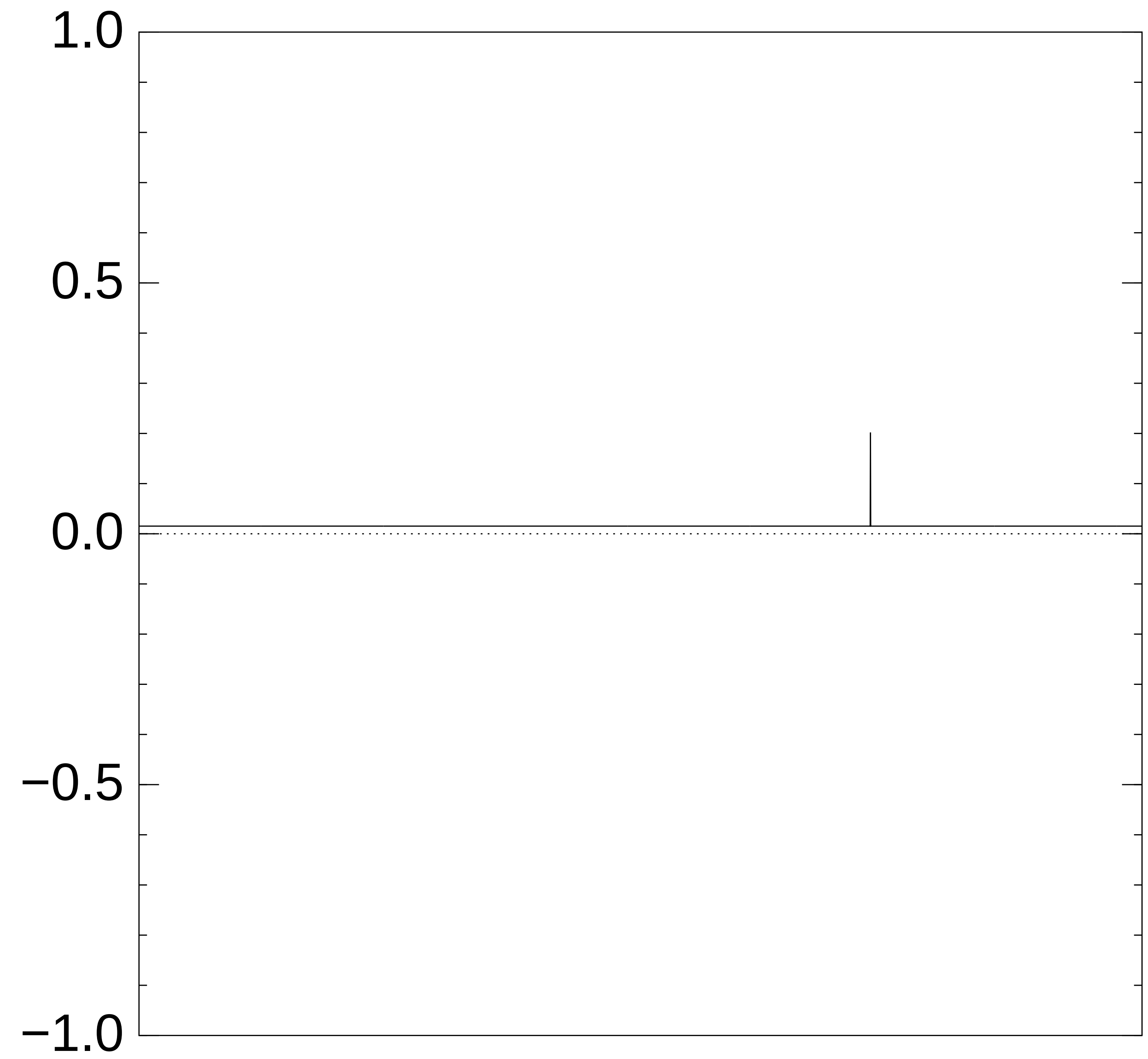
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $6 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

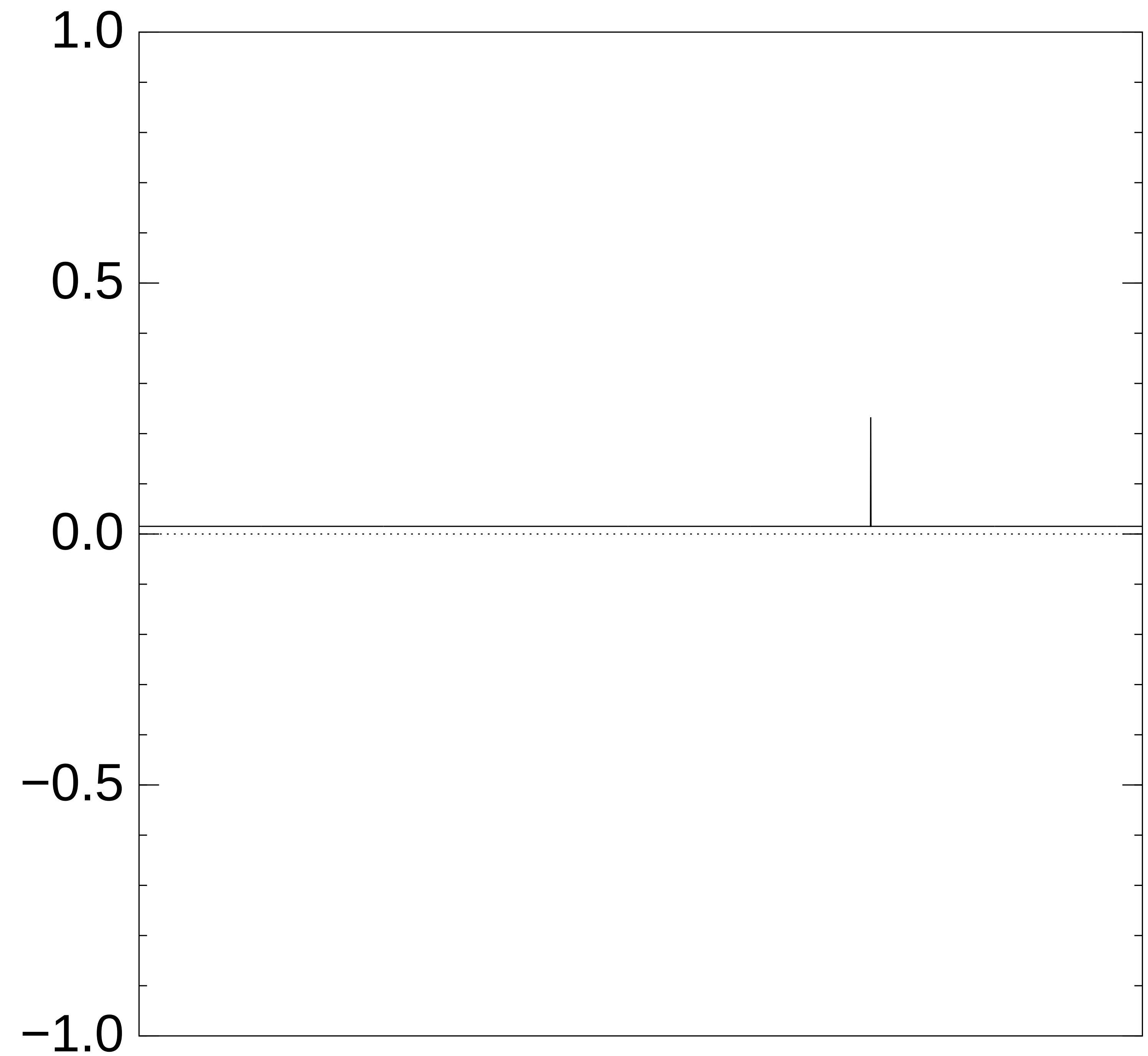
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $7 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

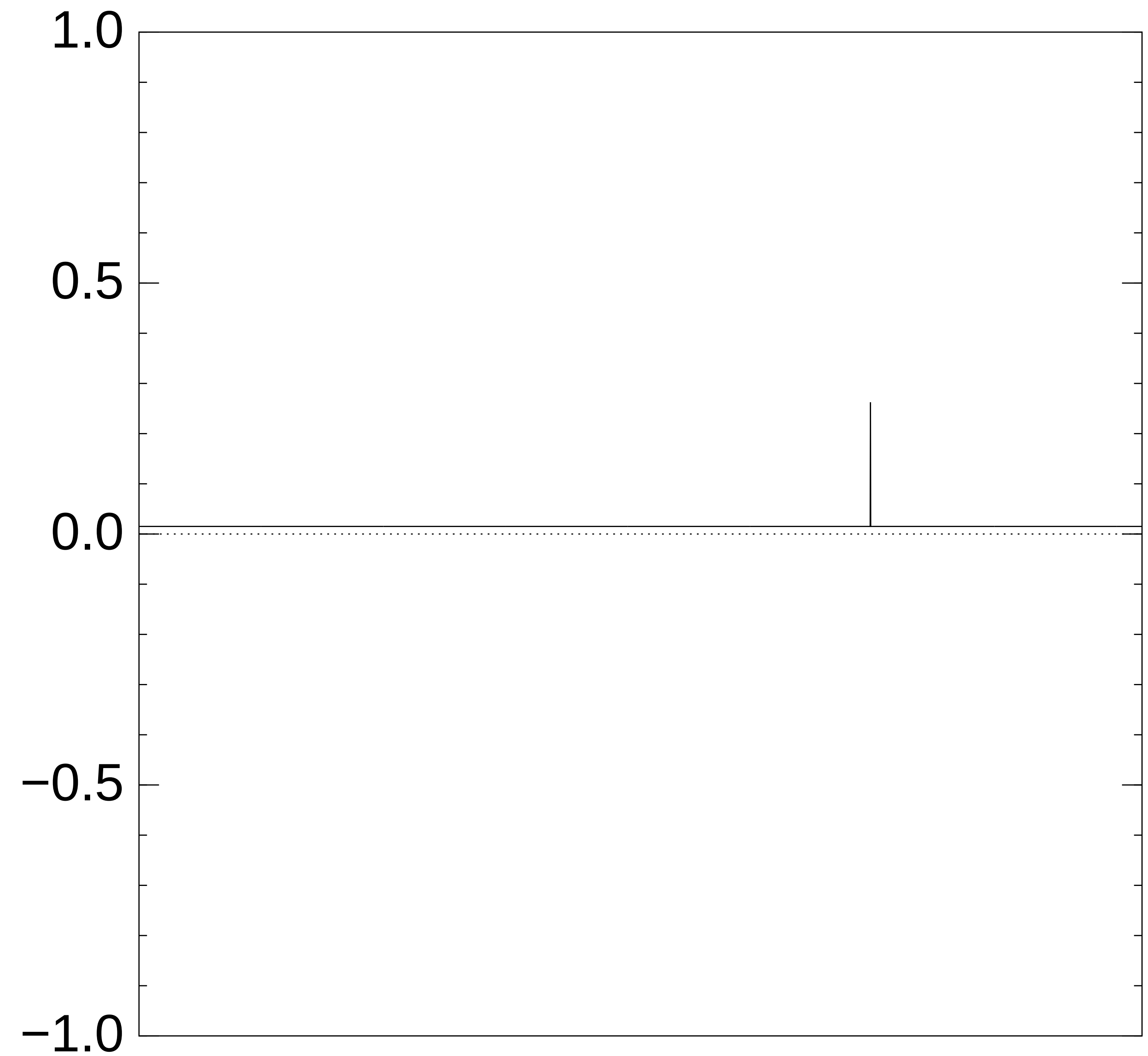
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $8 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

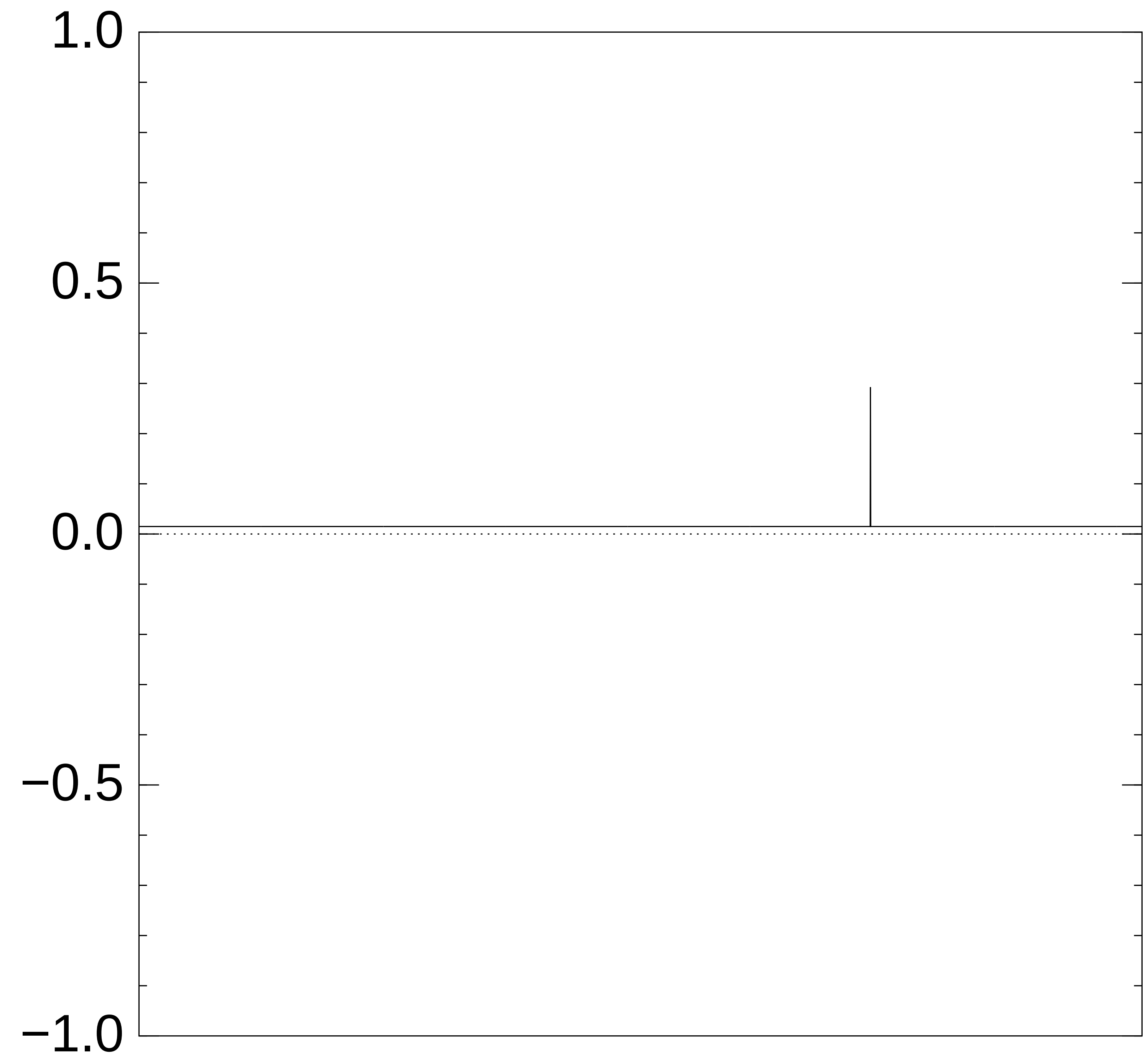
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $9 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

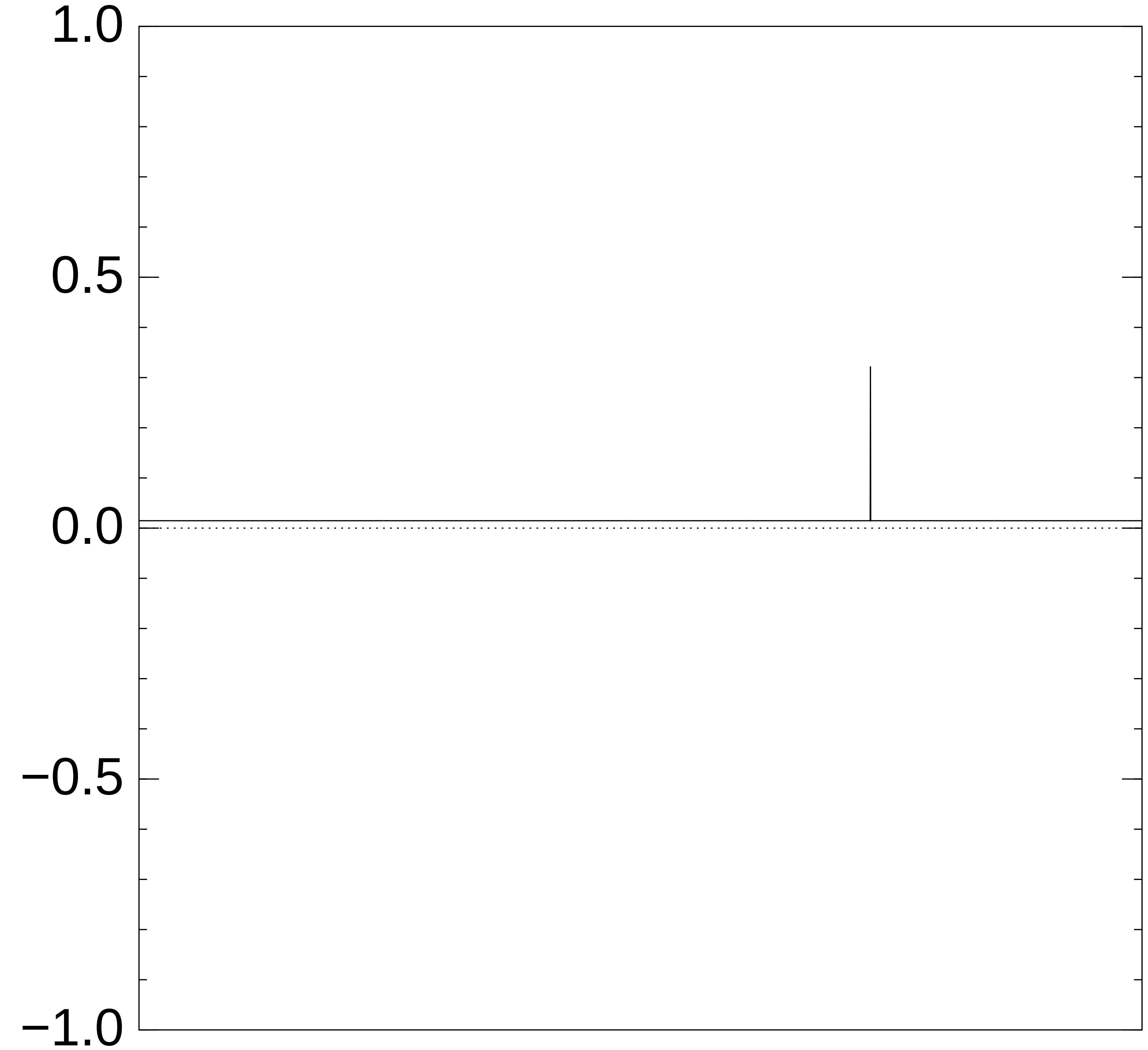
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $10 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

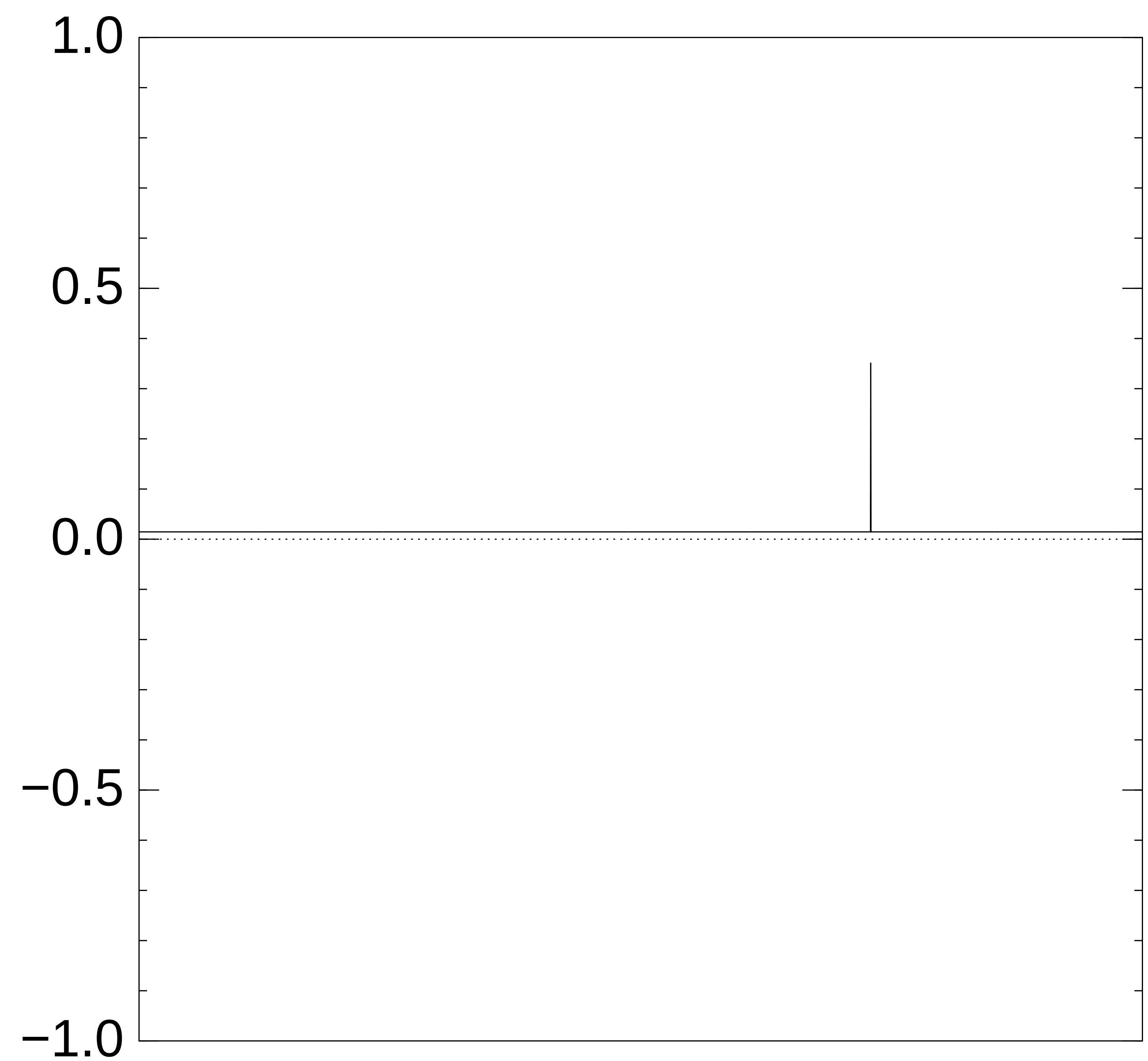
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $11 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

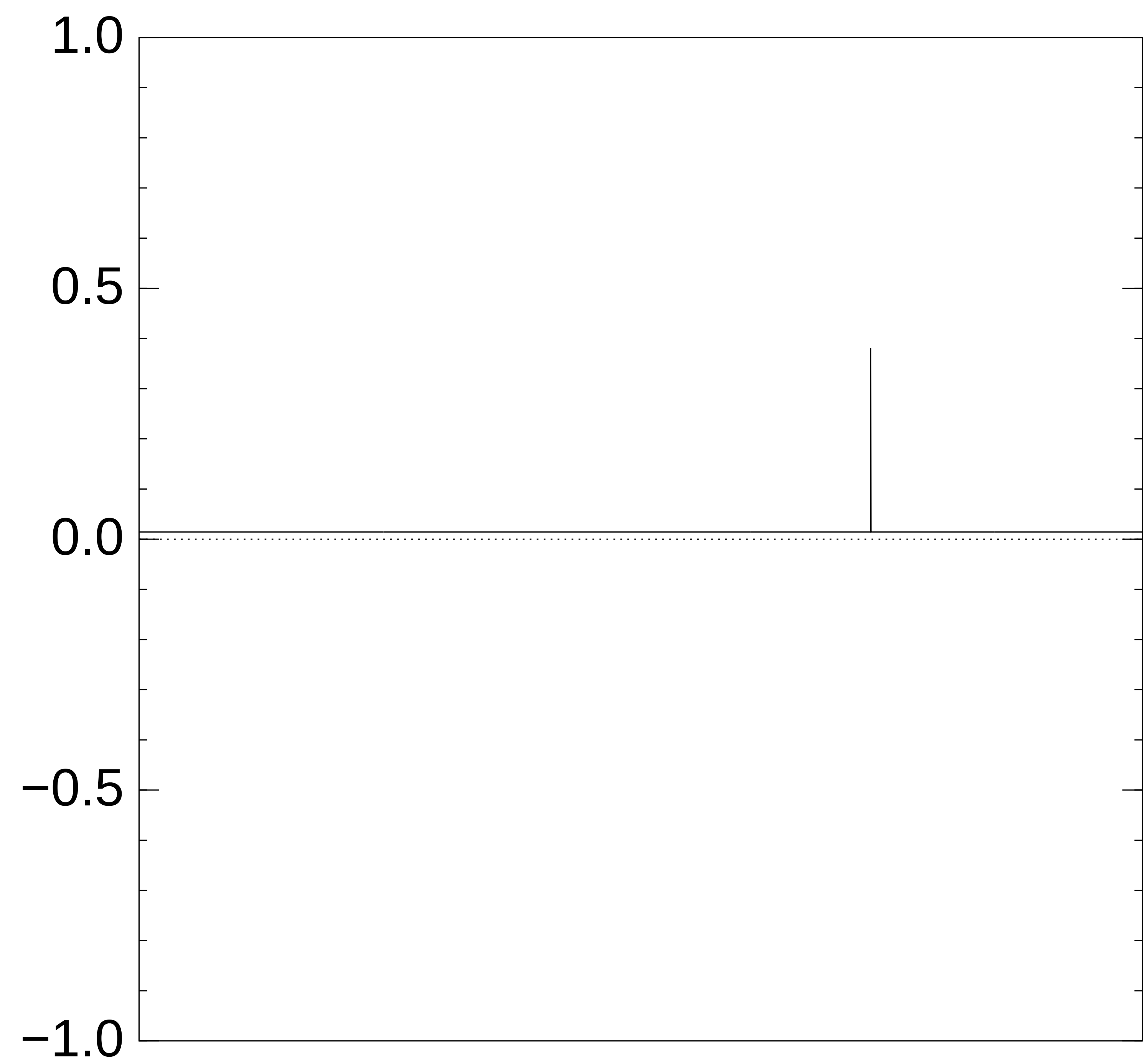
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $12 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

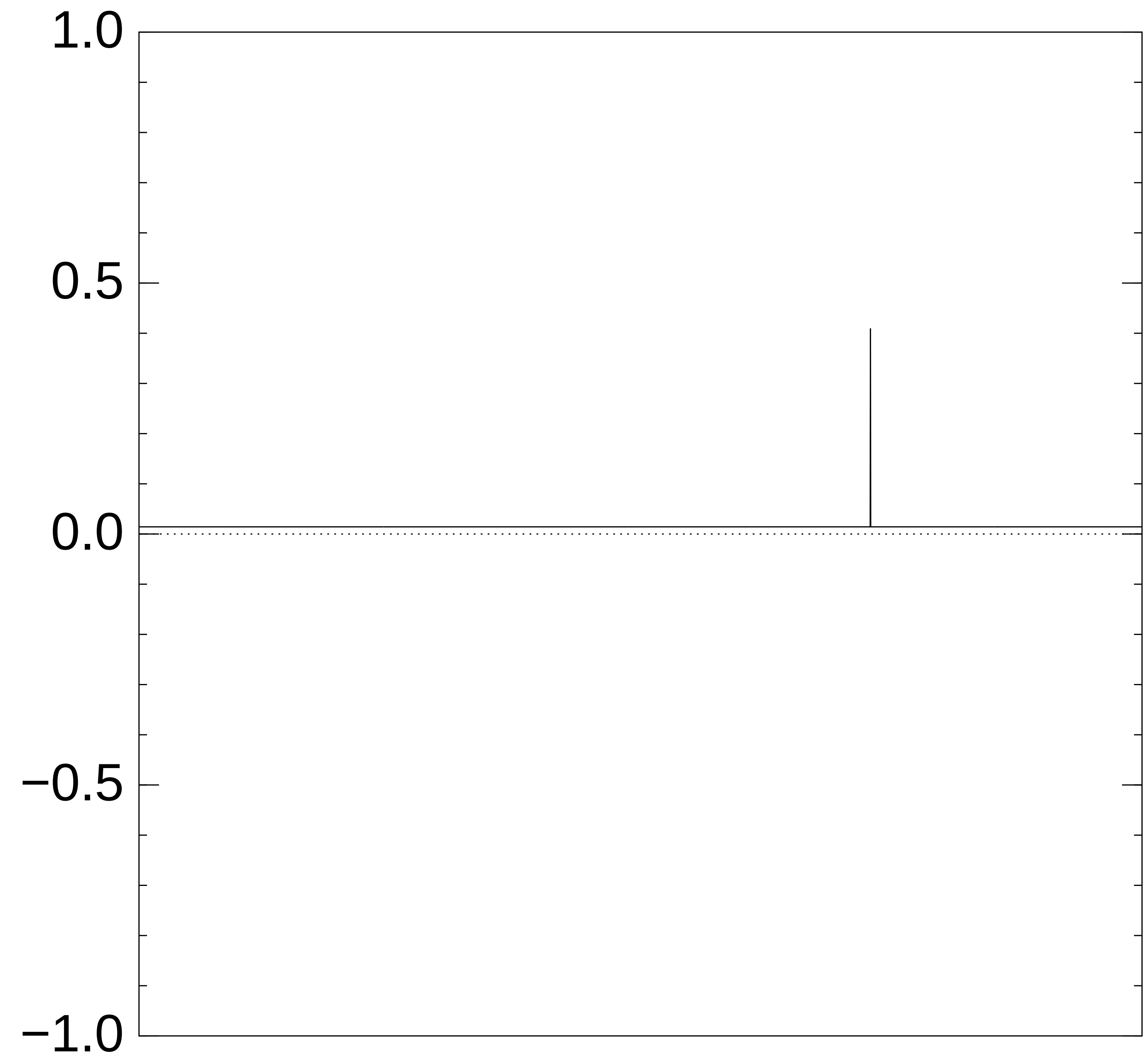
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion". Negate a around its average. This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits. With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $13 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

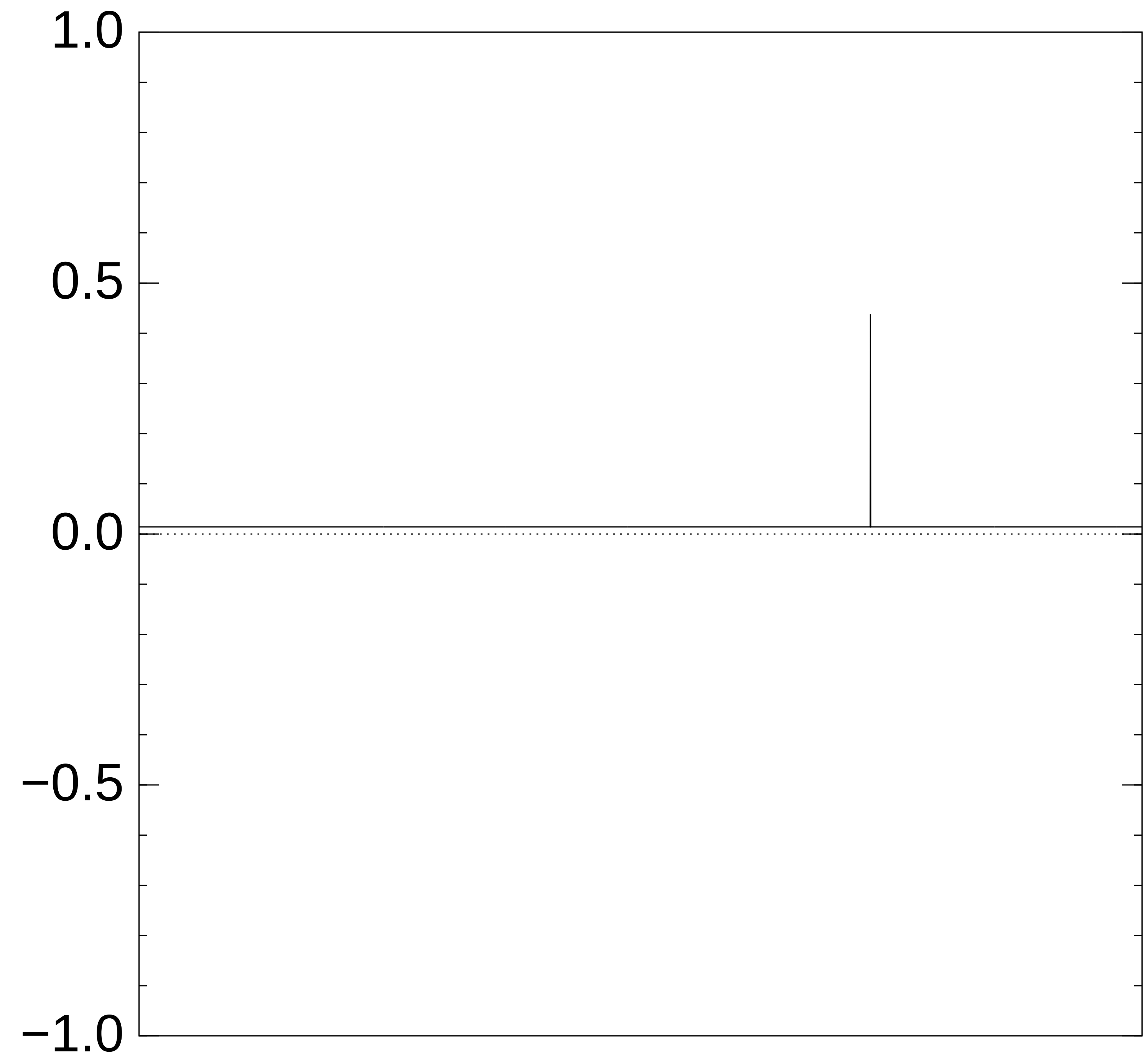
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $14 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

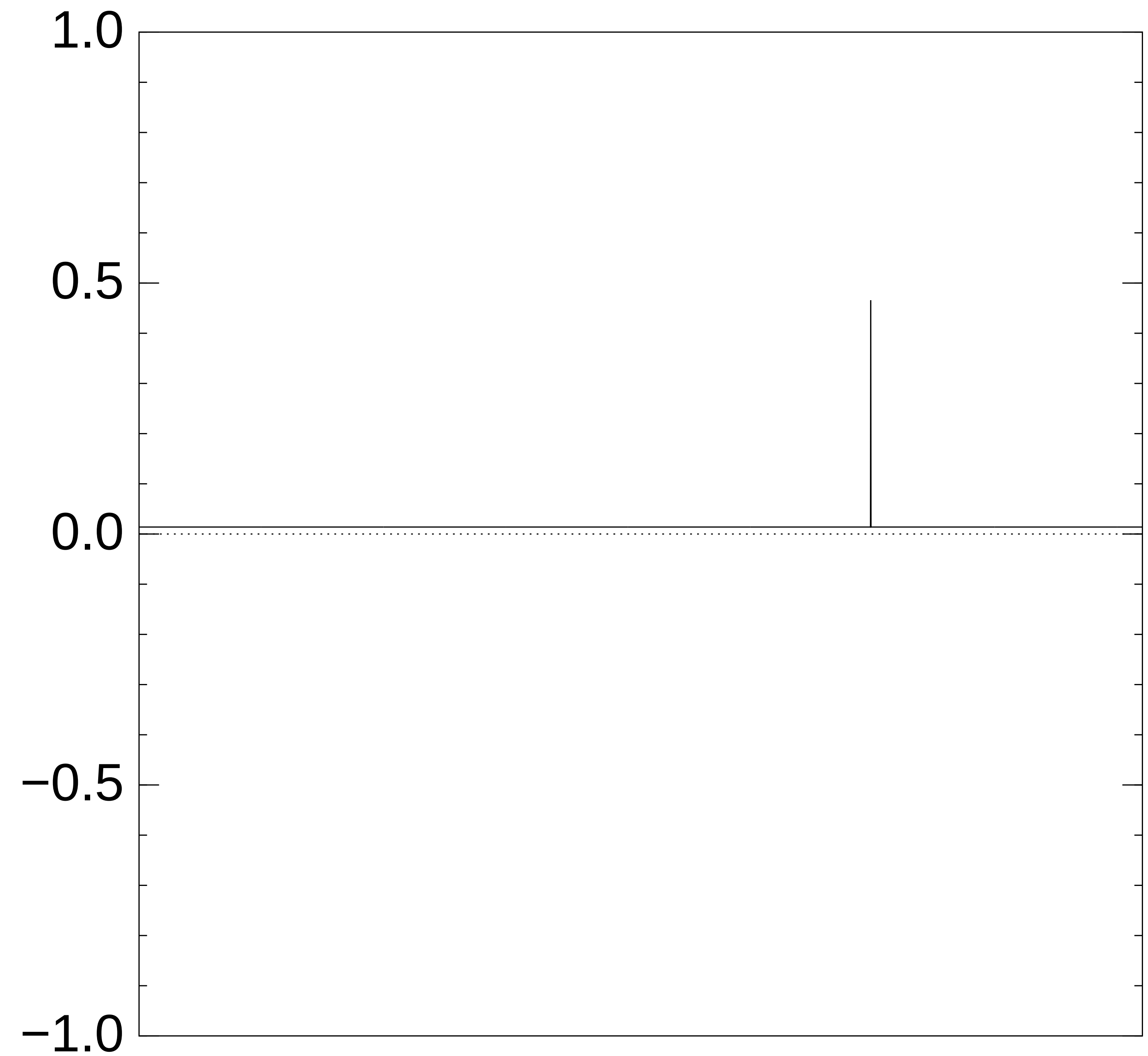
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $15 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

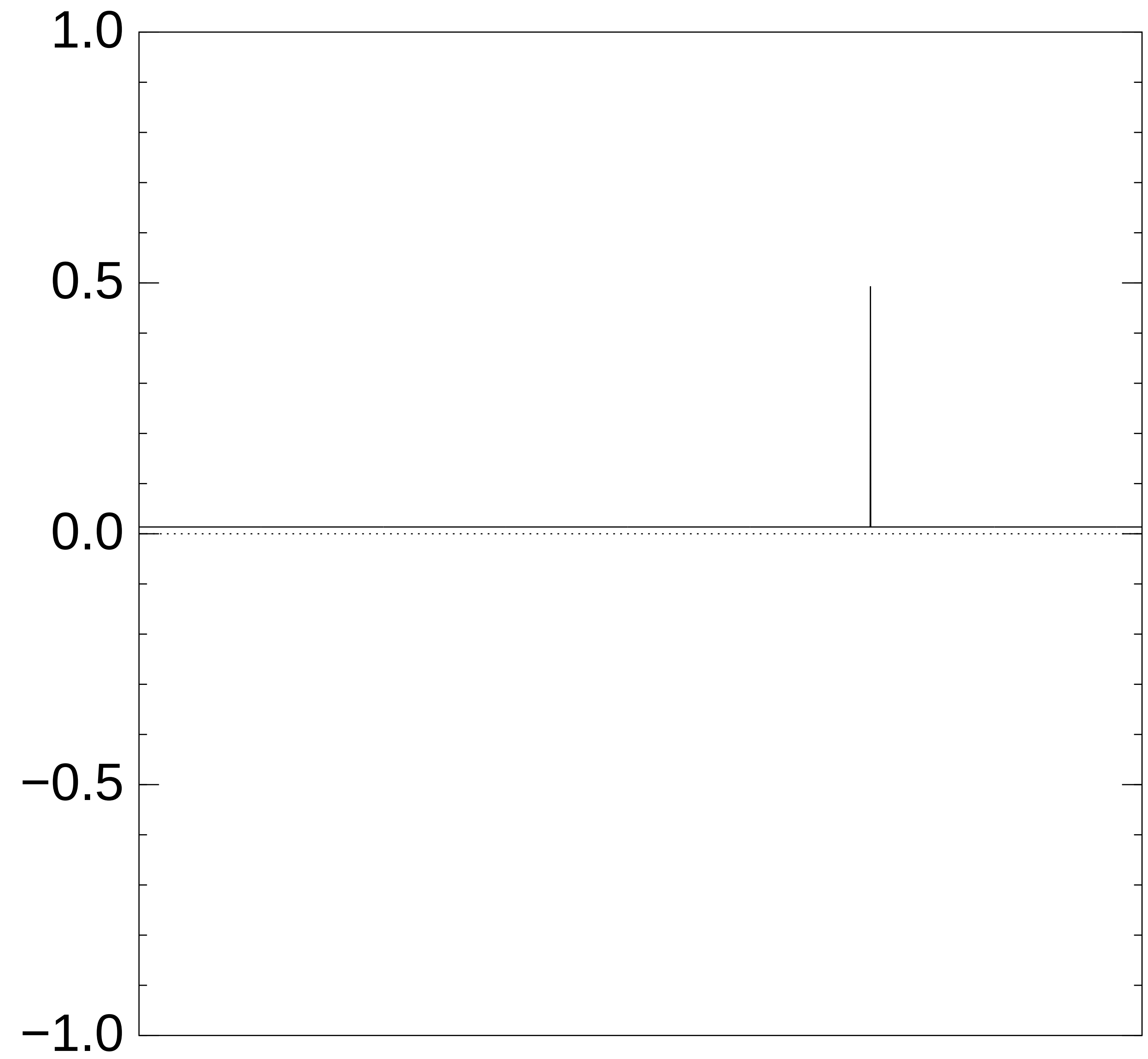
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $16 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

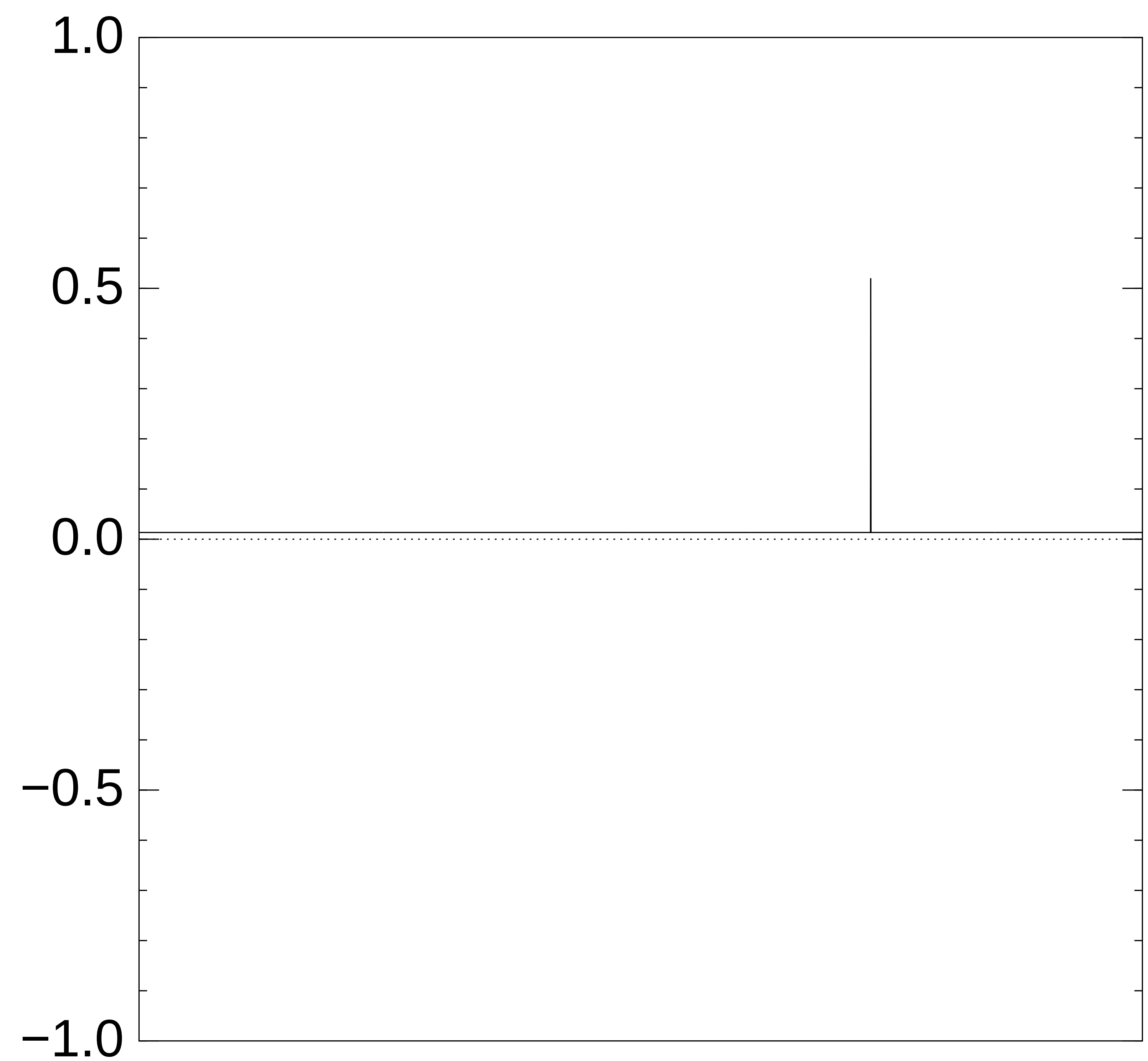
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $17 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

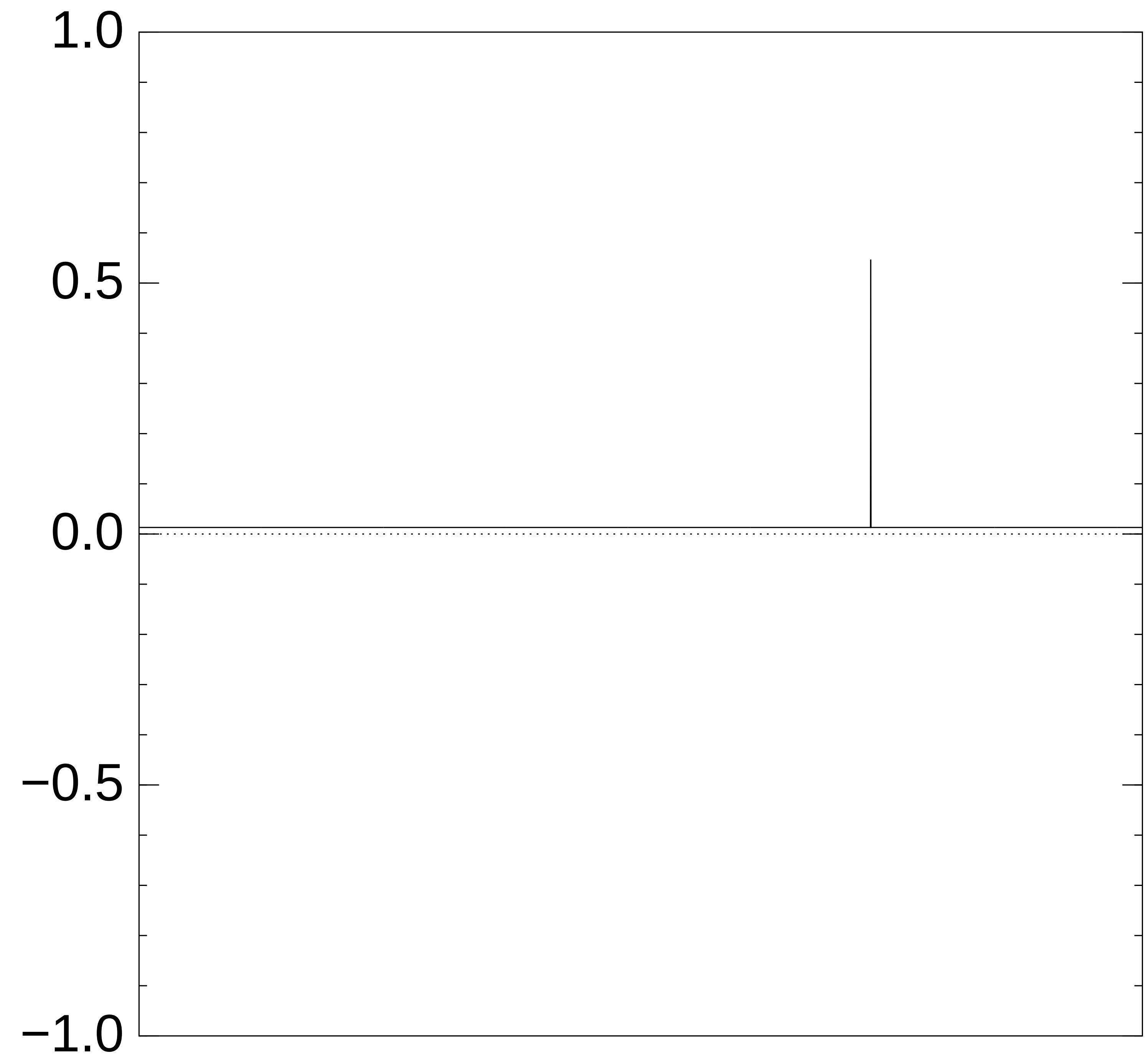
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $18 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

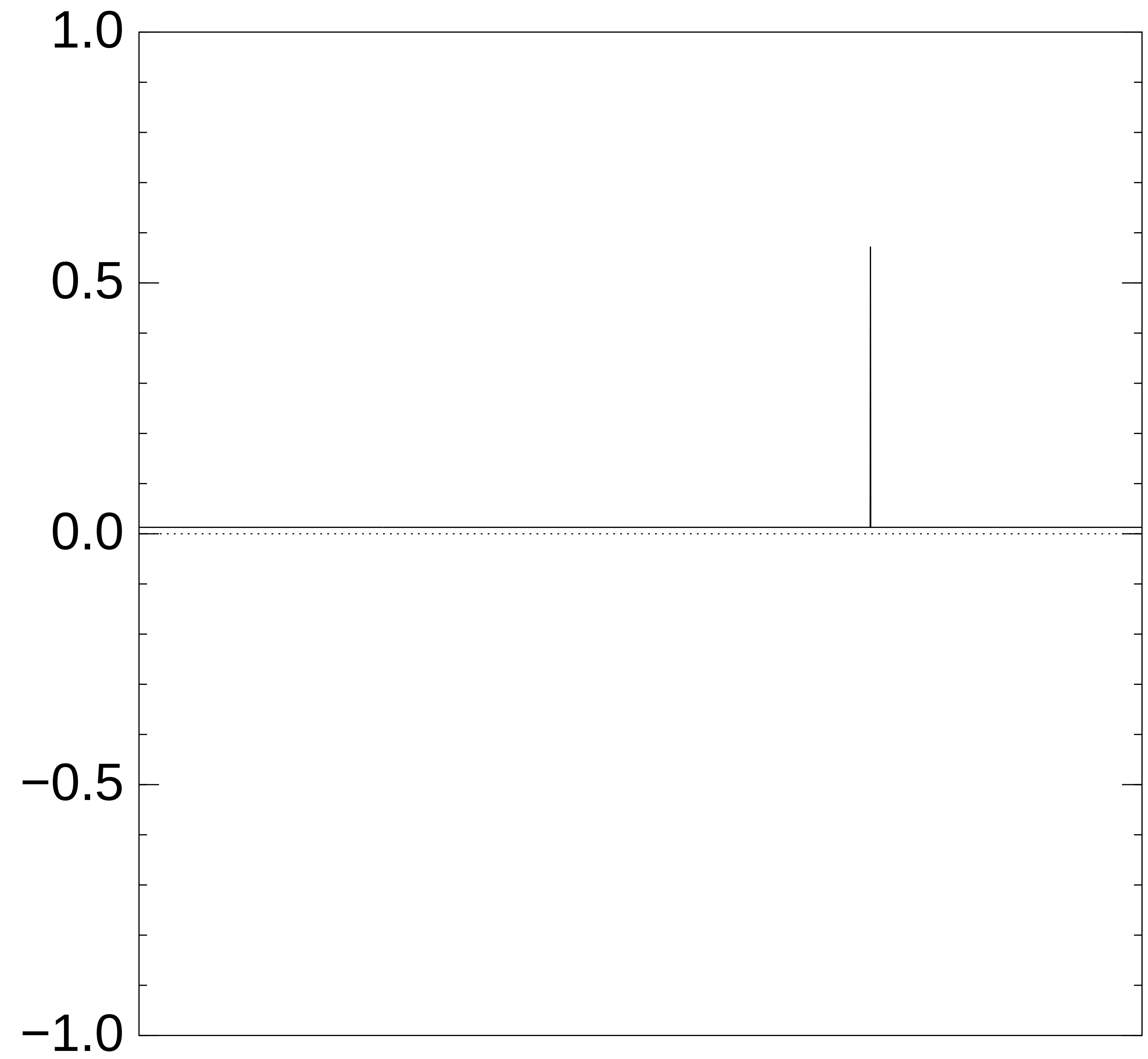
Step 1: Set $a \leftarrow b$ where $b_u = -a_u$ if $f(u) = 0$, $b_u = a_u$ otherwise. This is fast if f is fast.

Step 2: "Grover diffusion". Negate a around its average. This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits. With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $19 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

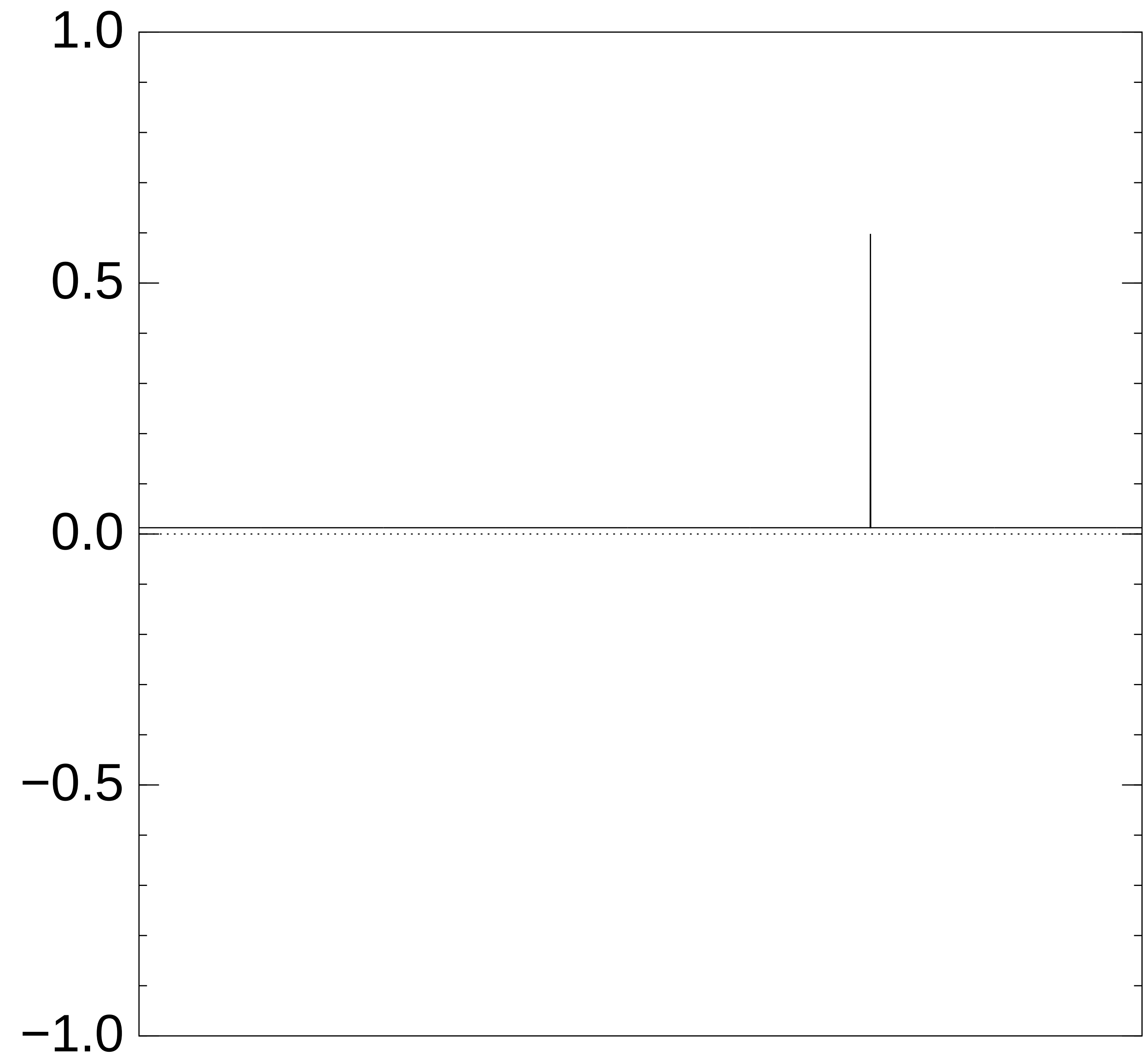
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $20 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

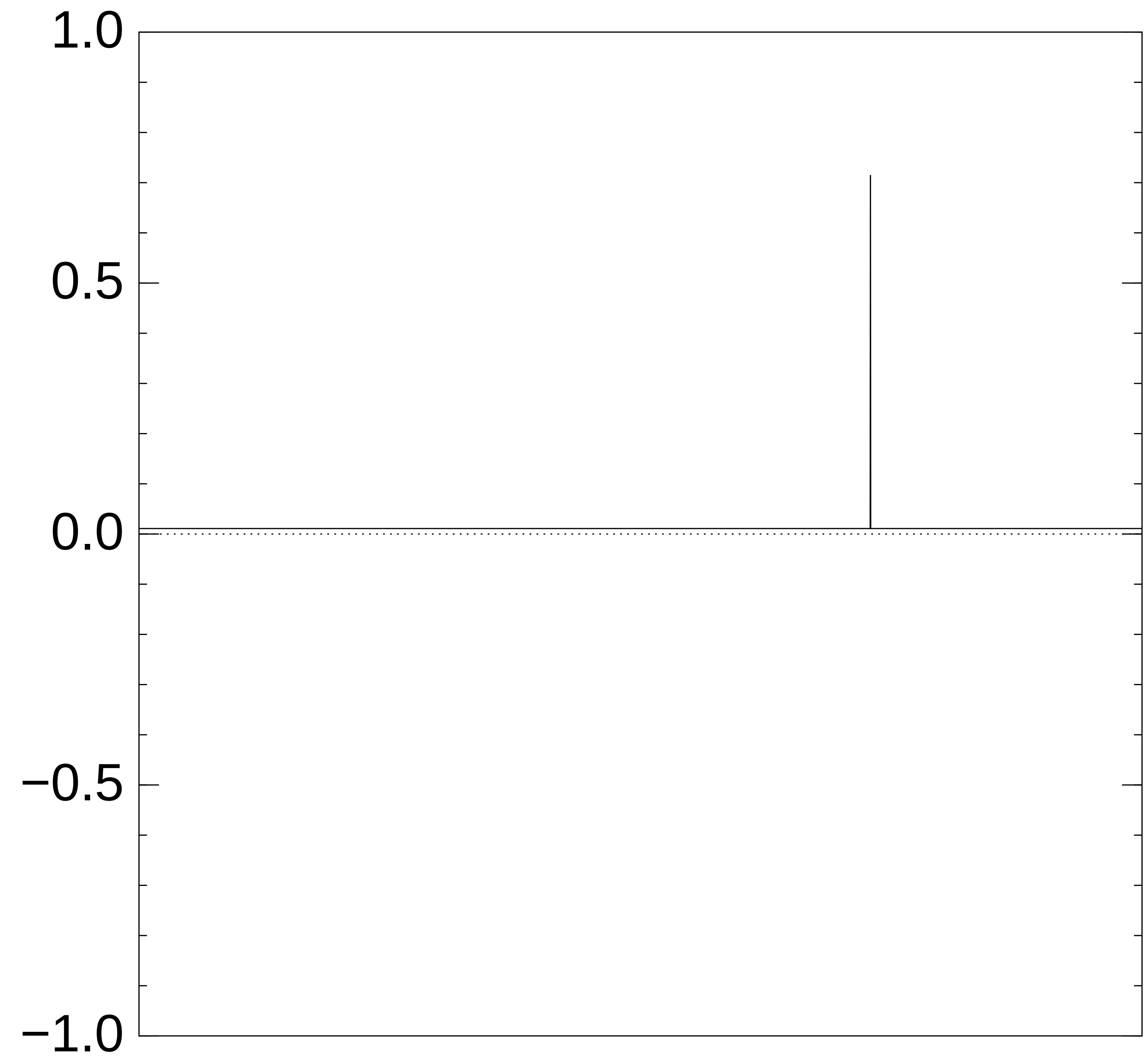
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $25 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

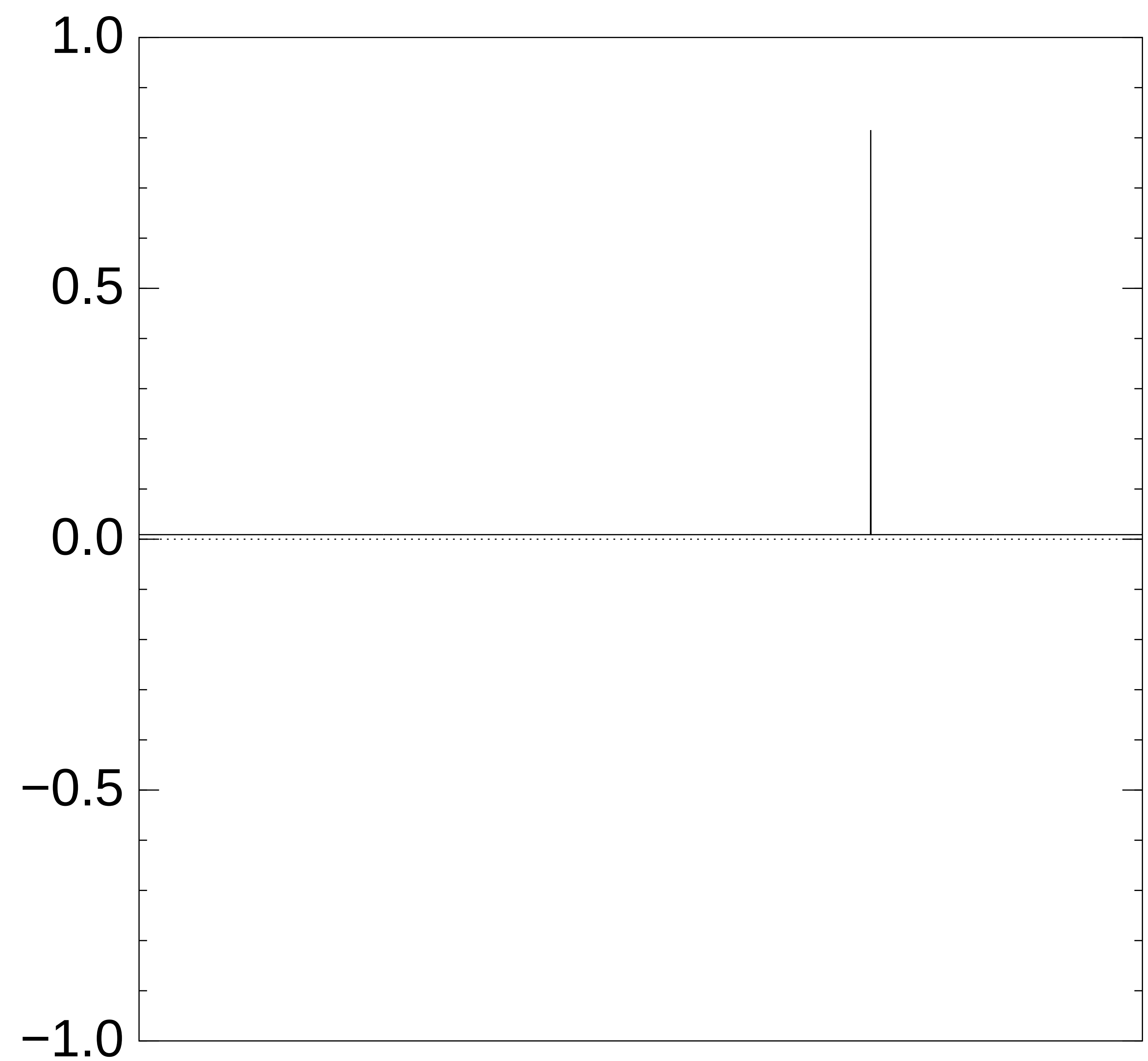
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $30 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

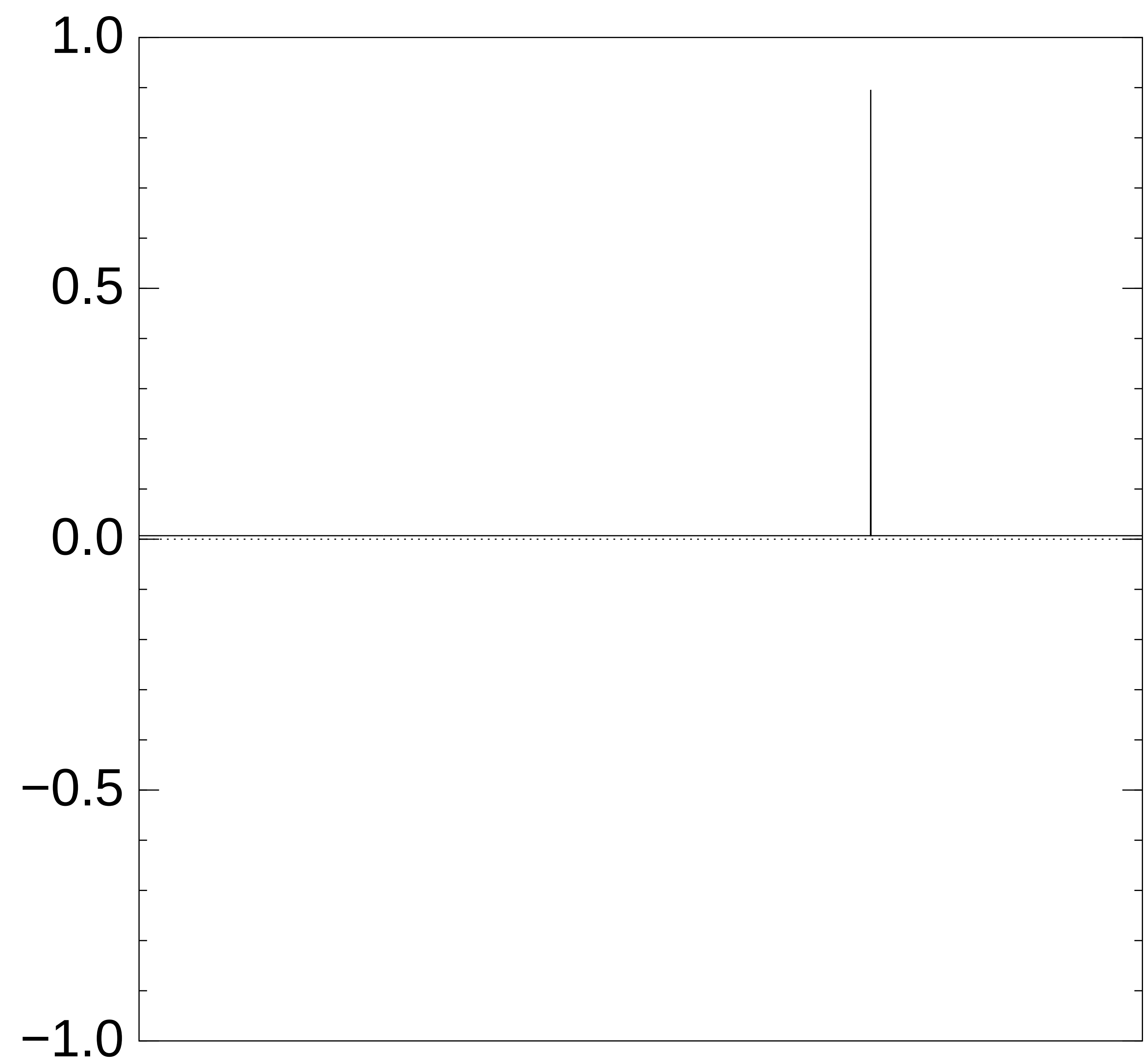
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $35 \times$ (Step 1 + Step 2):



Good moment to stop, measure.

Start from uniform superposition over n-bit strings u: each $a_u = 1$.

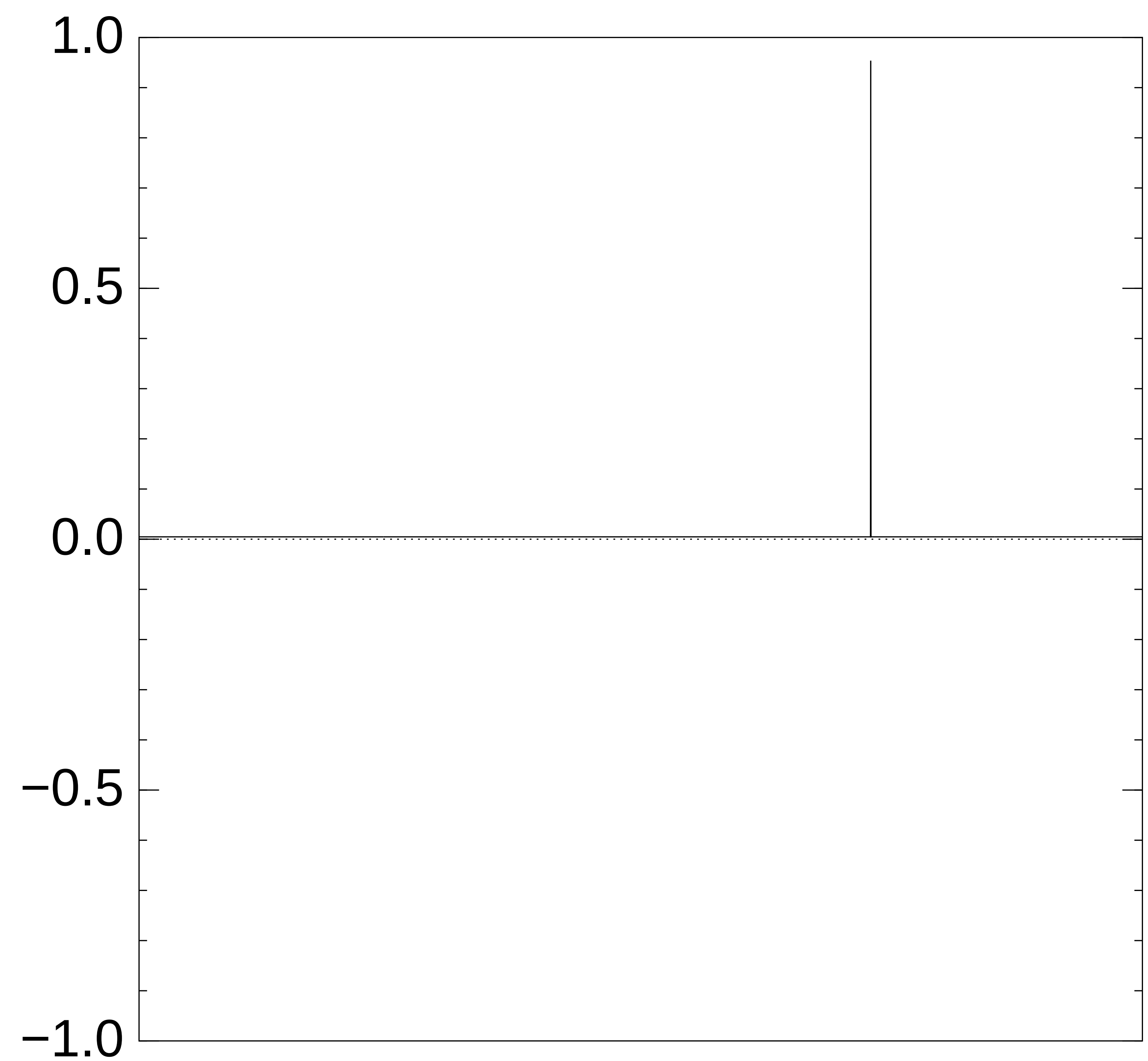
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $40 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

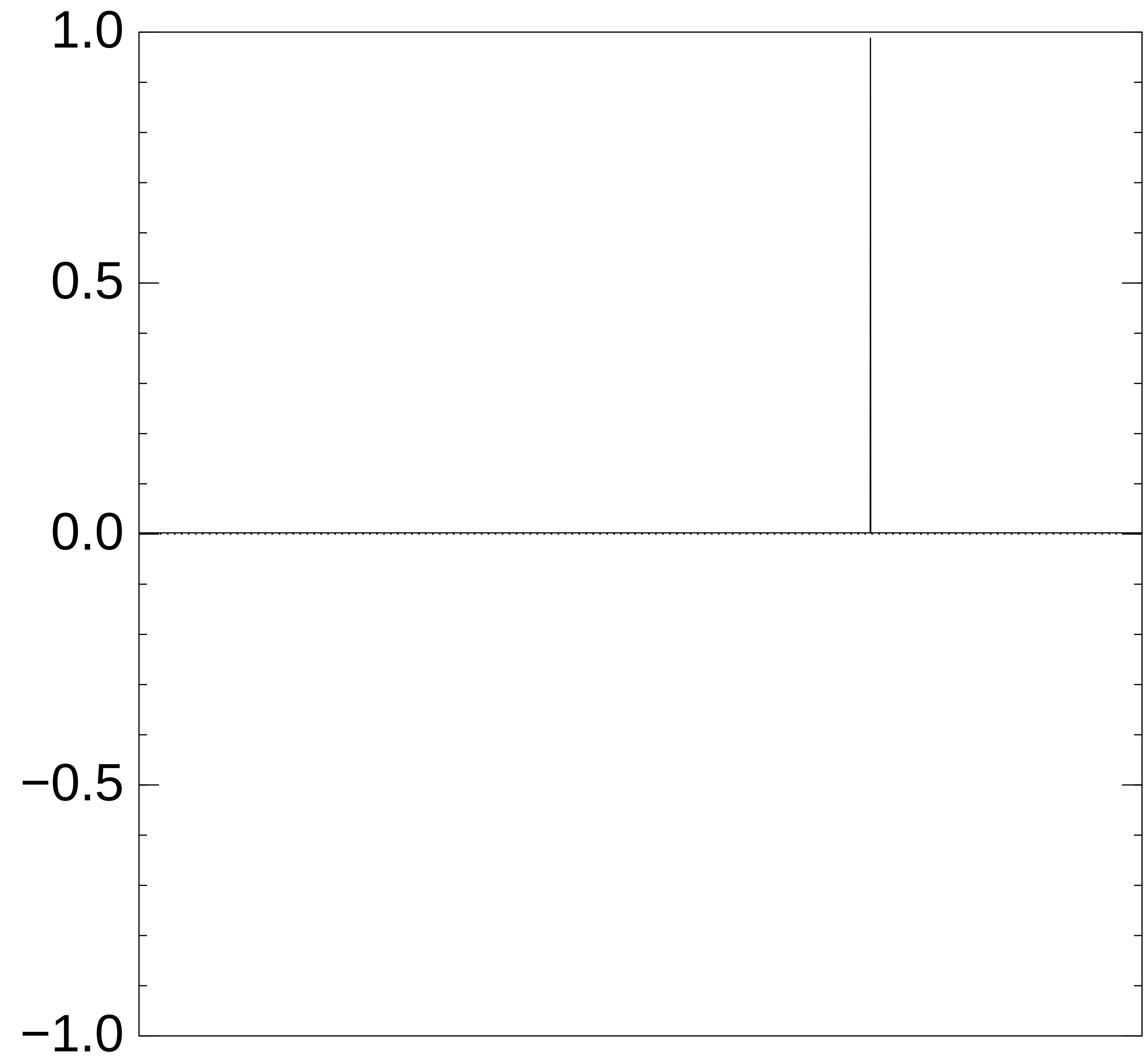
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $45 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

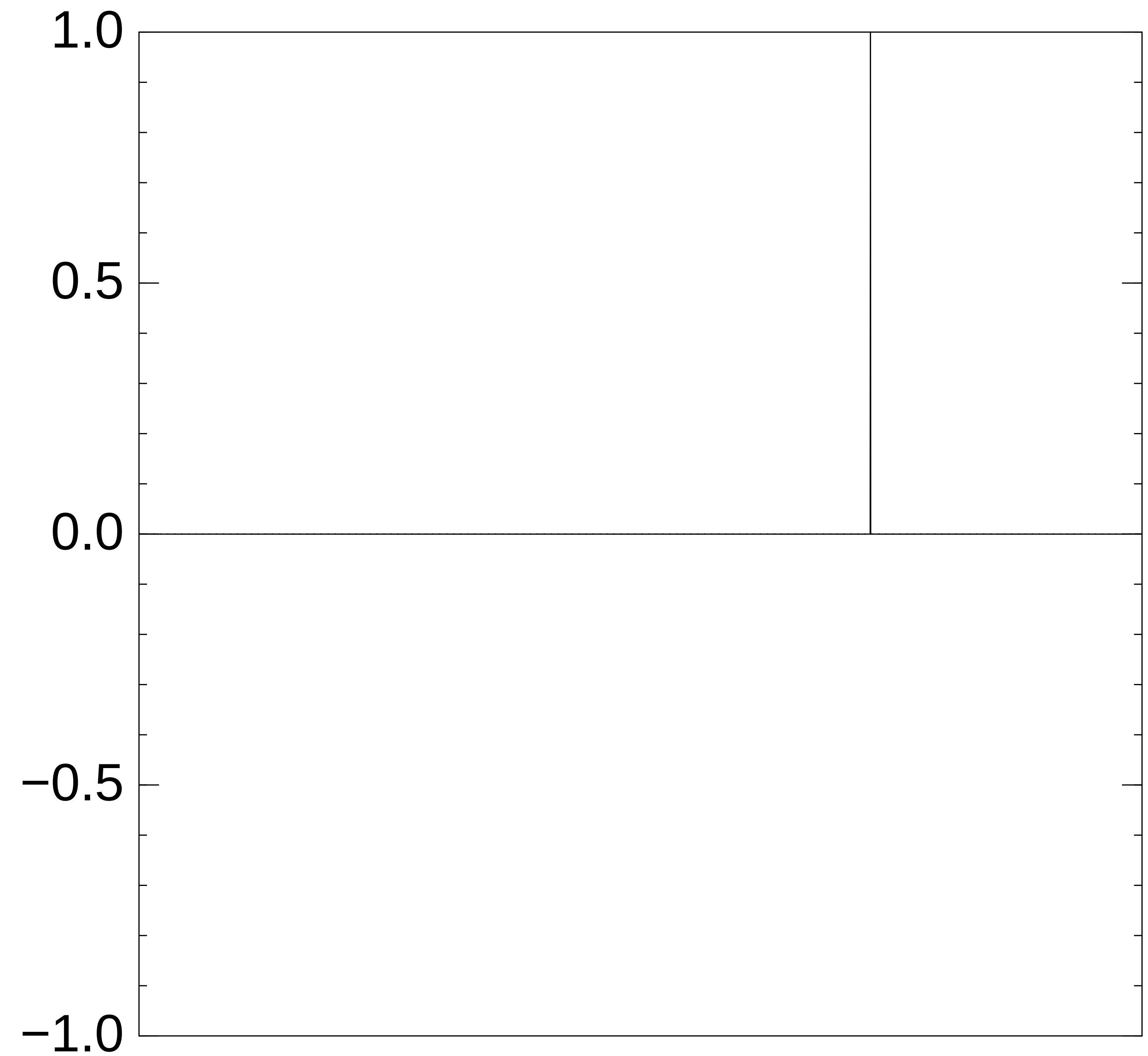
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $50 \times$ (Step 1 + Step 2):



Traditional stopping point.

Start from uniform superposition over n-bit strings u: each $a_u = 1$.

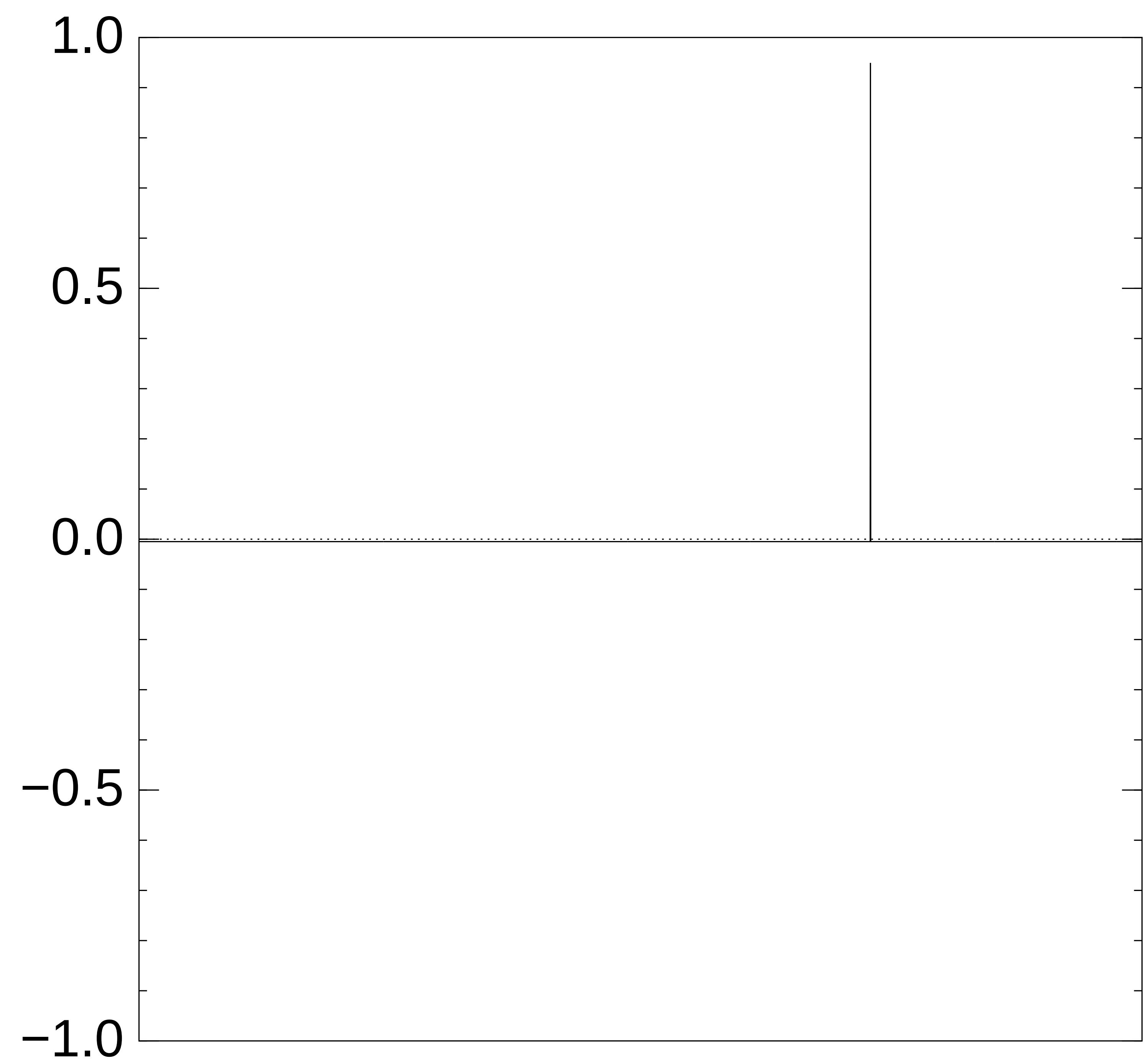
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $60 \times (\text{Step 1} + \text{Step 2})$:



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

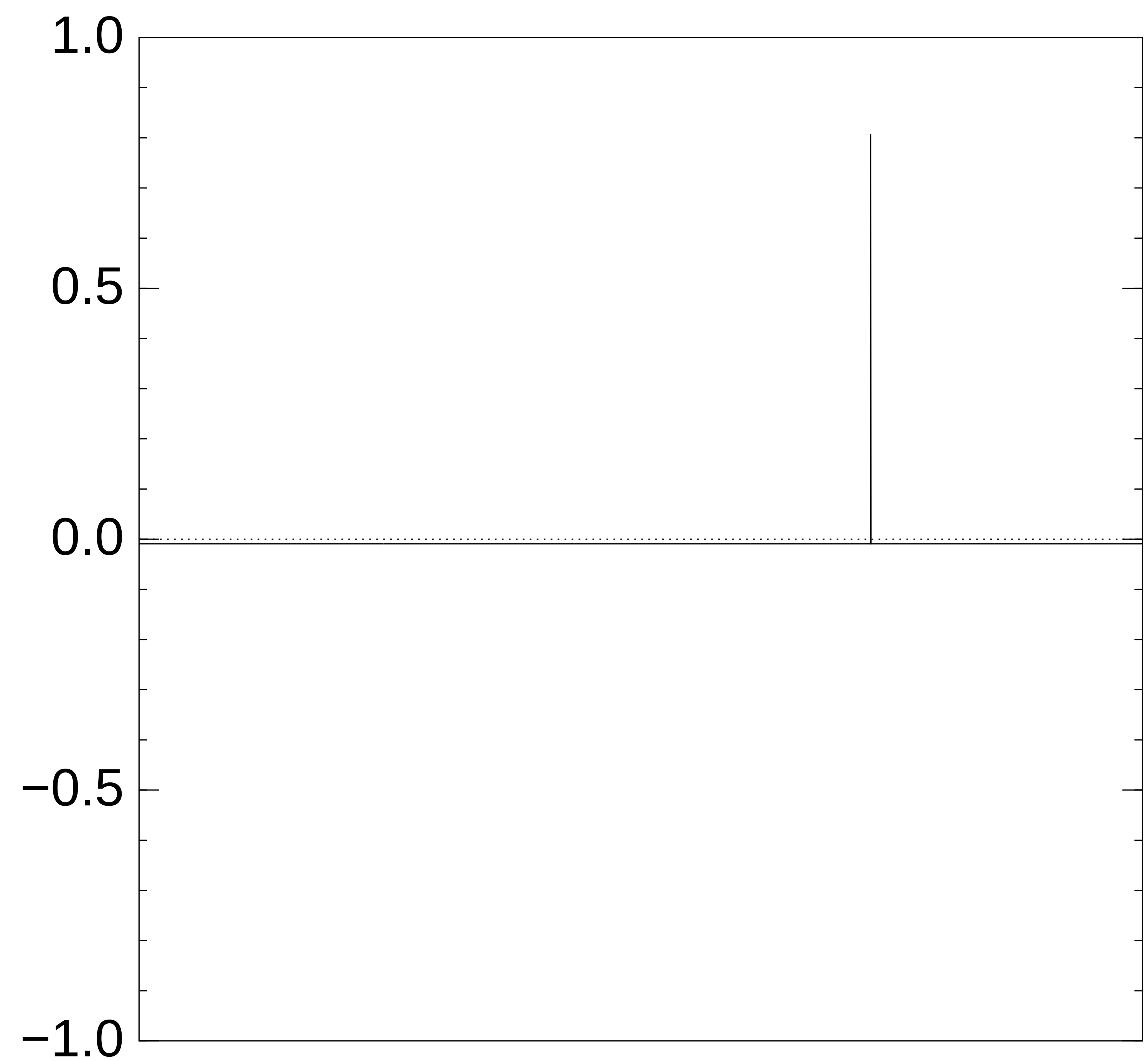
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $70 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

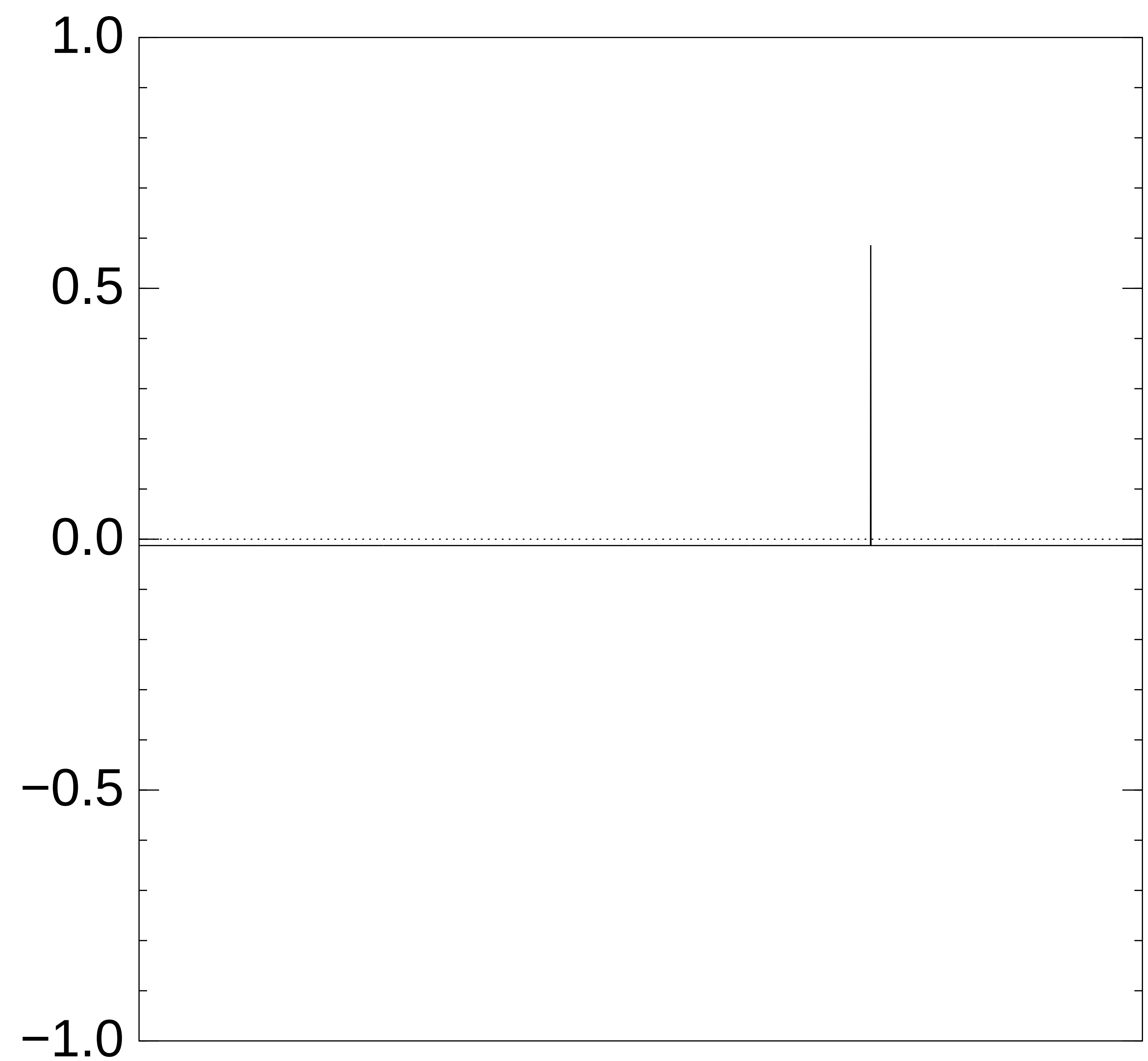
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $80 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

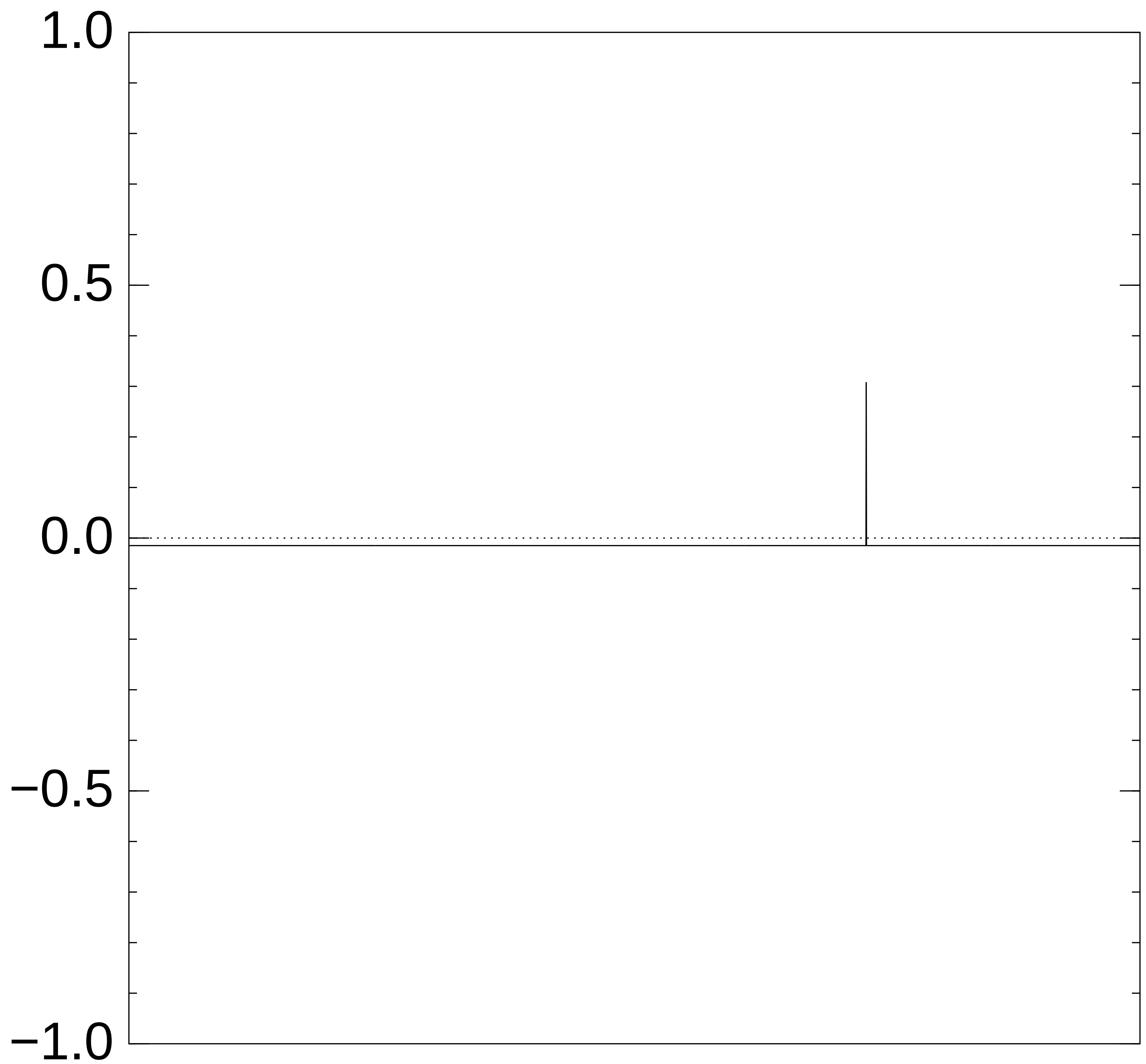
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $90 \times$ (Step 1 + Step 2):



Start from uniform superposition over n-bit strings u: each $a_u = 1$.

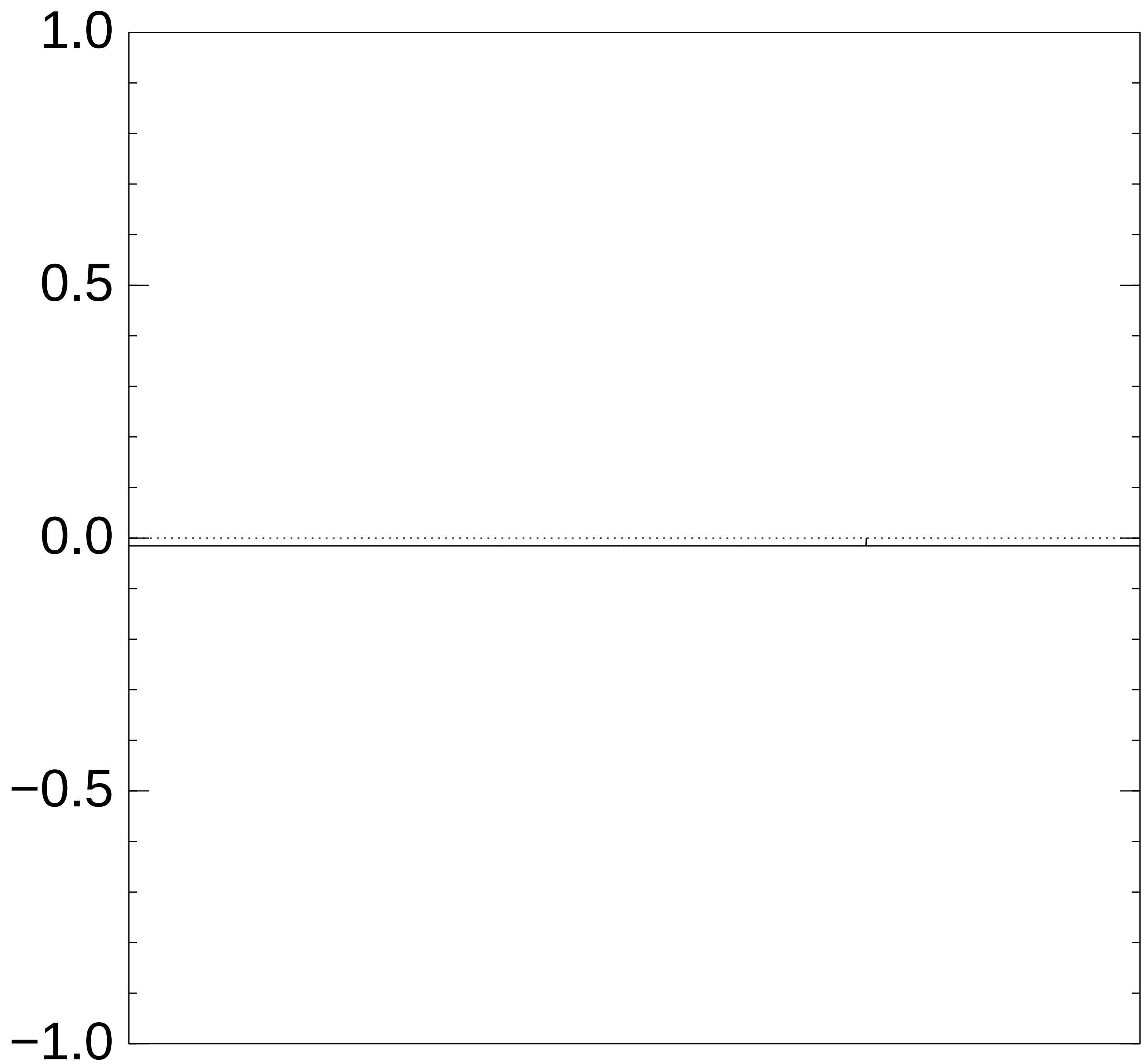
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast if f is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $100 \times$ (Step 1 + Step 2):



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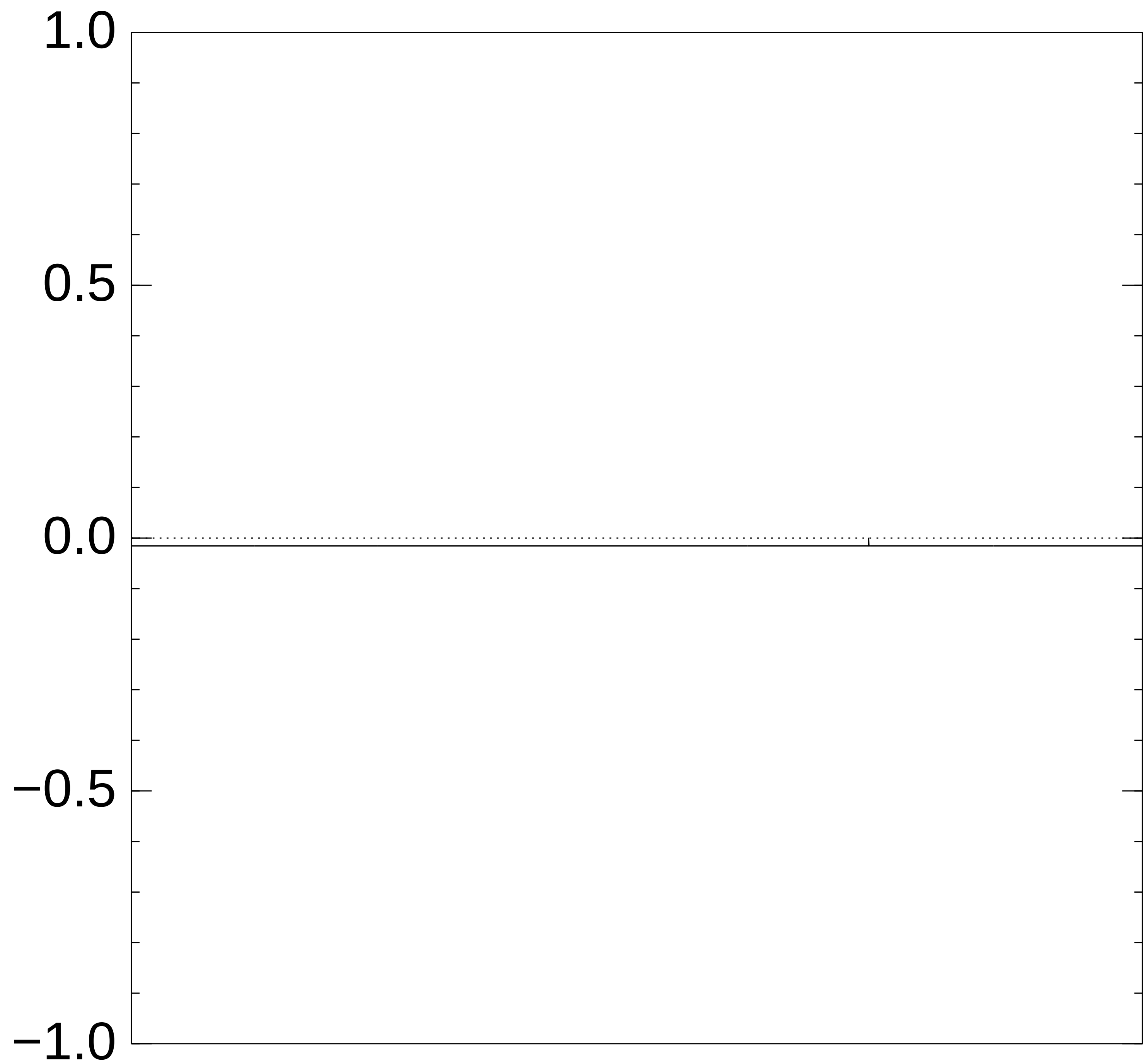

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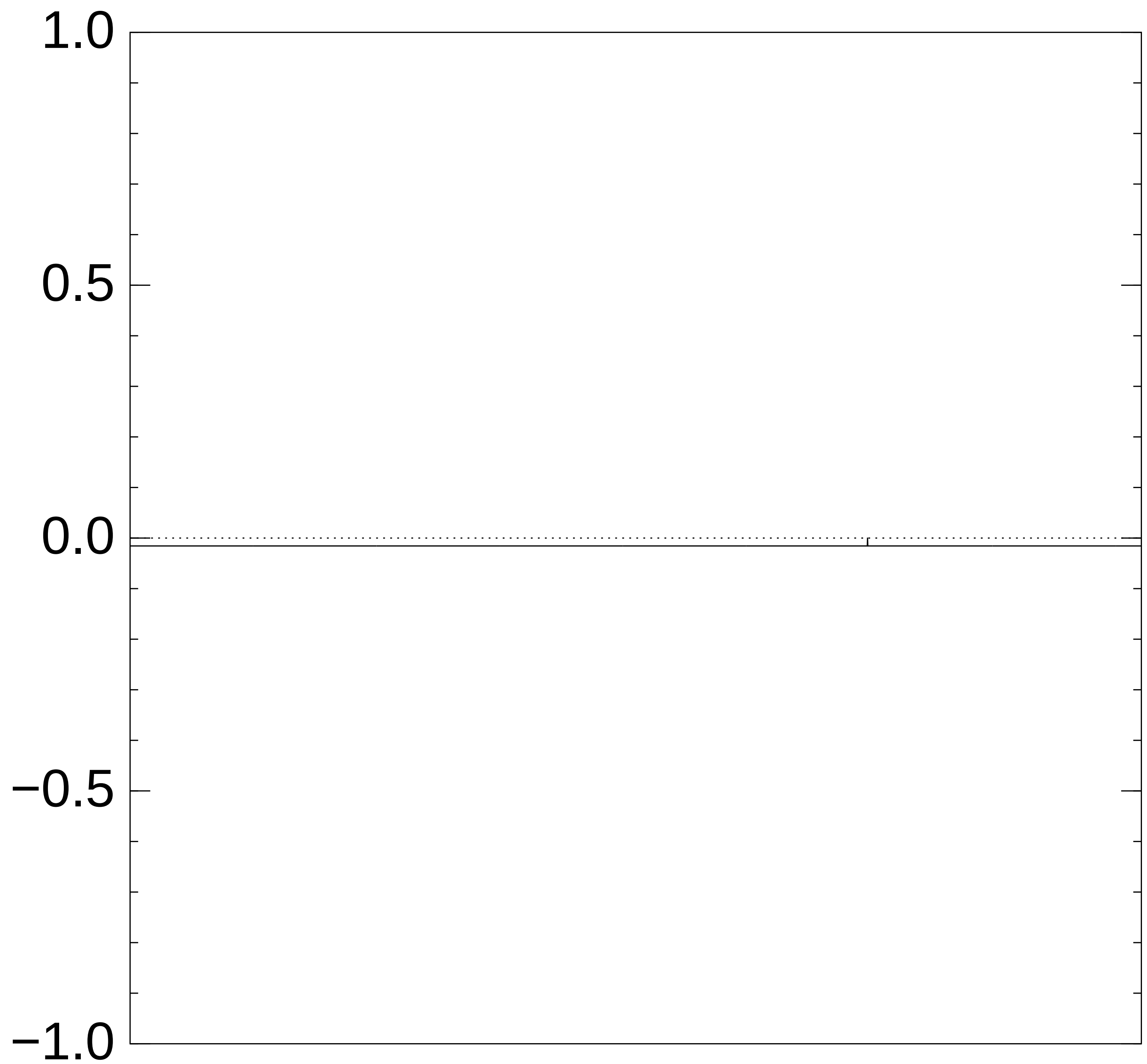

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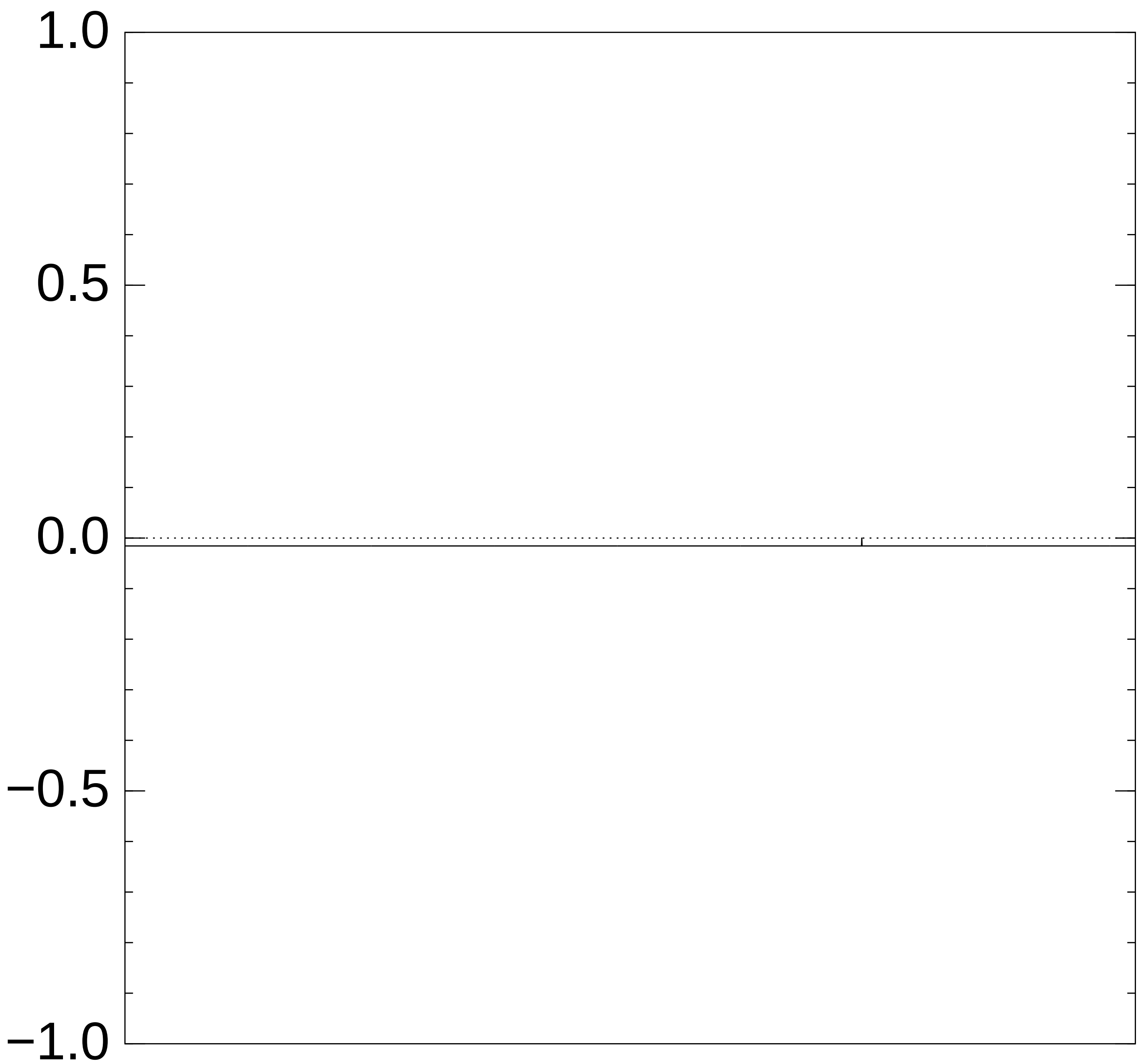

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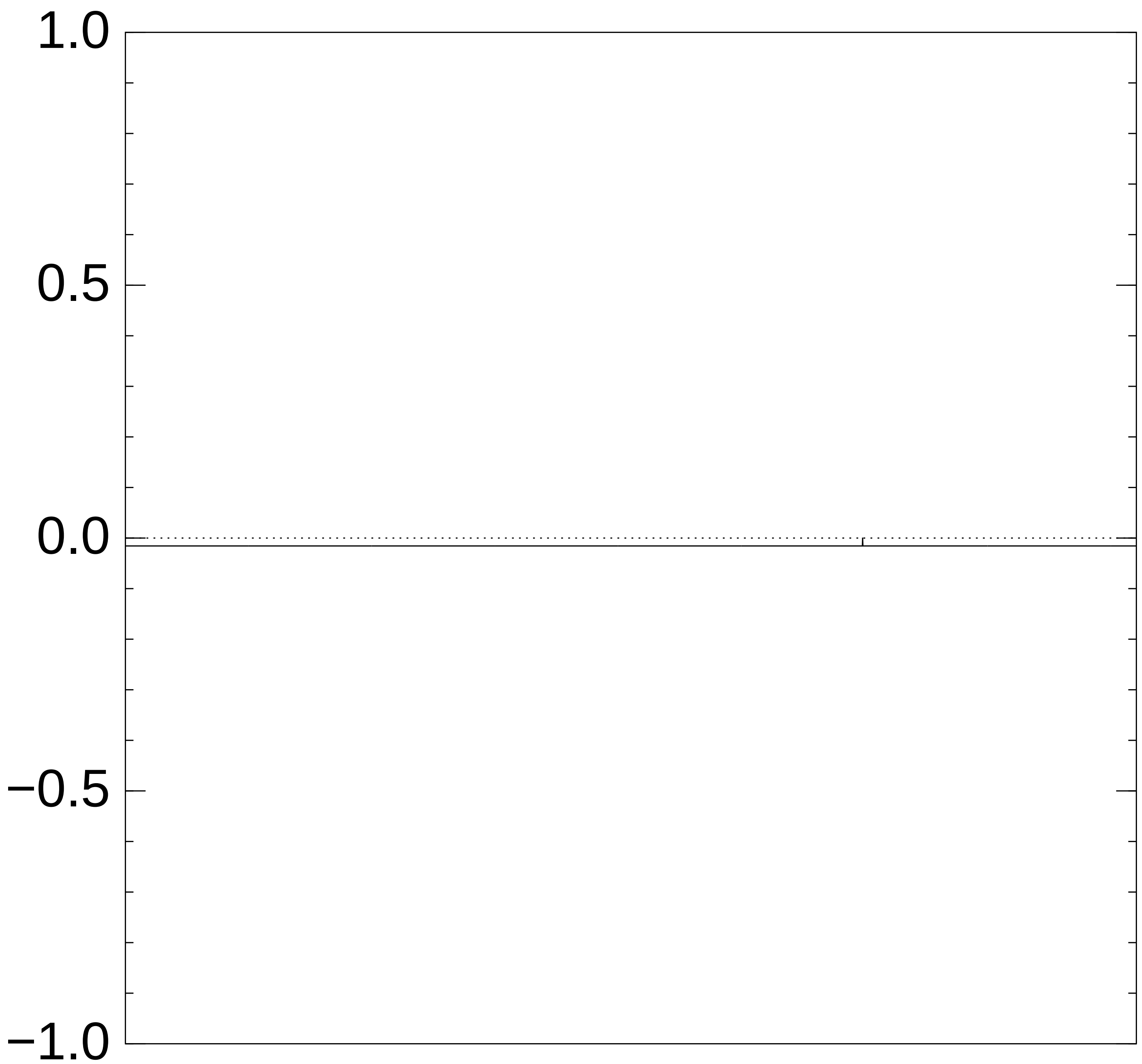

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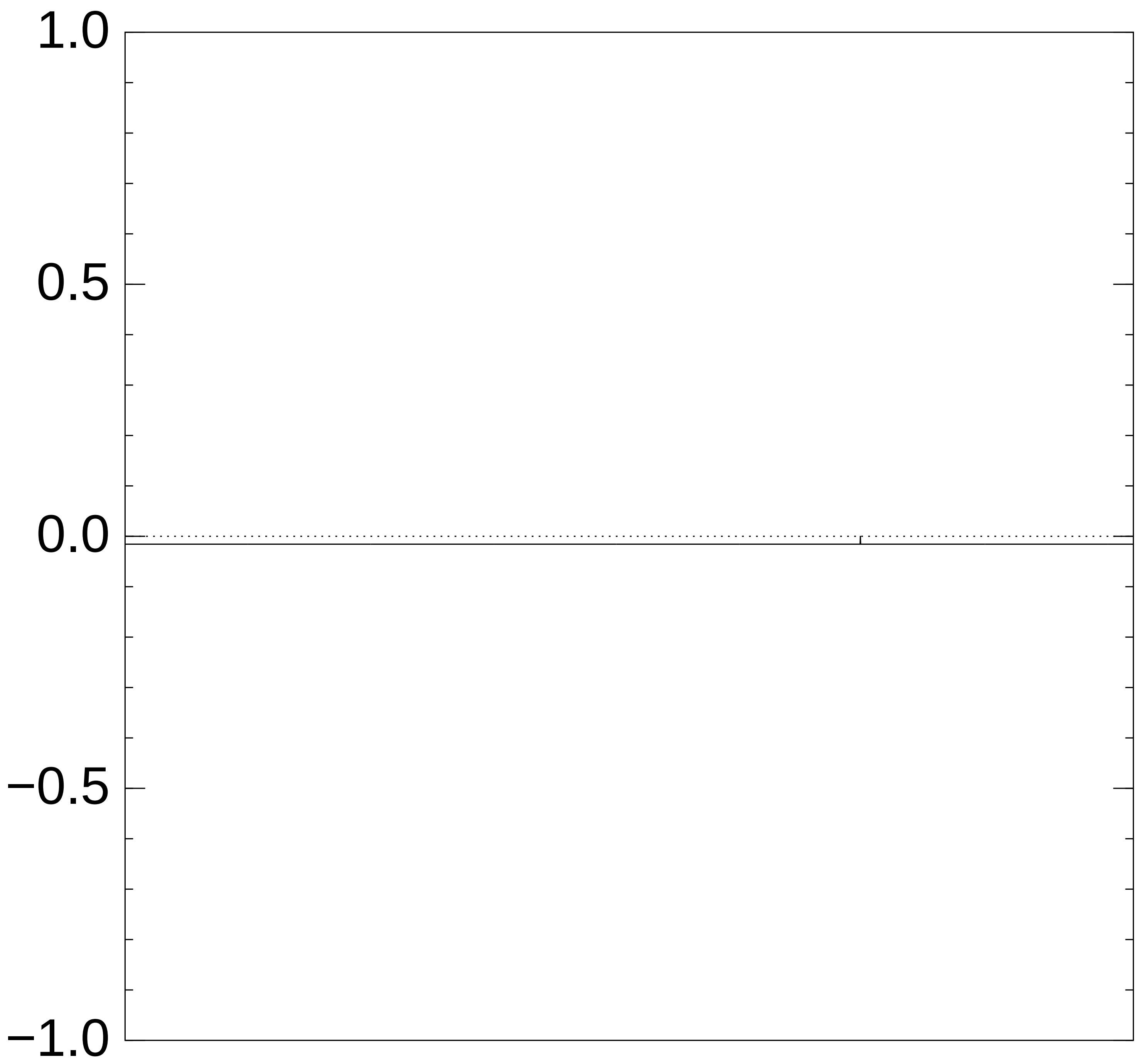

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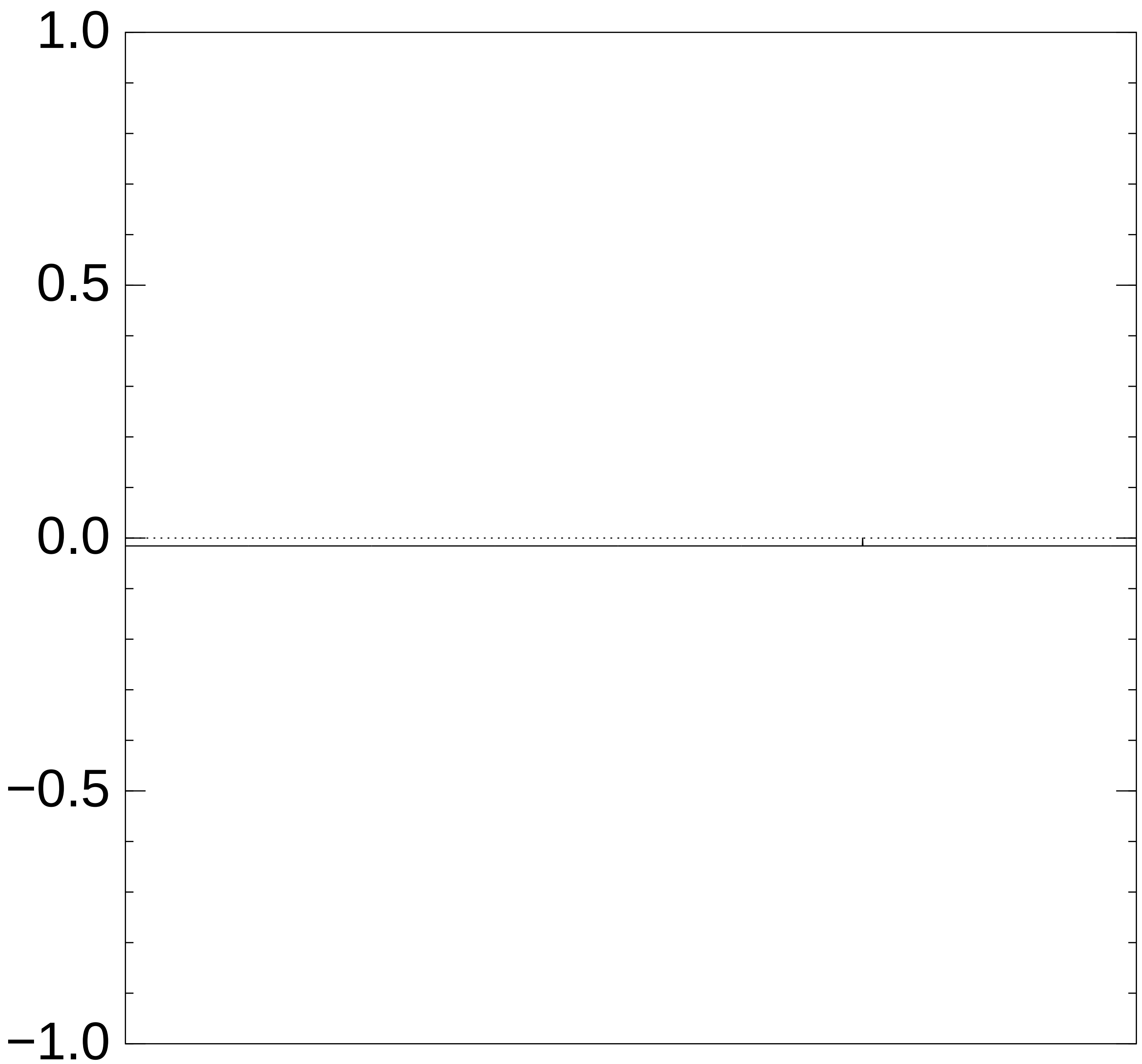

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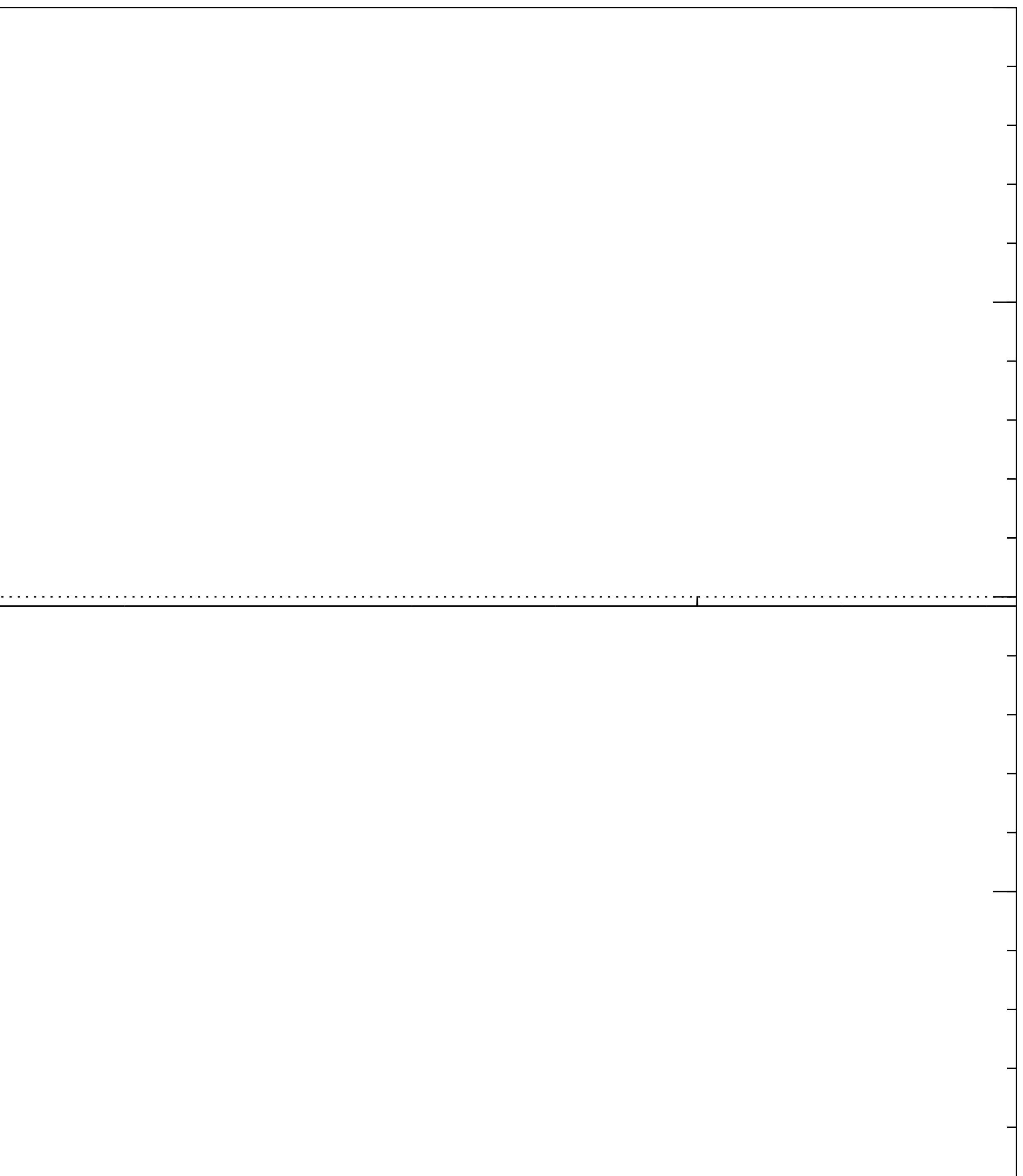

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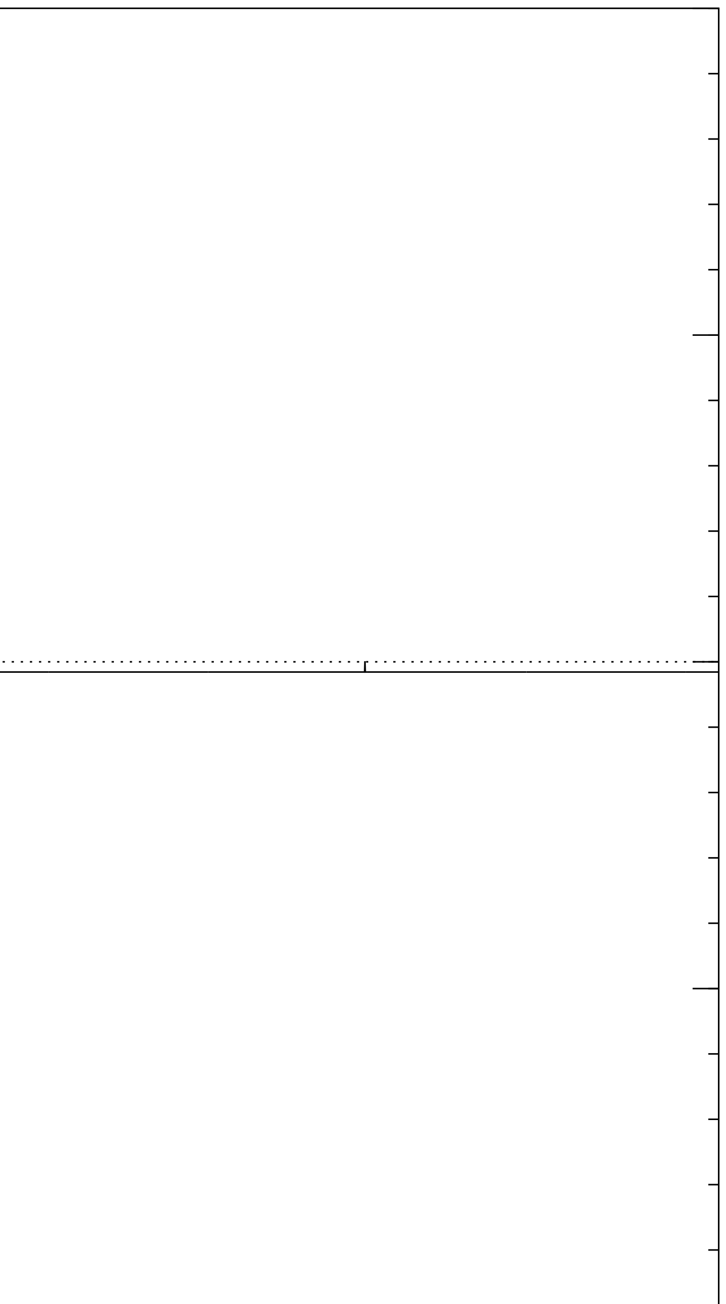

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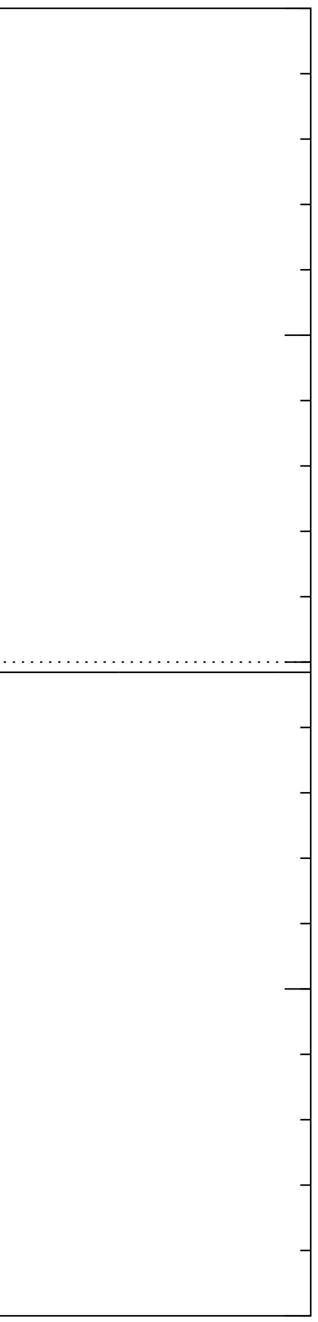

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Replace u by random neighb
repeat enough times for mix
check if good; keep repeatin

Quantum walk: (repetitions

What if there are many "good" values of f, not just value 0?

Can modify algorithm: instead of negating when $f(q) = 0$, negate when $g(f(q)) = 0$.

Or simply apply original Grover to the composition $q \mapsto g(f(q))$.

What if one doesn't know R?

Can modify algorithm. Or repeat original algorithm with sequence of guesses for R, starting with 2^n and decreasing exponentially. Approximation of R suffices.

More interesting generalization: quantum walks. Seems more powerful than original Grover.

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Ambainis distinctness algorithm.

Say f has 2^n inputs, exactly one collision $\{p, q\}$.
"Collision": $p \neq q$; $f(p) = f(q)$.
Problem: find this collision.

Generic non-quantum algorithm: nearly 2^n calls to f.
Ambainis, using quantum walk: $\approx 2^{2n/3}$ calls to f.

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The impact on cry

2021.12: A Firefox
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2021.12: A Firefox connection to `https://google.com` is encrypted and authenticated by AES-128-GCM, using a key exchanged by the X25519 ECDH system, with the key exchange signed by an ECDSA-NIST-P-256 key, with the signing key certified by an RSA-2048 key, which in turn is certified by an RSA-4096 key, which is trusted by Firefox. SHA-256, SHA-384 also appear.

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SHA-256, SHA-384 also appear.

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Similar cost to 2^{55} bit operations.
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2^{64} nanoseconds $=$ 585 years,
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To run $2^{10}\times$ faster,
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Latest developments:
see recent talk on S-unit att
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3. Kuperberg applications and optimizations. Interesting example: attacking CRS/CSIDH, isogeny-based systems for small non-interactive key exchange.

(This subexponential CRS attack prompted the development of SIKE. SIKE is smaller than CRS/CSIDH for *sufficiently large* security levels against known attacks, but cutoff is unclear. SIKE also opens up new attack avenues and doesn't provide non-interactive key exchange.)

4. Shor applications. Interesting example: discrete logarithms in groups related to number fields, combined with further techniques, led to a polynomial-time attack breaking usual "cyclotomic $h^+ = 1$" case of Gentry STOC 2009 "Fully homomorphic encryption using ideal lattices" and some newer lattice-based cryptosystems.

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7. Changing cryptosystems to enable attacks: e.g. "Please use your secret key on a quantum computer to decrypt the following superposition of ciphertexts."