Quantum algorithms

Daniel J. Bernstein

"Quantum algorithm"
means an algorithm that
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i.e. a sequence of instructions,
where each instruction is
in a quantum computer's
supported instruction set.

How do we know which instructions a quantum computer will support?

Quantum computer type 1 (QC1):
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"NOT gate", "Hadamard gate",
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Data stored in 4 qubits: a list of 16 numbers, not all zero. e.g.:

$(3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3)$.

The state of a computer

Data ("state") stored in 3 bits:

a list of 3 elements of $\{0, 1\}$.

e.g.: $(0, 0, 0)$.

e.g.: $(1, 1, 1)$.

e.g.: $(0, 1, 1)$.

Data stored in 64 bits:

a list of 64 elements of $\{0, 1\}$.

e.g.: $(1, 1, 1, 1, 1, 0, 0, 0, 1,$
$0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0,$
$0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1,$
$1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1,$
$0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0,$
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The state of a quantum computer

Data stored in 3 qubits:

a list of 8 numbers, not all zero.

e.g.: $(3, 1, 4, 1, 5, 9, 2, 6)$.

e.g.: $(-2, 7, -1, 8, 1, -8, -2, 8)$.

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Measuring n qubits

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"Quantum RNG."

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e.g.: Say 3 qubits have state

$(0, 0, 0, 0, 0, 1, 0, 0)$.

Measurement produces

$000 = 0$ with probability 0;

$001 = 1$ with probability 0;

$010 = 2$ with probability 0;

$011 = 3$ with probability 0;

$100 = 4$ with probability 0;

$101 = 5$ with probability 1;

$110 = 6$ with probability 0;

$111 = 7$ with probability 0.

5 is guaranteed outcome.

NOT gates

NOT_0 gate on 3 q

$(3, 1, 4, 1, 5, 9, 2, 6$

$(1, 3, 1, 4, 9, 5, 6, 2$

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NOT_0 gate on 4 qubits:
$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto (1,3,1,4,9,5,6,2,3,5,8,5,7,9,3,9)$.

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Measurement produces
$000 = 0$ with probability 0;
$001 = 1$ with probability 0;
$010 = 2$ with probability 0;
$011 = 3$ with probability 0;
$100 = 4$ with probability 0;
$101 = 5$ with probability 1;
$110 = 6$ with probability 0;
$111 = 7$ with probability 0.

5 is guaranteed outcome.

<u>NOT gates</u>

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NOT_1 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(4, 1, 3, 1, 2, 6, 5, 9)$.

e.g.: Say 3 qubits have state
$(0, 0, 0, 0, 0, 1, 0, 0)$.

Measurement produces
$000 = 0$ with probability 0;
$001 = 1$ with probability 0;
$010 = 2$ with probability 0;
$011 = 3$ with probability 0;
$100 = 4$ with probability 0;
$101 = 5$ with probability 1;
$110 = 6$ with probability 0;
$111 = 7$ with probability 0.

5 is guaranteed outcome.

NOT gates

NOT_0 gate on 3 qubits:
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NOT_1 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(4, 1, 3, 1, 2, 6, 5, 9)$.

NOT_2 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(5, 9, 2, 6, 3, 1, 4, 1)$.

y 3 qubits have state
0, 0, 1, 0, 0).

ement produces
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with probability 0;
with probability 0;
with probability 0;
with probability 0;
with probability 1;
with probability 0;
with probability 0.

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NOT gates

NOT_0 gate on 3 qubits:
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$(1, 3, 1, 4, 9, 5, 6, 2)$.

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$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
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$(5, 9, 2, 6, 3, 1, 4, 1)$.

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NOT gates

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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(5, 9, 2, 6, 3, 1, 4, 1)$.

state

(1, 0, 0, 0, 0, 0, 0, 0

(0, 1, 0, 0, 0, 0, 0, 0

(0, 0, 1, 0, 0, 0, 0, 0

(0, 0, 0, 1, 0, 0, 0, 0

(0, 0, 0, 0, 1, 0, 0, 0

(0, 0, 0, 0, 0, 1, 0, 0

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Operation on quan
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NOT gates

NOT_0 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(5, 9, 2, 6, 3, 1, 4, 1)$.

state	measure
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state
NOT_0, swapping pairs.
Operation after measuremen
flipping bit 0 of result.
Flip: output is not input.

NOT gates

NOT_0 gate on 3 qubits:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(1, 3, 1, 4, 9, 5, 6, 2)$.

NOT_0 gate on 4 qubits:
$(3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$
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$(4, 1, 3, 1, 2, 6, 5, 9)$.

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$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(5, 9, 2, 6, 3, 1, 4, 1)$.

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

tes

ate on 3 qubits:

$1, 5, 9, 2, 6) \mapsto$

$4, 9, 5, 6, 2)$.

ate on 4 qubits:

$5,9,2,6,5,3,5,8,9,7,9,3) \mapsto$

$9,5,6,2,3,5,8,5,7,9,3,9)$.

ate on 3 qubits:

$1, 5, 9, 2, 6) \mapsto$

$1, 2, 6, 5, 9)$.

ate on 3 qubits:

$1, 5, 9, 2, 6) \mapsto$

$6, 3, 1, 4, 1)$.

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controll

e.g. $\mathrm{C}_1\mathrm{N}$

$(3, 1, 4,$

$(3, 1, 1,$

ubits:

) $\mapsto$

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ubits:

3,5,8,9,7,9,3) $\mapsto$

5,8,5,7,9,3,9).

ubits:

) $\mapsto$

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ubits:

) $\mapsto$

).

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (C

e.g. C_1NOT_0:
$(3,1,4,1,5,9,2,6$
$(3,1,1,4,5,9,6,2$

$,3) \mapsto$
$,9).$

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state: NOT_0, swapping pairs.

Operation after measurement: flipping bit 0 of result.

Flip: output is not input.

<u>Controlled-NOT (CNOT) ga</u>

e.g. $\mathrm{C}_1\mathrm{NOT}_0$:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2).$

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (CNOT) gates

e.g. $\mathrm{C}_1\mathrm{NOT}_0$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
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Operation on quantum state:
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Controlled-NOT (CNOT) gates

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$(3,1,4,1,5,9,2,6) \mapsto$
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Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

state	measurement
$(1, 0, 0, 0, 0, 0, 0, 0)$	000
$(0, 1, 0, 0, 0, 0, 0, 0)$	001
$(0, 0, 1, 0, 0, 0, 0, 0)$	010
$(0, 0, 0, 1, 0, 0, 0, 0)$	011
$(0, 0, 0, 0, 1, 0, 0, 0)$	100
$(0, 0, 0, 0, 0, 1, 0, 0)$	101
$(0, 0, 0, 0, 0, 0, 1, 0)$	110
$(0, 0, 0, 0, 0, 0, 0, 1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (CNOT) gates

e.g. $\mathrm{C}_1\mathrm{NOT}_0$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\mathrm{C}_2\mathrm{NOT}_0$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

state	measurement
$(1,0,0,0,0,0,0,0)$	000
$(0,1,0,0,0,0,0,0)$	001
$(0,0,1,0,0,0,0,0)$	010
$(0,0,0,1,0,0,0,0)$	011
$(0,0,0,0,1,0,0,0)$	100
$(0,0,0,0,0,1,0,0)$	101
$(0,0,0,0,0,0,1,0)$	110
$(0,0,0,0,0,0,0,1)$	111

Operation on quantum state:
NOT_0, swapping pairs.
Operation after measurement:
flipping bit 0 of result.
Flip: output is not input.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,9,5,6,2)$.

e.g. C_0NOT_2:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,9,4,6,5,1,2,1)$.

state	measurement
$0,0,0,0,0)$	000
$0,0,0,0,0)$	001
$0,0,0,0,0)$	010
$1,0,0,0,0)$	011
$0,1,0,0,0)$	100
$0,0,1,0,0)$	101
$0,0,0,1,0)$	110
$0,0,0,0,1)$	111

on on quantum state:
swapping pairs.
on after measurement:
bit 0 of result.
tput is not input.

Controlled-NOT (CNOT) gates

e.g. $\mathrm{C}_1\mathrm{NOT}_0$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. $\mathrm{C}_2\mathrm{NOT}_0$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,9,5,6,2)$.

e.g. $\mathrm{C}_0\mathrm{NOT}_2$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,9,4,6,5,1,2,1)$.

Toffoli g

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e.g. $\mathrm{C}_2\mathrm{C}$
$(3,1,4,1$
$(3,1,4,1$

measurement

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0) 001

0) 010

0) 011

0) 100

0) 101

0) 110

1) 111

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Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:

flipping bit 0 *if* bit 1 is set; i.e.,

$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. C_0NOT_2:

$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$

$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as CC

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e.g. $C_2C_1NOT_0$:

$(3, 1, 4, 1, 5, 9, 2, 6$

$(3, 1, 4, 1, 5, 9, 6, 2$

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Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
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e.g. C_2NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. C_0NOT_2:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as CCNOT gate
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e.g. $C_2C_1NOT_0$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,1,4,5,9,6,2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,9,5,6,2)$.

e.g. C_0NOT_2:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,9,4,6,5,1,2,1)$.

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:
$(3,1,4,1,5,9,2,6) \mapsto$
$(3,1,4,1,5,9,6,2)$.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. C_0NOT_2:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

Controlled-NOT (CNOT) gates

e.g. C_1NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 1, 4, 5, 9, 6, 2)$.

Operation after measurement:
flipping bit 0 *if* bit 1 is set; i.e.,
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

e.g. C_2NOT_0:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 9, 5, 6, 2)$.

e.g. C_0NOT_2:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 9, 4, 6, 5, 1, 2, 1)$.

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $C_0C_1NOT_2$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 6, 5, 9, 2, 1)$.

ed-NOT (CNOT) gates

IOT$_0$:

1, 5, 9, 2, 6) $\mapsto$

4, 5, 9, 6, 2).

on after measurement:

bit 0 *if* bit 1 is set; i.e.,

$q_0) \mapsto (q_2, q_1, q_0 \oplus q_1)$.

IOT$_0$:

1, 5, 9, 2, 6) $\mapsto$

1, 9, 5, 6, 2).

IOT$_2$:

1, 5, 9, 2, 6) $\mapsto$

6, 5, 1, 2, 1).

Toffoli gates

Also known as CCNOT gates:
controlled-controlled-NOT gates.

e.g. $C_2C_1NOT_0$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 1, 5, 9, 6, 2)$.

Operation after measurement:
$(q_2, q_1, q_0) \mapsto (q_2, q_1, q_0 \oplus q_1 q_2)$.

e.g. $C_0C_1NOT_2$:
$(3, 1, 4, 1, 5, 9, 2, 6) \mapsto$
$(3, 1, 4, 6, 5, 9, 2, 1)$.

More sh

Combine

to build

CNOT) gates

) $\mapsto$

).

easurement:

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$(3, 1, 4, 6, 5, 9, 2, 1)$.

More shuffling

Combine NOT, C
to build other per

ates

nt:

i.e.,

q_1).

Toffoli gates

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More shuffling

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Toffoli gates

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More shuffling

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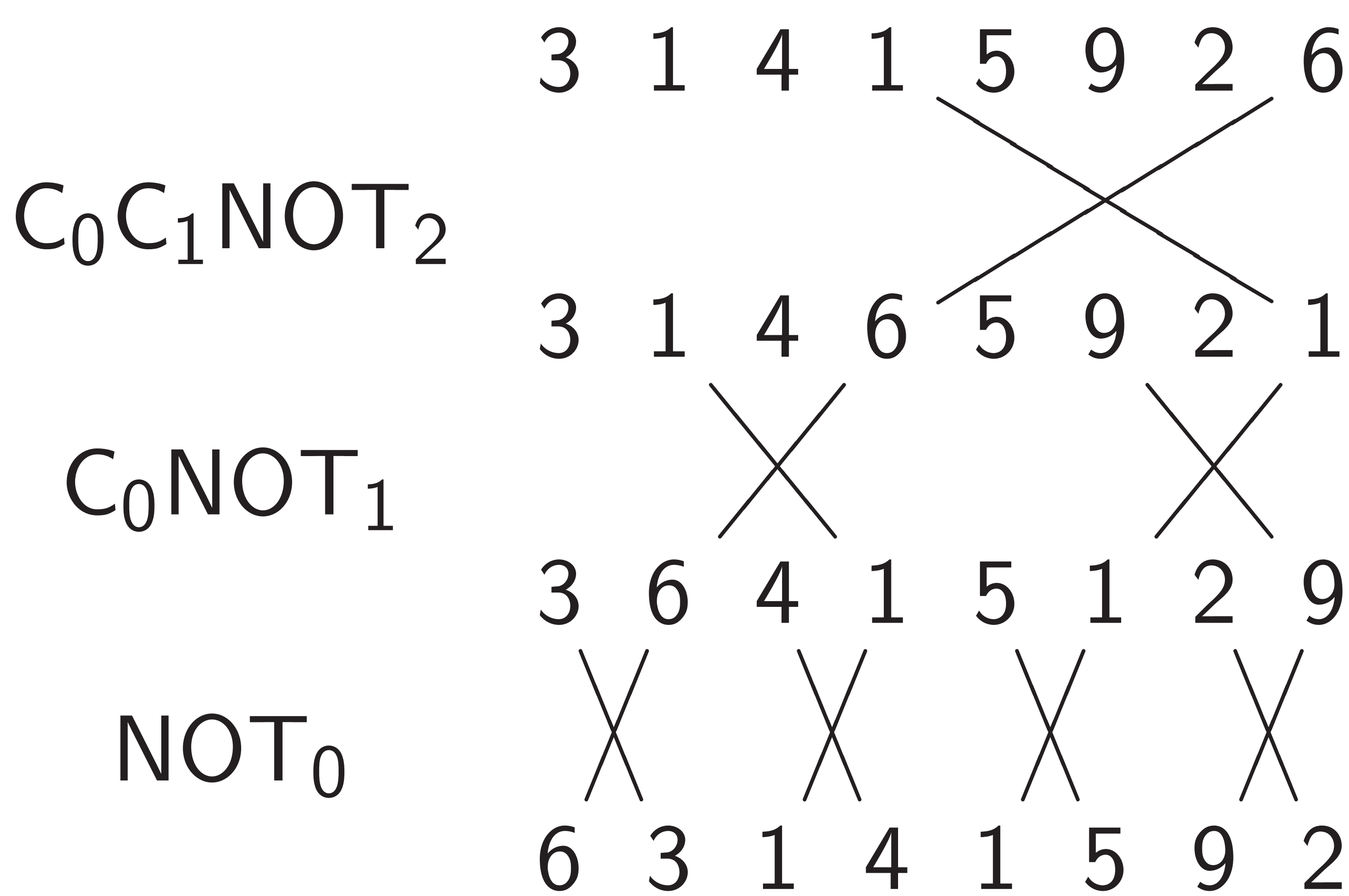
e.g. series of gates to rotate 8 positions by distance 1:



gates

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C_1NOT_2:
$1, 5, 9, 2, 6) \mapsto$
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More shuffling

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to build other permutations.

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rotate 8 positions by distance 1:

	3	1	4	1	5	9	2	6
$C_0C_1NOT_2$								
	3	1	4	6	5	9	2	1
C_0NOT_1								
	3	6	4	1	5	1	2	9
NOT_0								
	6	3	1	4	1	5	9	2

Hadama

Hadama

$(a, b) \mapsto$

3 1
4 2

NOT gates:
ed-NOT gates.

$) \mapsto$
$).$

easurement:
$, q_1, q_0 \oplus q_1 q_2).$

$) \mapsto$
$).$

More shuffling

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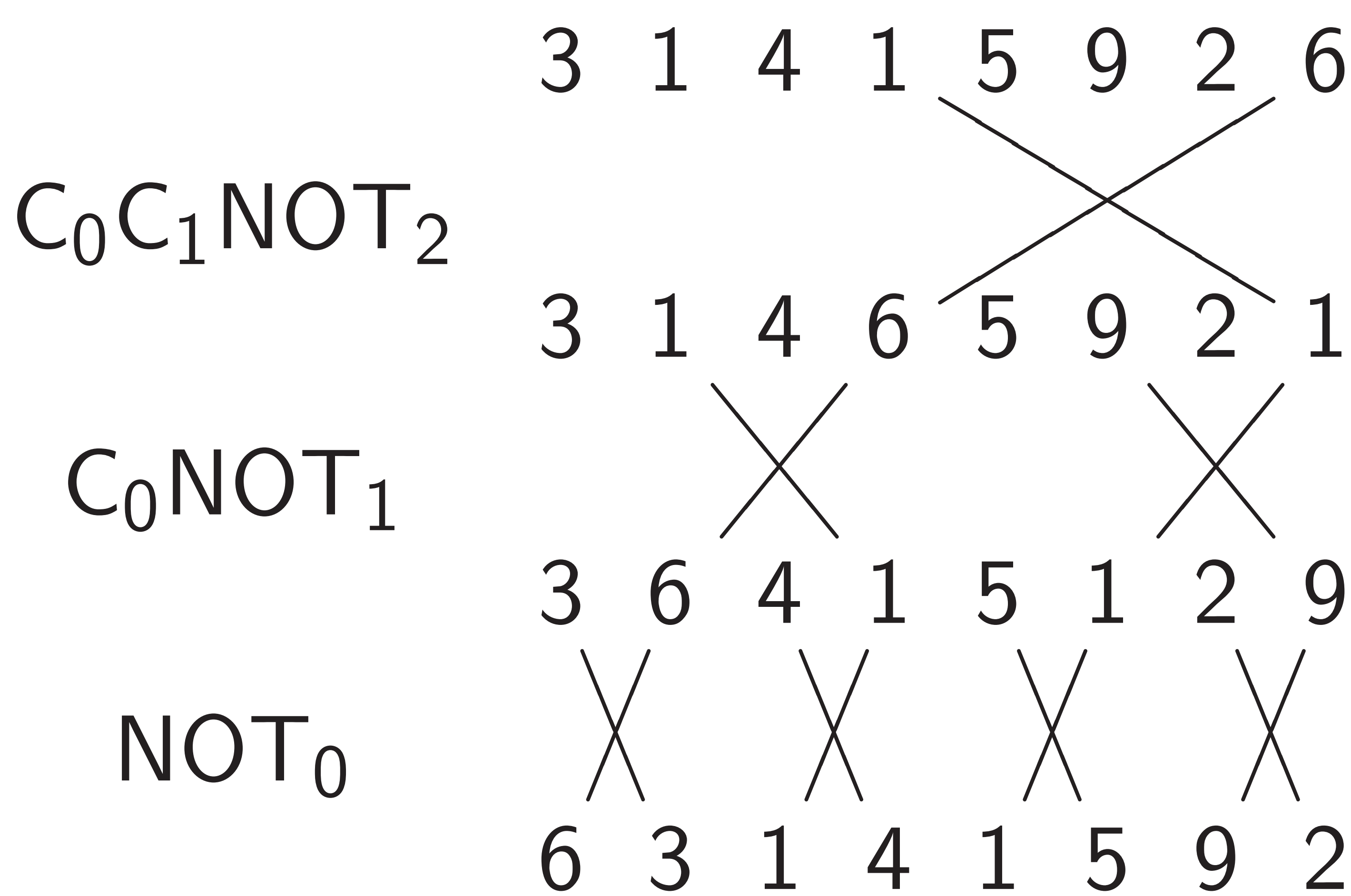
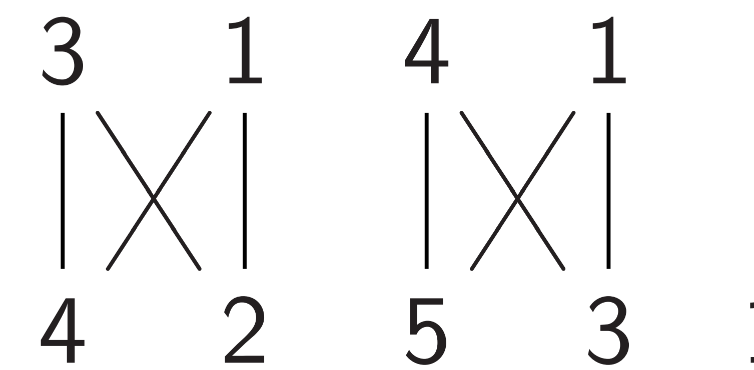


Hadamard gates

Hadamard$_0$:

$(a, b) \mapsto (a + b, a$



es:

gates.

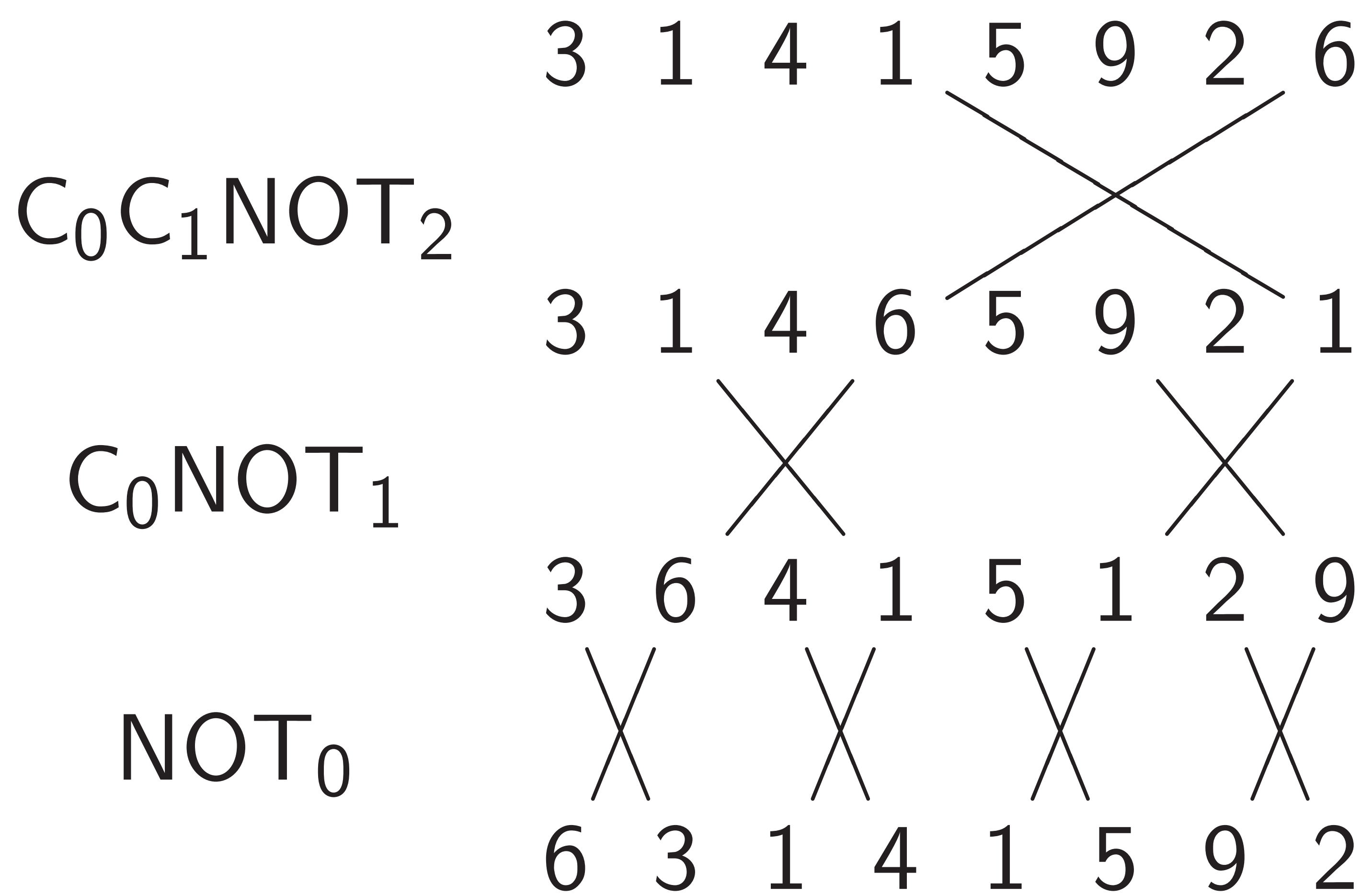
nt:

$q_1 q_2)$.

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

e.g. series of gates to rotate 8 positions by distance 1:



Hadamard gates

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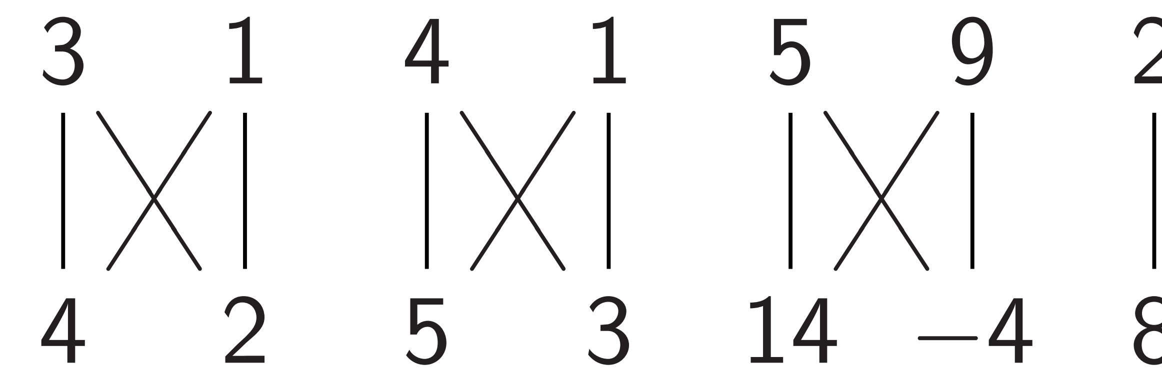

More shuffling

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rotate 8 positions by distance 1:

	3	1	4	1	5	9	2	6
$C_0C_1NOT_2$								
	3	1	4	6	5	9	2	1
C_0NOT_1								
	3	6	4	1	5	1	2	9
NOT_0								
	6	3	1	4	1	5	9	2

Hadamard gates

Hadamard_0:

$(a, b) \mapsto (a + b, a - b)$.

3	1	4	1	5	9	2	6
4	2	5	3	14	−4	8	−4

More shuffling

Combine NOT, CNOT, Toffoli to build other permutations.

e.g. series of gates to rotate 8 positions by distance 1:

3 1 4 1 5 9 2 6

$C_0C_1NOT_2$

3 1 4 6 5 9 2 1

C_0NOT_1

3 6 4 1 5 1 2 9

NOT_0

6 3 1 4 1 5 9 2

Hadamard gates

Hadamard_0:

$(a, b) \mapsto (a + b, a - b)$.

3 1 4 1 5 9 2 6

4 2 5 3 14 −4 8 −4

Hadamard_1:

$(a, b, c, d) \mapsto$
$(a + c, b + d, a - c, b - d)$.

3 1 4 1 5 9 2 6

7 2 −1 0 7 15 3 3

uffling

e NOT, CNOT, Toffoli
other permutations.

es of gates to
positions by distance 1:

3 1 4 1 5 9 2 6

OT$_2$

3 1 4 6 5 9 2 1

T$_1$

3 6 4 1 5 1 2 9

$_0$

6 3 1 4 1 5 9 2

Hadamard gates

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3 1 4 1 5 9 2 6

7 2 −1 0 7 15 3 3

Some us

Hadama

3 1

4 2

2 4

6 −2

NOT, Toffoli
mutations.

s to
by distance 1:

4 1 5 9 2 6

4 6 5 9 2 1

4 1 5 1 2 9

1 4 1 5 9 2

Hadamard gates

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3 1 4 1 5 9 2 6

7 2 -1 0 7 15 3 3

Some uses of Had

Hadamard$_0$, NOT

3 1 4 1

4 2 5 3

2 4 3 5

6 -2 8 -2

foli

ce 1:

9 2 6

9 2 1

1 2 9

5 9 2

Hadamard gates

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3	1	4	1	5	9	2	6
7	2	−1	0	7	15	3	3

Some uses of Hadamard gat

Hadamard$_0$, NOT$_0$, Hadama

3	1	4	1	5	9	2
4	2	5	3	14	−4	8
2	4	3	5	−4	14	−
6	−2	8	−2	10	−18	4

Hadamard gates

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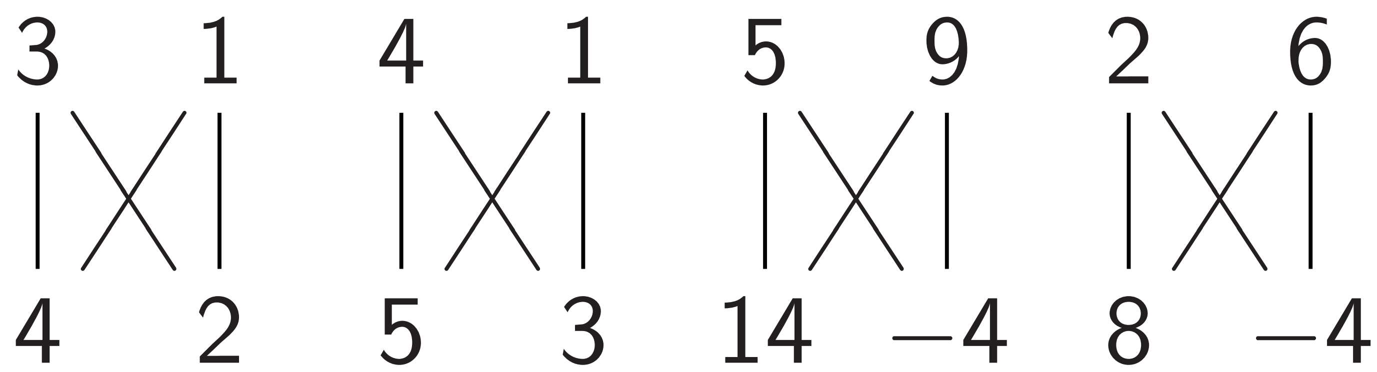

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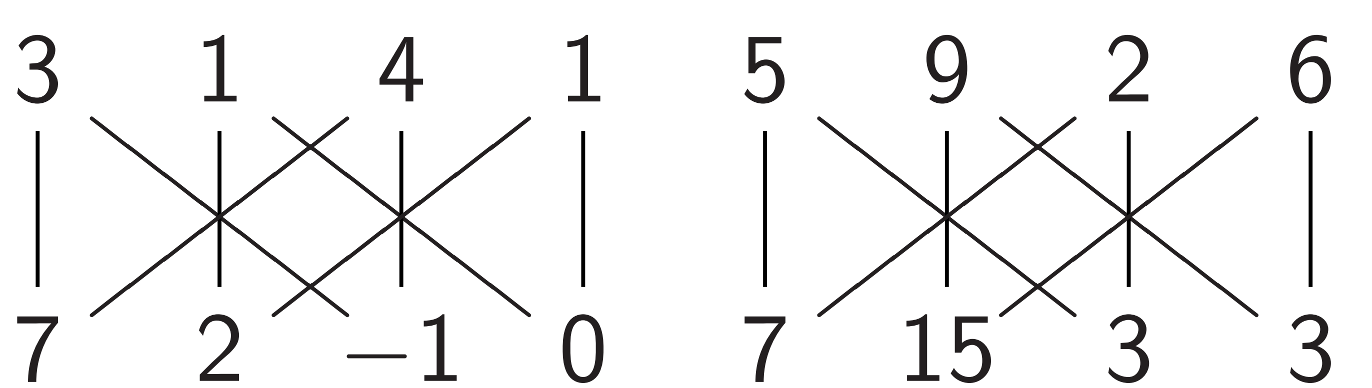
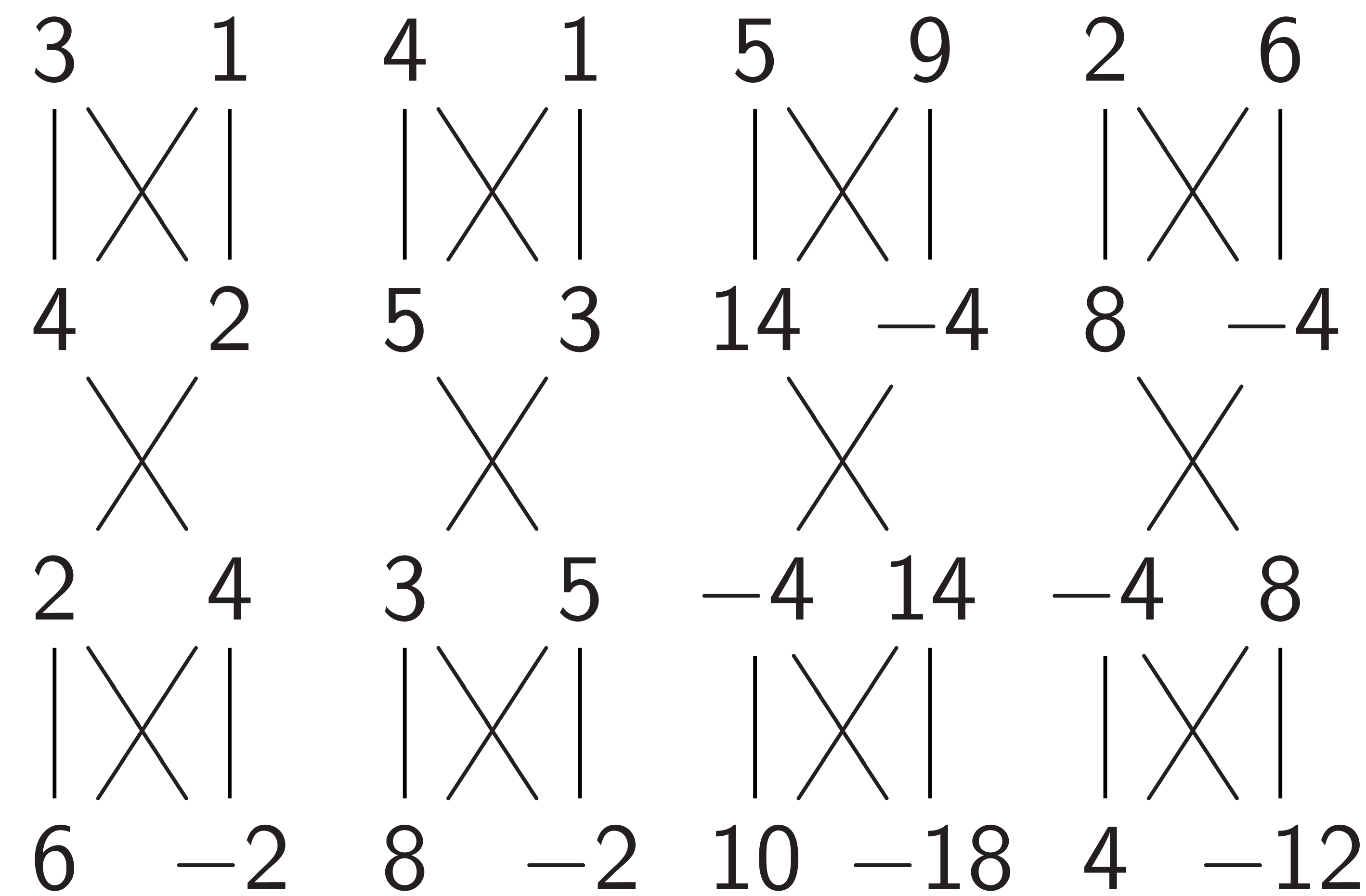
$(a + c, b + d, a - c, b - d)$.



Some uses of Hadamard gates

Hadamard$_0$, NOT$_0$, Hadamard$_0$:



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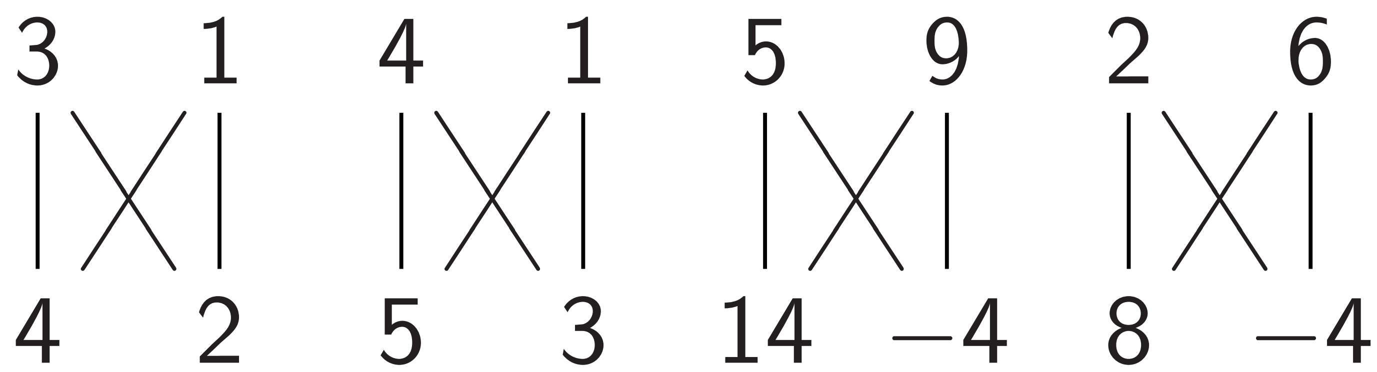

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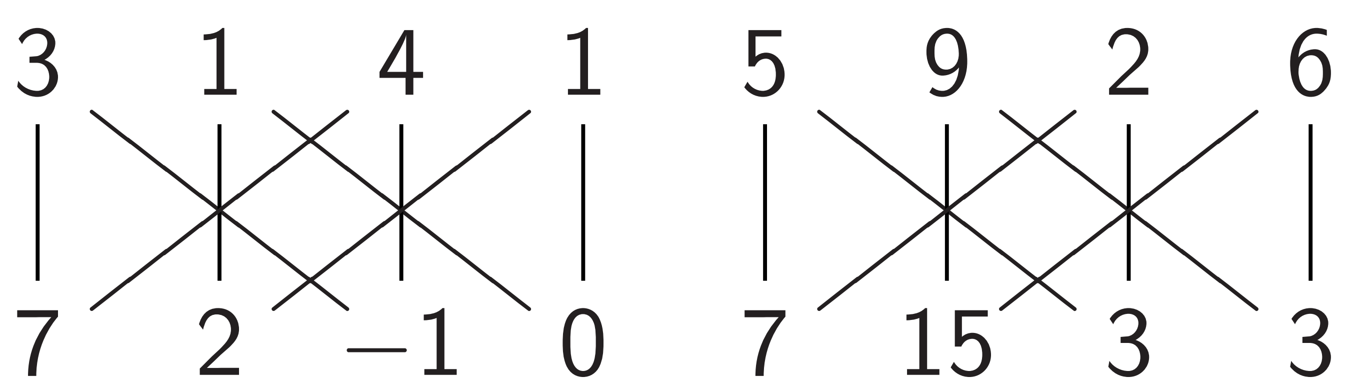
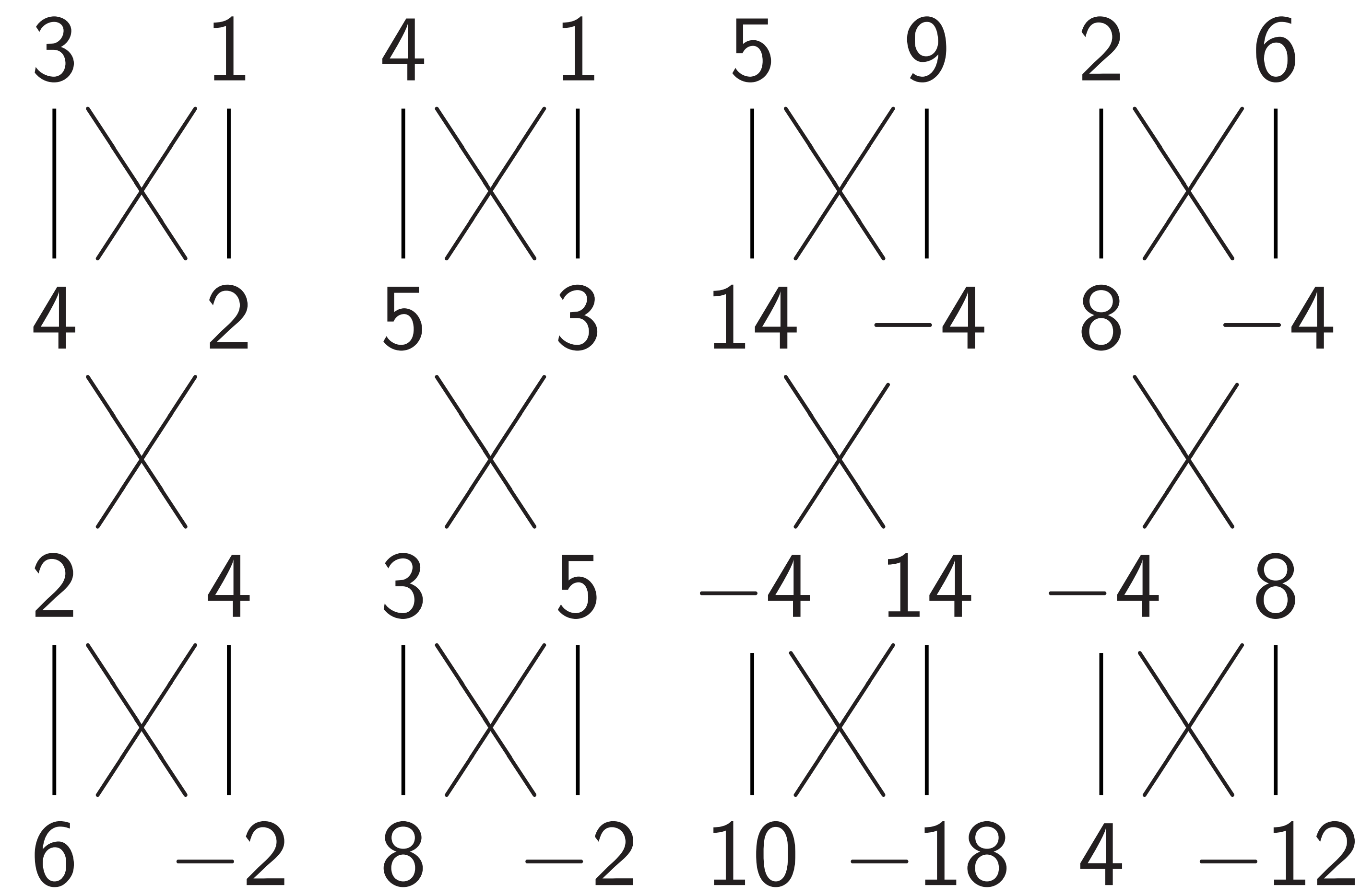


Some uses of Hadamard gates

Hadamard$_0$, NOT$_0$, Hadamard$_0$:



“Multiply each amplitude by 2.”
This is not physically observable.

Hadamard gates

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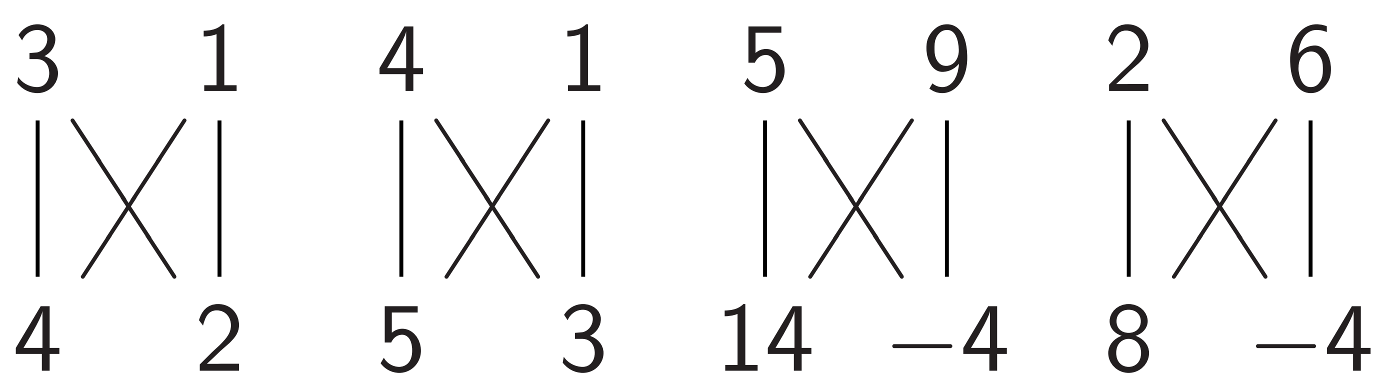

Hadamard$_1$:

$(a, b, c, d) \mapsto$
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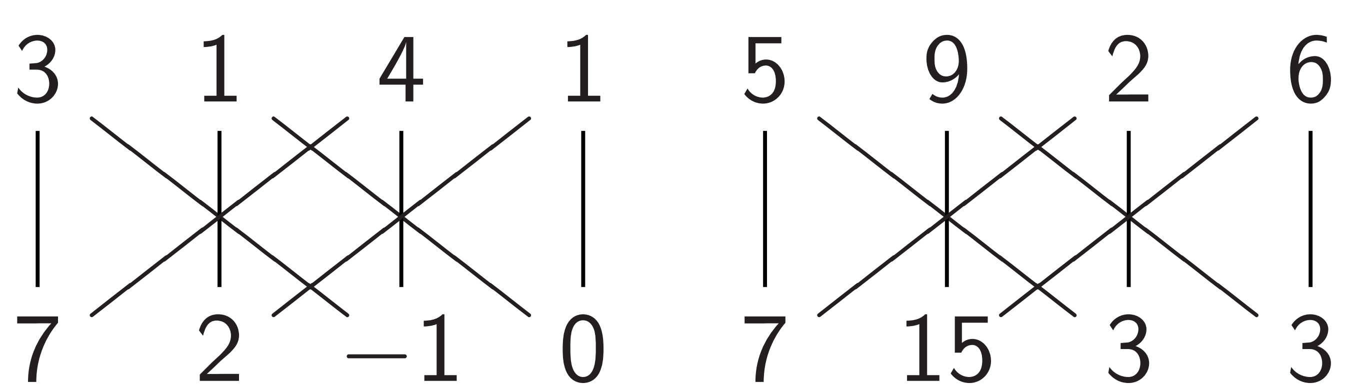

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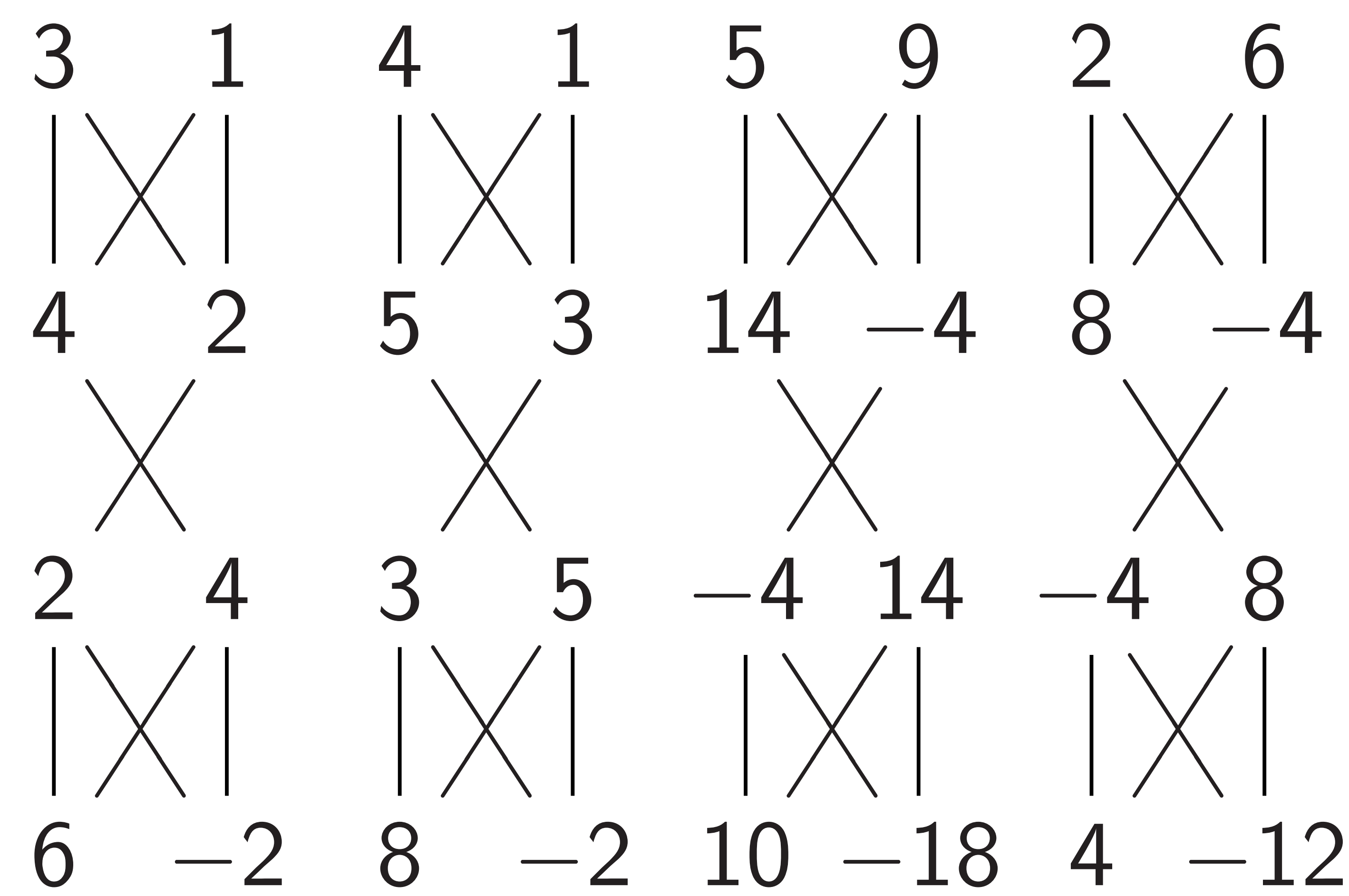

"Multiply each amplitude by 2."
This is not physically observable.

"Negate amplitude if q_0 is set."
No effect on measuring *now*.

rd gates

rd$_0$:

$\mapsto (a + b, a - b)$.

4 1 5 9 2 6

5 3 14 −4 8 −4

rd$_1$:

$d) \mapsto$

$b + d, a - c, b - d)$.

4 1 5 9 2 6

−1 0 7 15 3 3

Some uses of Hadamard gates

Hadamard$_0$, NOT$_0$, Hadamard$_0$:

3 1 4 1 5 9 2 6

4 2 5 3 14 −4 8 −4

2 4 3 5 −4 14 −4 8

6 −2 8 −2 10 −18 4 −12

"Multiply each amplitude by 2."
This is not physically observable.

"Negate amplitude if q_0 is set."
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Fancier

"Negate

Assumes

C_0C_1NO

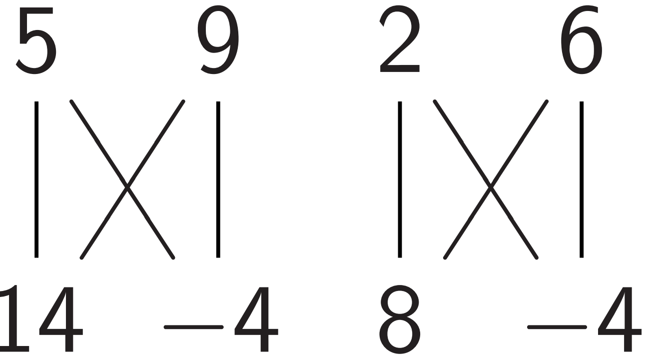
Hadama

NOT

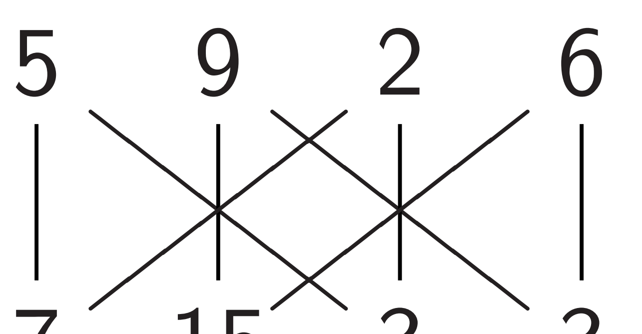
Hadama

C_0C_1NO

$- b)$.

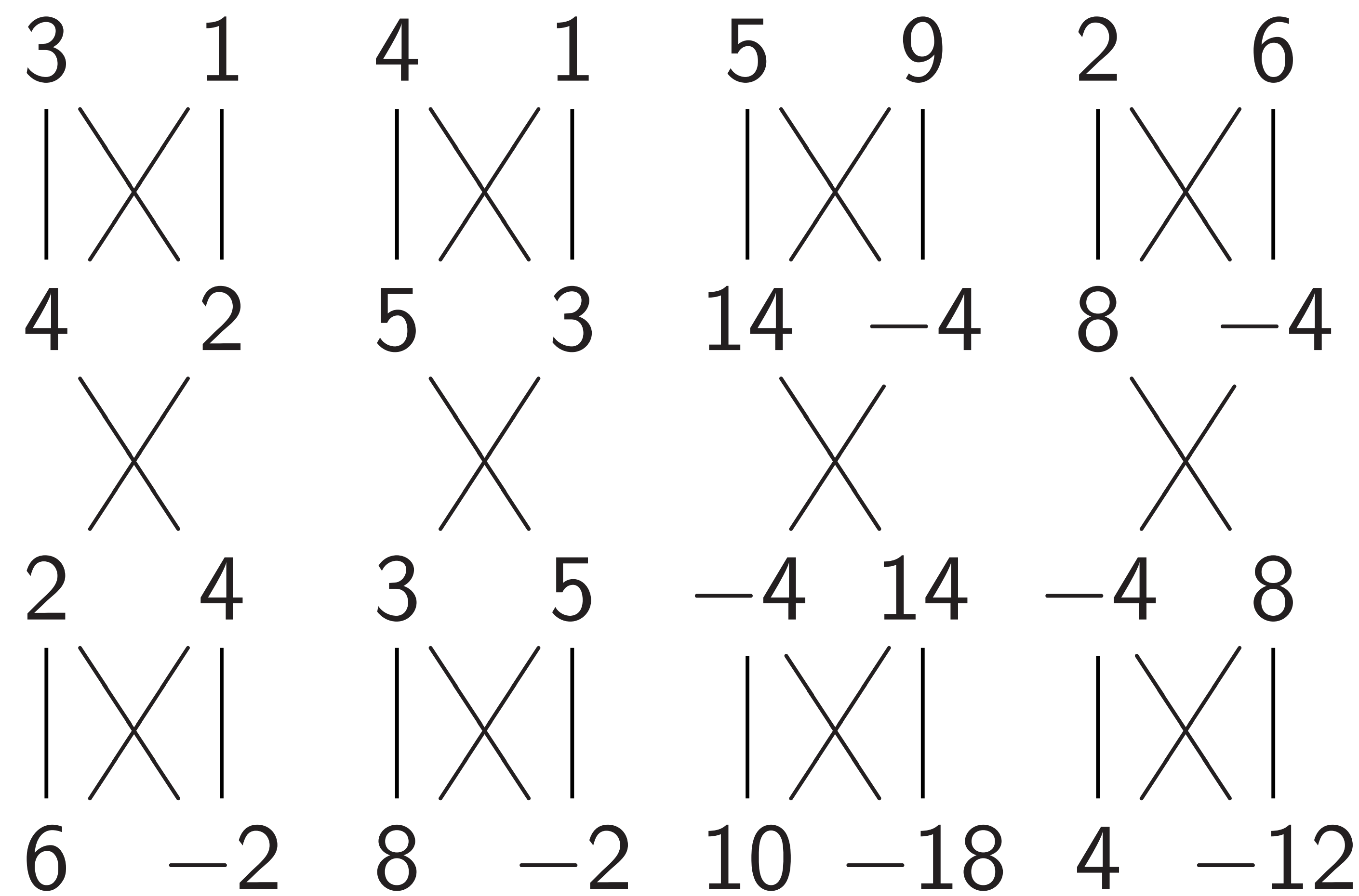


$c, b - d)$.



Some uses of Hadamard gates

Hadamard$_0$, NOT$_0$, Hadamard$_0$:



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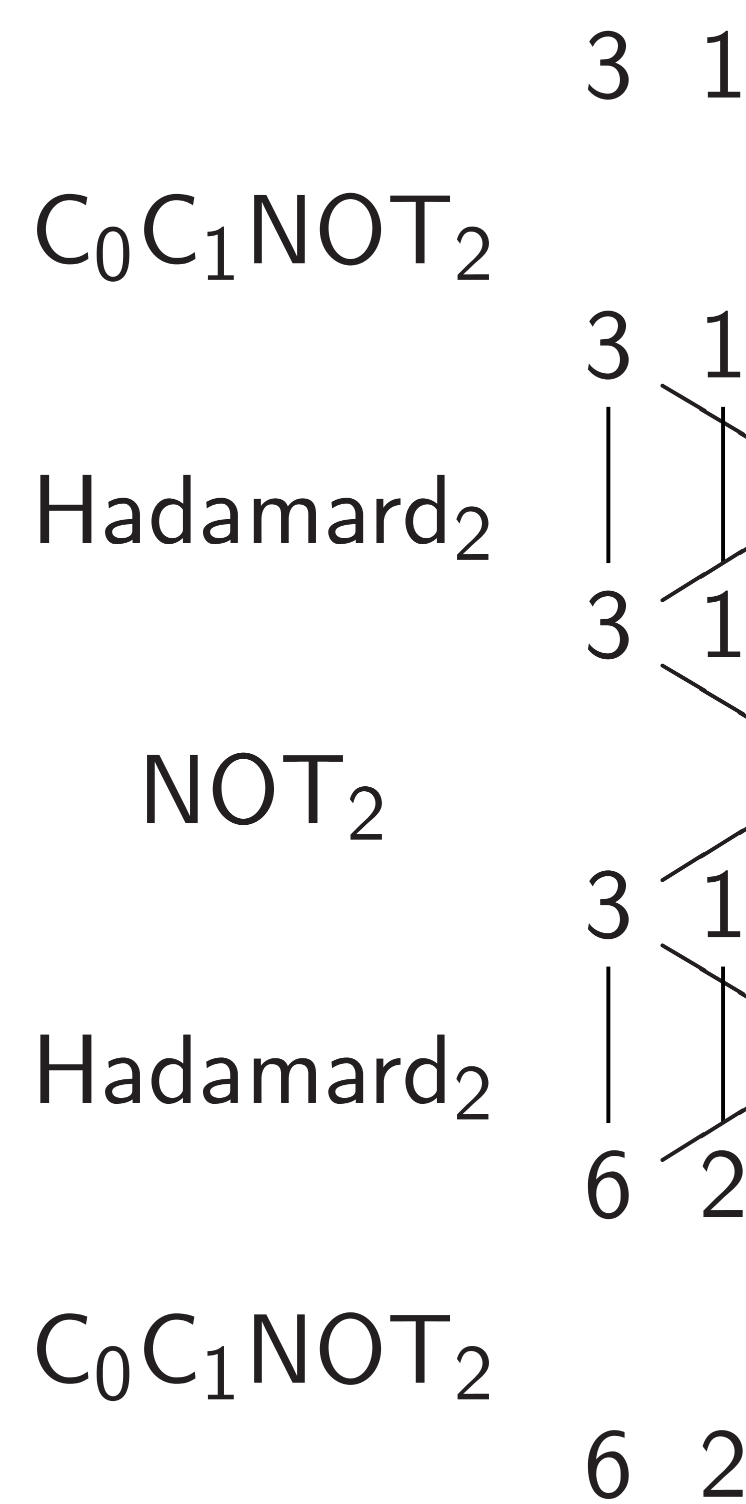
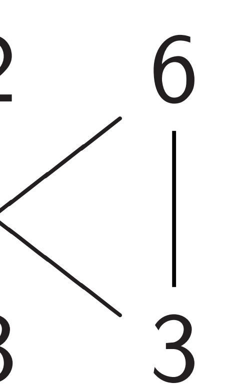
Fancier example:

“Negate amplitud

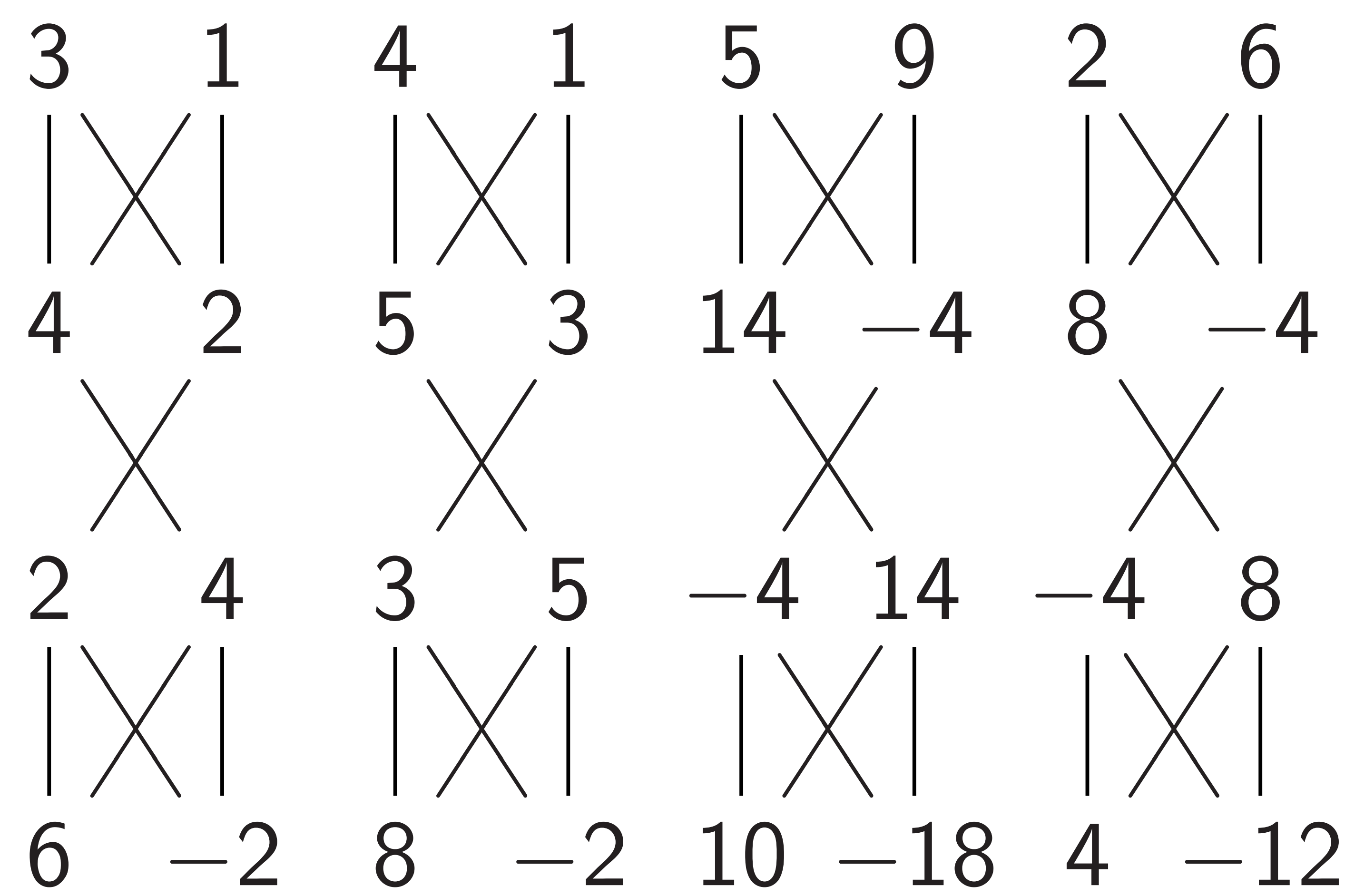
Assumes $q_2 = 0$:





Some uses of Hadamard gates

Hadamard_0, NOT_0, Hadamard_0:



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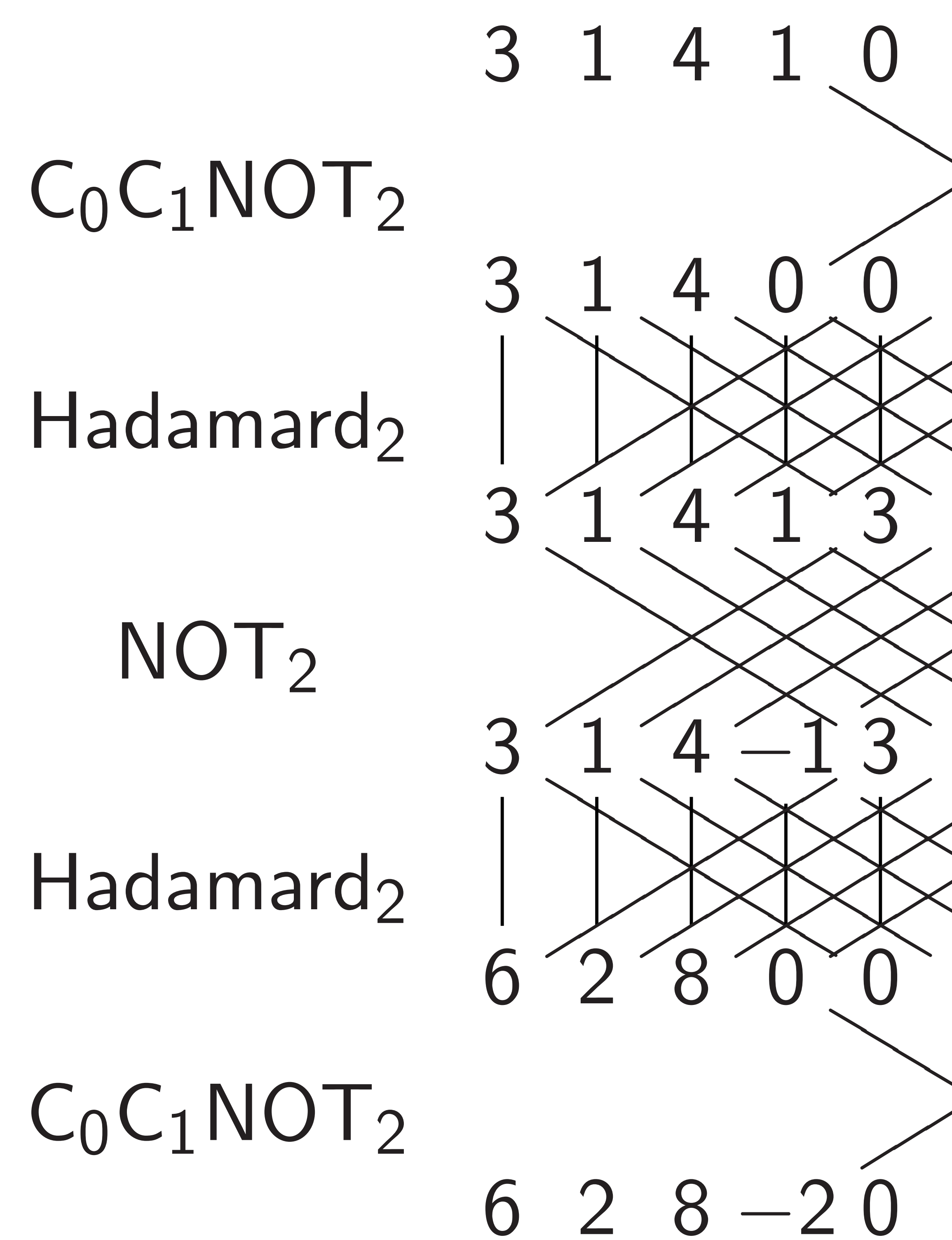
"Negate amplitude if $q_0 q_1$ is

Assumes $q_2 = 0$: "ancilla" c



Some uses of Hadamard gates

Hadamard$_0$, NOT$_0$, Hadamard$_0$:

$$\begin{array}{cccccccc}
3 & 1 & 4 & 1 & 5 & 9 & 2 & 6 \\
4 & 2 & 5 & 3 & 14 & -4 & 8 & -4 \\
2 & 4 & 3 & 5 & -4 & 14 & -4 & 8 \\
6 & -2 & 8 & -2 & 10 & -18 & 4 & -12
\end{array}$$

"Multiply each amplitude by 2."
This is not physically observable.

"Negate amplitude if q_0 is set."
No effect on measuring *now*.

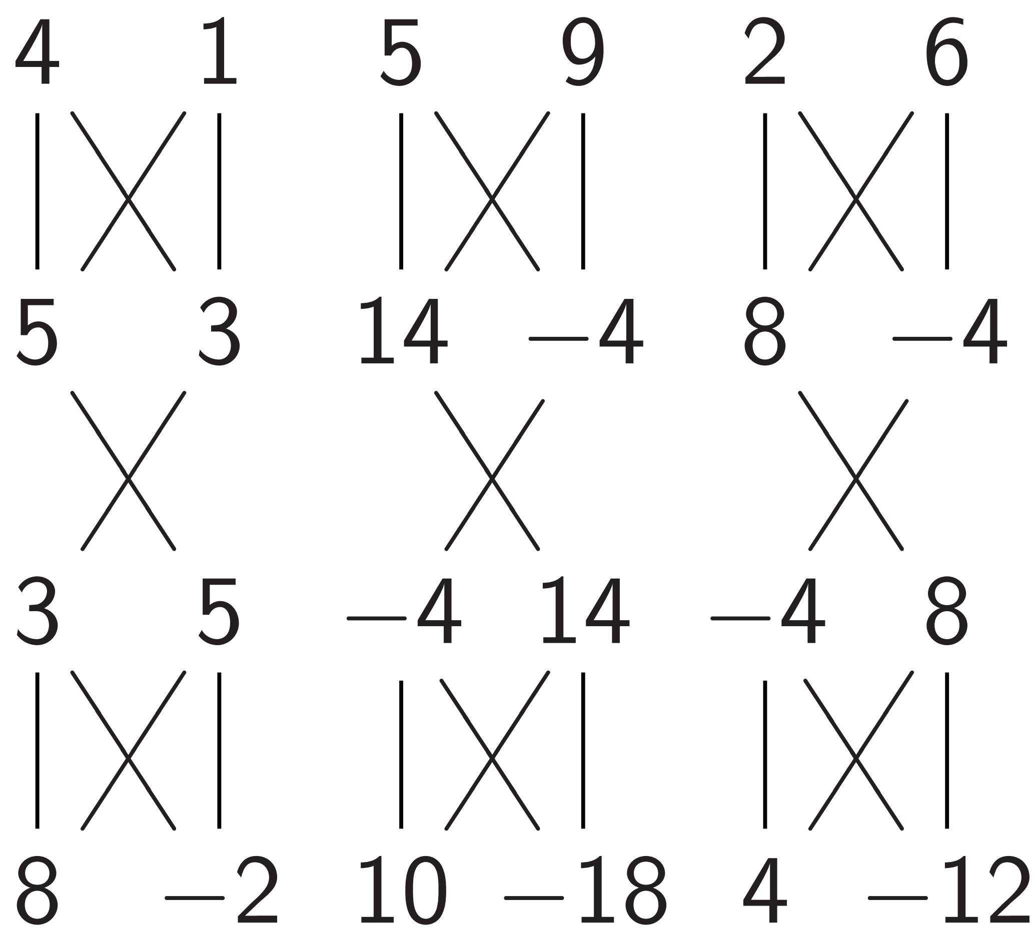
Fancier example:

"Negate amplitude if $q_0 q_1$ is set."
Assumes $q_2 = 0$: "ancilla" qubit.

$$\begin{array}{l|cccccccc}
 & 3 & 1 & 4 & 1 & 0 & 0 & 0 & 0 \\
\mathrm{C_0C_1NOT_2} & & & & & & & & \\
 & 3 & 1 & 4 & 0 & 0 & 0 & 0 & 1 \\
\mathrm{Hadamard_2} & & & & & & & & \\
 & 3 & 1 & 4 & 1 & 3 & 1 & 4 & -1 \\
\mathrm{NOT_2} & & & & & & & & \\
 & 3 & 1 & 4 & -1 & 3 & 1 & 4 & 1 \\
\mathrm{Hadamard_2} & & & & & & & & \\
 & 6 & 2 & 8 & 0 & 0 & 0 & 0 & -2 \\
\mathrm{C_0C_1NOT_2} & & & & & & & & \\
 & 6 & 2 & 8 & -2 & 0 & 0 & 0 & 0
\end{array}$$

ses of Hadamard gates

rd$_0$, NOT$_0$, Hadamard$_0$:



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not physically observable.

amplitude if q_0 is set.”
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Fancier example:

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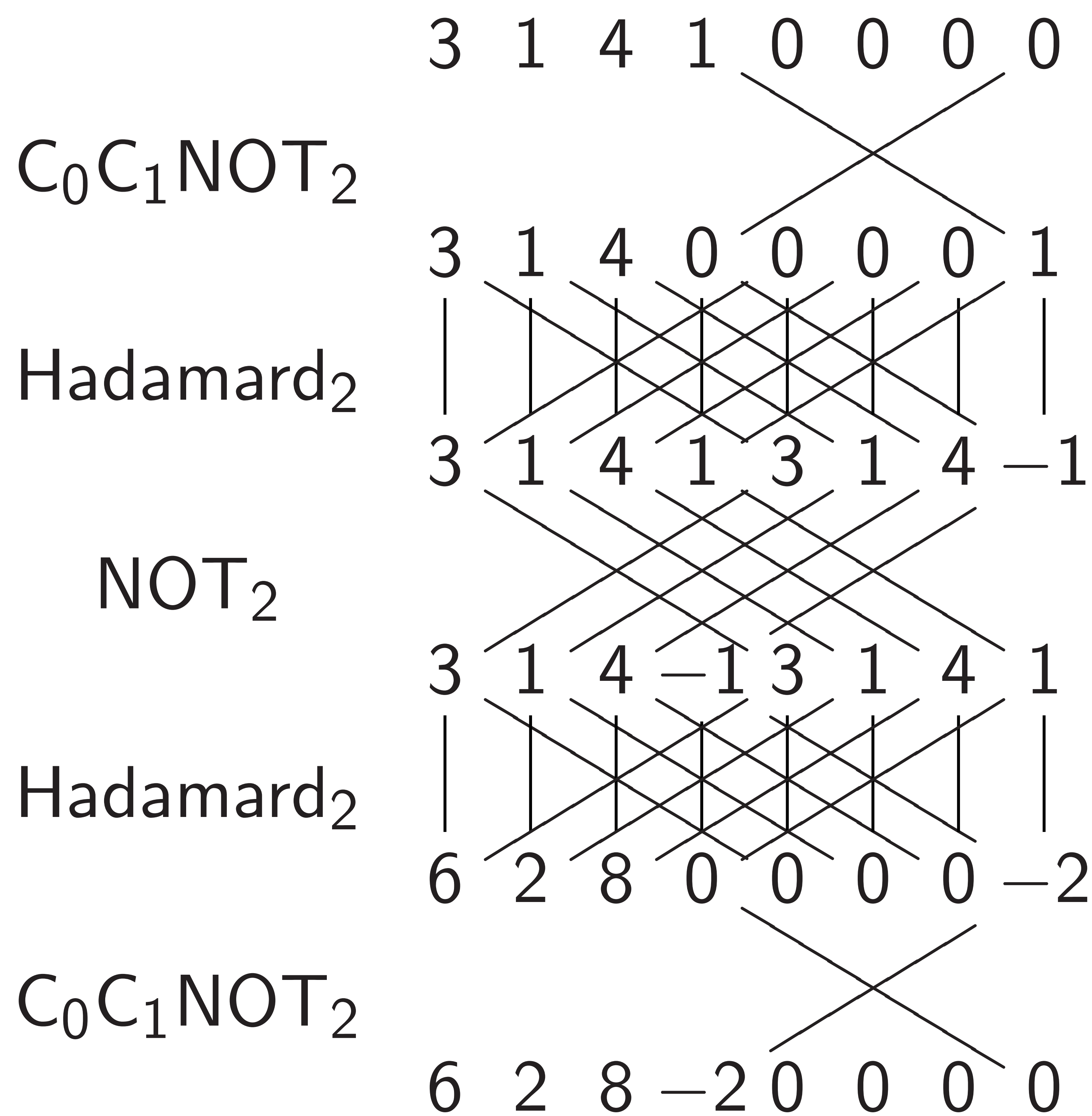


Affects r
amplituc
$(3, 1, 4, 1$

amard gates

$_0$, Hadamard$_0$:

5 9 2 6

14 −4 8 −4

−4 14 −4 8

10 −18 4 −12

nplitude by 2.”

ally observable.

e if q_0 is set.”

uring *now*.

Fancier example:

“Negate amplitude if $q_0 q_1$ is set.”

Assumes $q_2 = 0$: “ancilla” qubit.

3 1 4 1 0 0 0 0

$C_0C_1NOT_2$

3 1 4 0 0 0 0 1

Hadamard$_2$

3 1 4 1 3 1 4 −1

NOT$_2$

3 1 4 −1 3 1 4 1

Hadamard$_2$

6 2 8 0 0 0 0 −2

$C_0C_1NOT_2$

6 2 8 −2 0 0 0 0

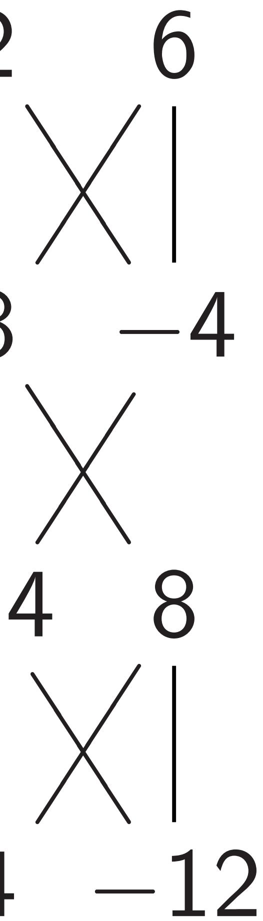
Affects measureme

amplitude around

$(3, 1, 4, 1) \mapsto (1.5,$

es

ard$_0$:



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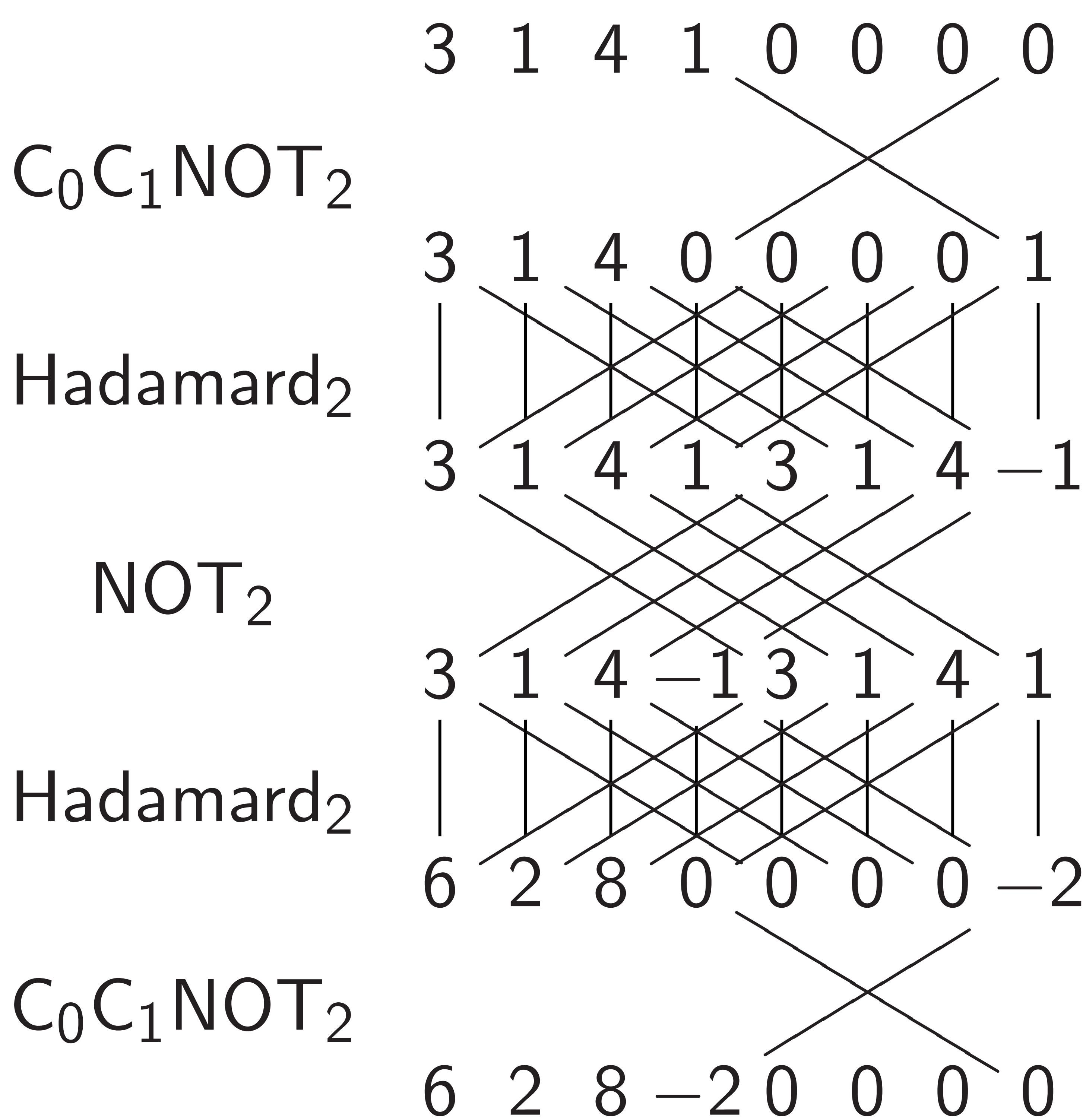


Affects measurements: “Neg

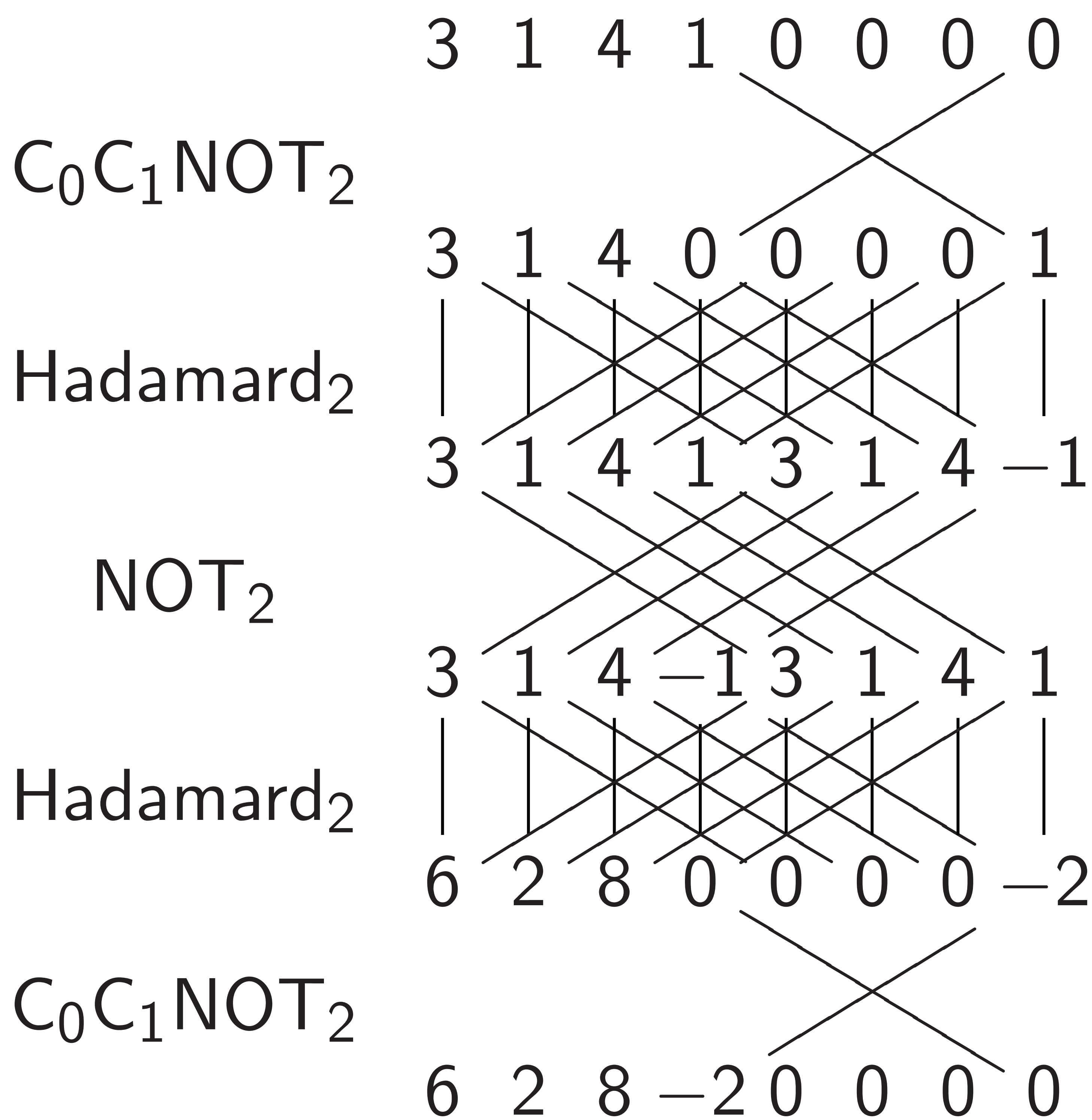
amplitude around its averag

$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3$

Fancier example:

"Negate amplitude if $q_0 q_1$ is set."

Assumes $q_2 = 0$: "ancilla" qubit.



Affects measurements: "Negate amplitude around its average."

$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

Fancier example:

"Negate amplitude if $q_0 q_1$ is set."

Assumes $q_2 = 0$: "ancilla" qubit.

	3	1	4	1	0	0	0	0
$\mathrm{C_0C_1NOT_2}$								
	3	1	4	0	0	0	0	1
$\mathrm{Hadamard_2}$								
	3	1	4	1	3	1	4	−1
$\mathrm{NOT_2}$								
	3	1	4	−1	3	1	4	1
$\mathrm{Hadamard_2}$								
	6	2	8	0	0	0	0	−2
$\mathrm{C_0C_1NOT_2}$								
	6	2	8	−2	0	0	0	0

Affects measurements: "Negate amplitude around its average."

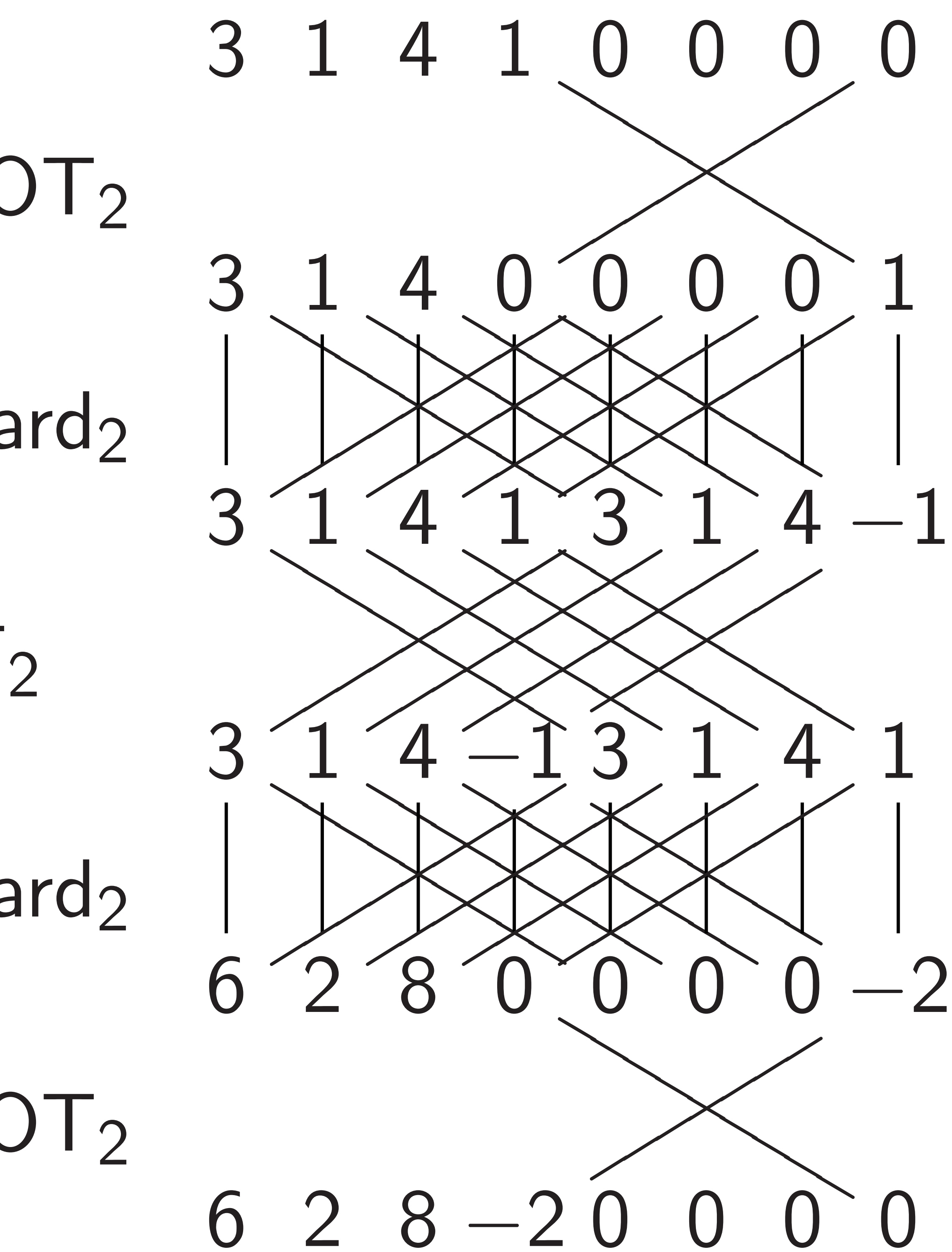
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

	3	1	4	1	0	0	0	0
$\mathrm{H_0}$								
	4	2	5	3	0	0	0	0
$\mathrm{H_1}$								
	9	5	−1	−1	0	0	0	0
...								
	−9	5	−1	−1	0	0	0	0
$\mathrm{H_0}$								
	−4	−14	−2	0	0	0	0	0
$\mathrm{H_1}$								
	−6	−14	−2	−14	0	0	0	0

example:

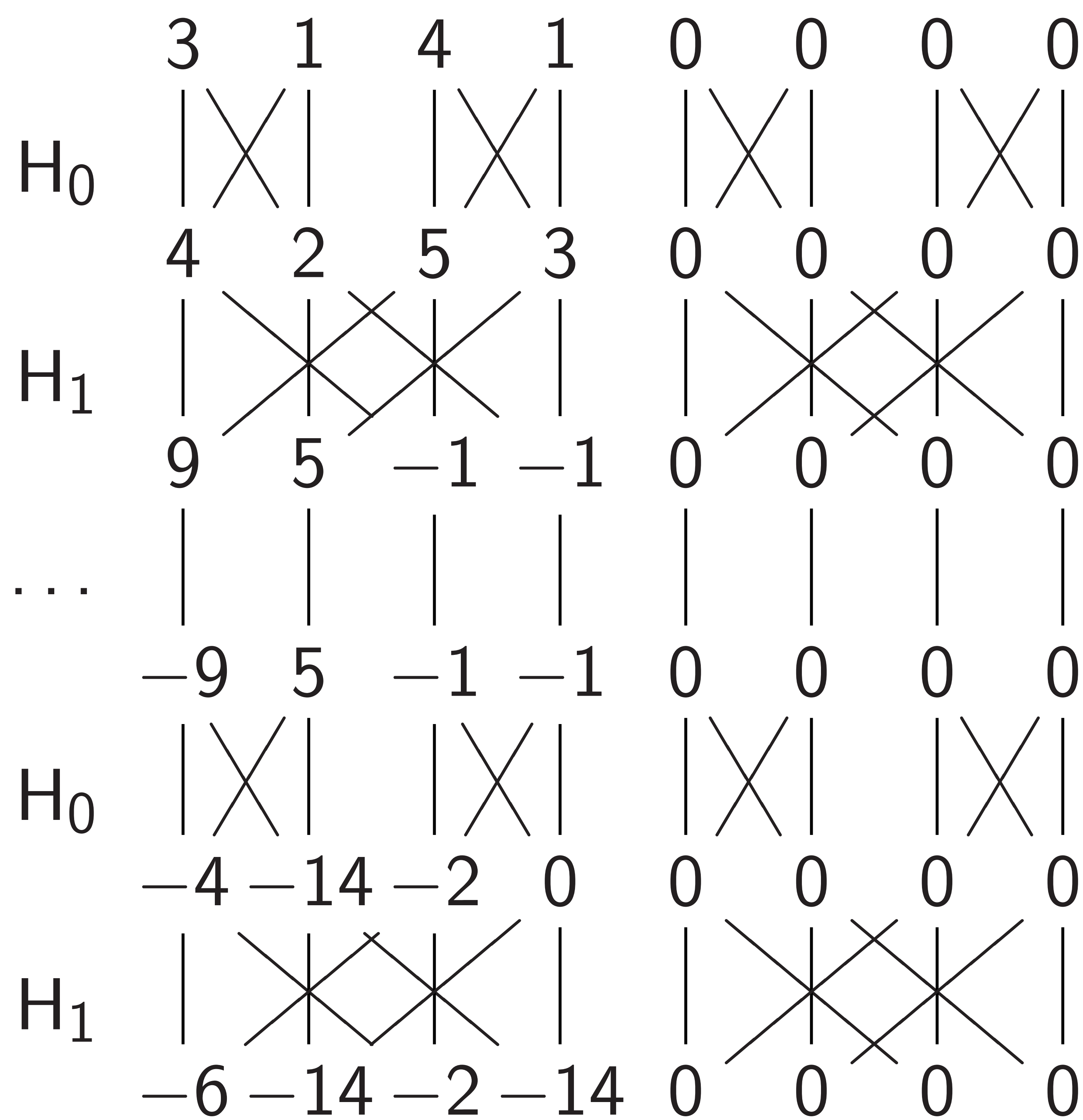
amplitude if $q_0 q_1$ is set."

s $q_2 = 0$: "ancilla" qubit.



Affects measurements: "Negate amplitude around its average."

$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



Simon's

Assumpt

- Given

can ef

- Nonze
- $f(u) =$
- f has

Goal: Fi

e if $q_0 q_1$ is set."
"ancilla" qubit.

4	1	0	0	0	0
4	0	0	0	0	1
4	1	3	1	4	−1
4	−1	3	1	4	1
8	0	0	0	0	−2
8	−2	0	0	0	0

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

	3	1	4	1	0	0	0	0
H_0								
	4	2	5	3	0	0	0	0
H_1								
	9	5	−1	−1	0	0	0	0
...								
	−9	5	−1	−1	0	0	0	0
H_0								
	−4	−14	−2	0	0	0	0	0
H_1								
	−6	−14	−2	−14	0	0	0	0

Simon's algorithm

Assumptions:

- Given any $u \in \{$
  can efficiently co
- Nonzero $s \in \{0,$
- $f(u) = f(u \oplus s)$
- f has no other c

Goal: Figure out s

s set.”

qubit.

0	0	0
0	0	1
1	4	−1
1	4	1
0	0	−2
0	0	0

Affects measurements: “Negate amplitude around its average.”

$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

	3	1	4	1	0	0	0	0
H_0								
	4	2	5	3	0	0	0	0
H_1								
	9	5	−1	−1	0	0	0	0
. . .								
	−9	5	−1	−1	0	0	0	0
H_0								
	−4	−14	−2	0	0	0	0	0
H_1								
	−6	−14	−2	−14	0	0	0	0

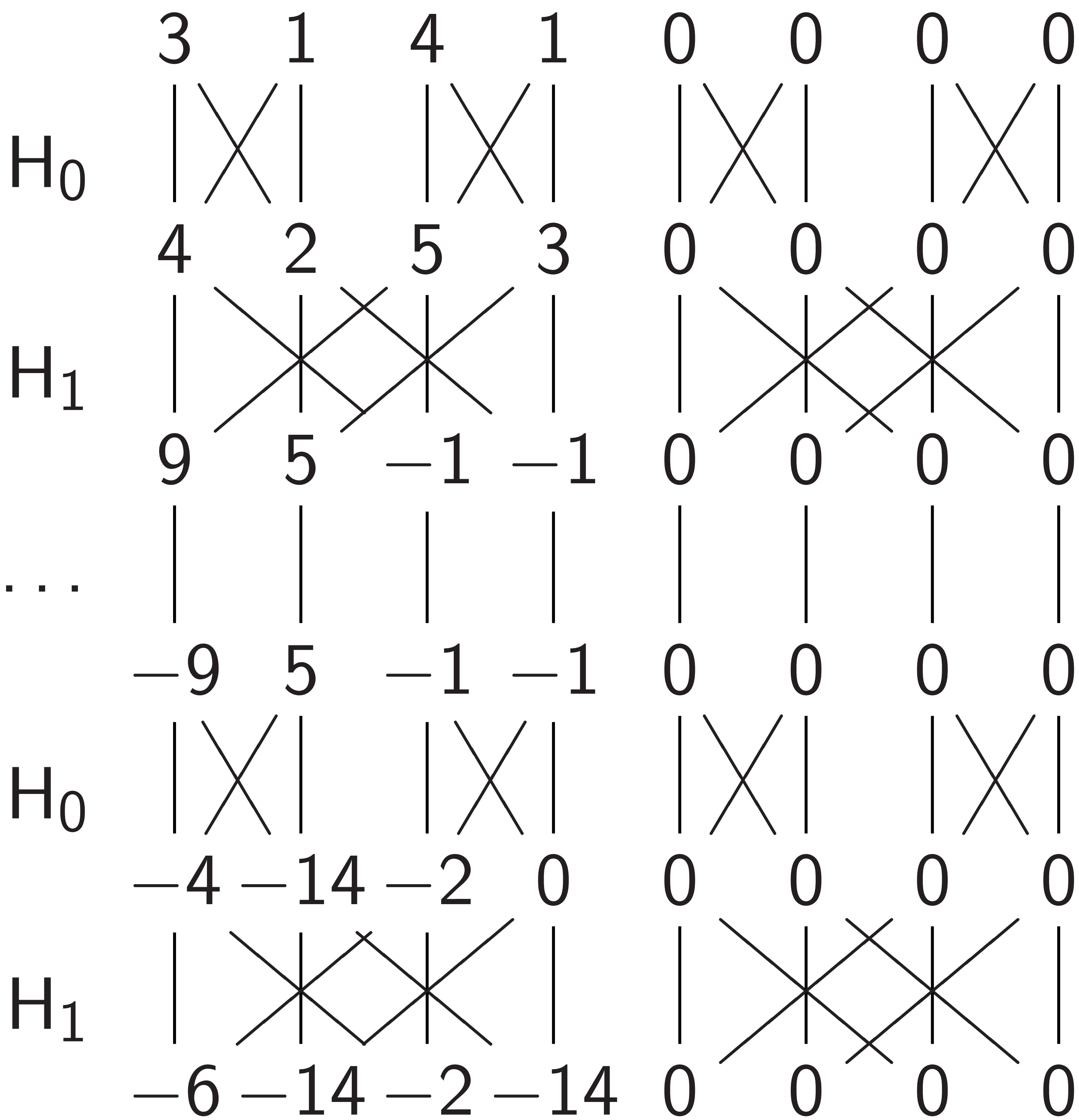
<u>Simon’s algorithm</u>

Assumptions:

- Given any $u \in \{0, 1\}^n$, can efficiently compute $f($
- Nonzero $s \in \{0, 1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.



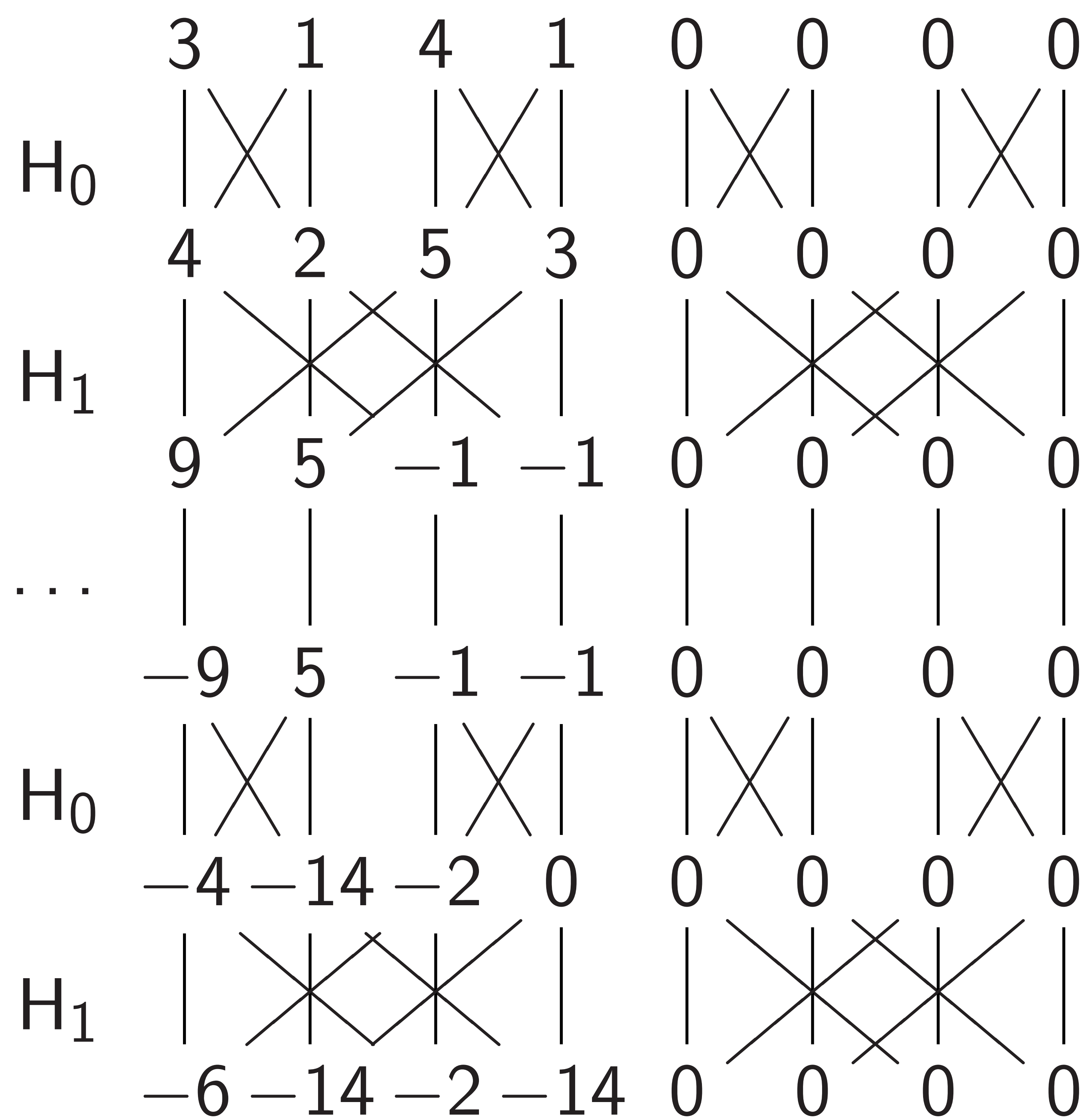
Simon's algorithm

Assumptions:

- Given any $u \in \{0, 1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0, 1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Affects measurements: "Negate amplitude around its average."
$(3,1,4,1) \mapsto (1.5,3.5,0.5,3.5)$.



Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Affects measurements: "Negate amplitude around its average."
$(3, 1, 4, 1) \mapsto (1.5, 3.5, 0.5, 3.5)$.

	3	1	4	1	0	0	0	0
H_0								
	4	2	5	3	0	0	0	0
H_1								
	9	5	−1	−1	0	0	0	0
$\cdots$								
	−9	5	−1	−1	0	0	0	0
H_0								
	−4	−14	−2	0	0	0	0	0
H_1								
	−6	−14	−2	−14	0	0	0	0

Simon's algorithm

Assumptions:

- Given any $u \in \{0, 1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0, 1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

measurements: "Negate

de around its average."

1) $\mapsto$ (1.5, 3.5, 0.5, 3.5).

1	4	1	0	0	0	0
2	5	3	0	0	0	0
5	−1	−1	0	0	0	0
5	−1	−1	0	0	0	0
−14	−2	0	0	0	0	0
−14	−2	−14	0	0	0	0

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example

Step 1.

1, 0, 0,

0, 0, 0,

0, 0, 0,

0, 0, 0,

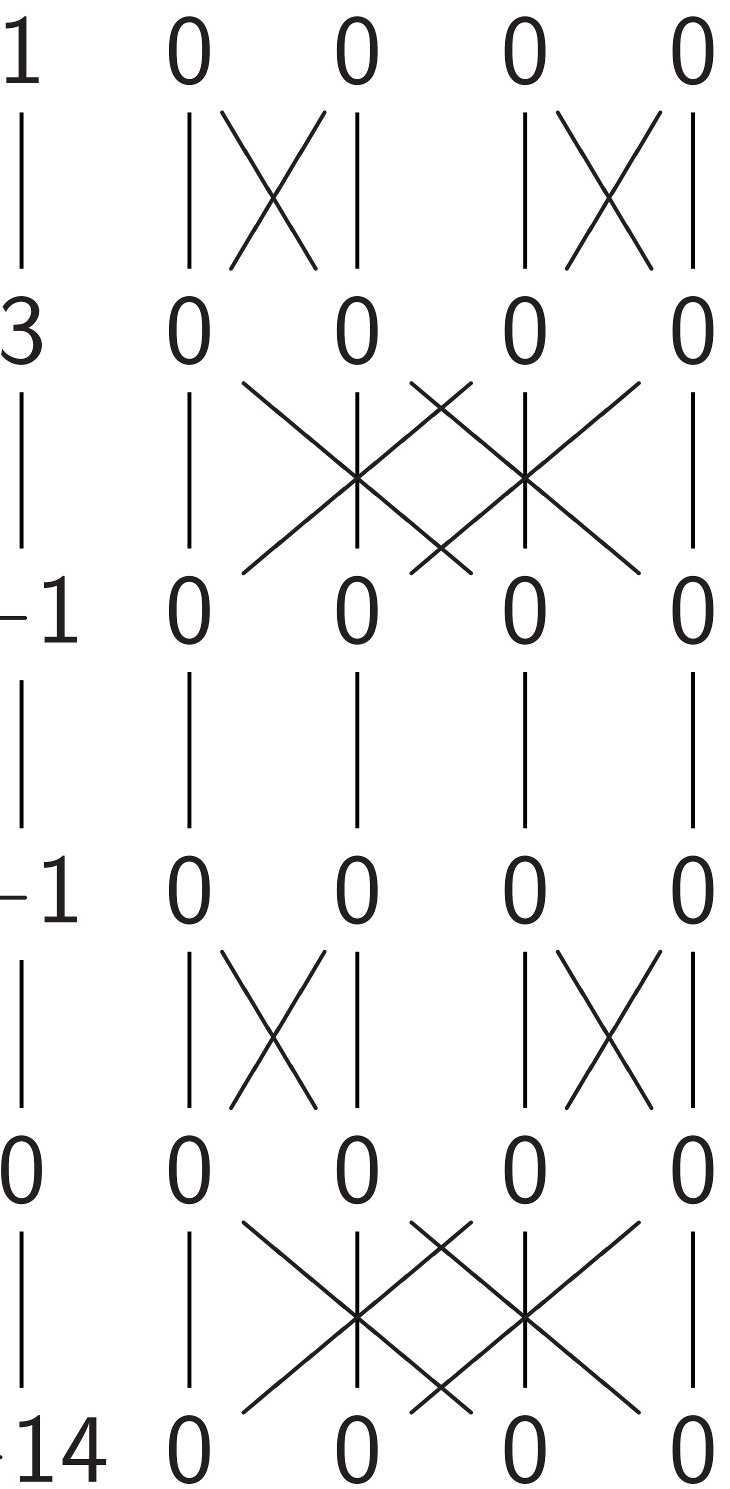
0, 0, 0,

0, 0, 0,

0, 0, 0,

0, 0, 0,

ents: "Negate
its average."
3.5, 0.5, 3.5).



Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon

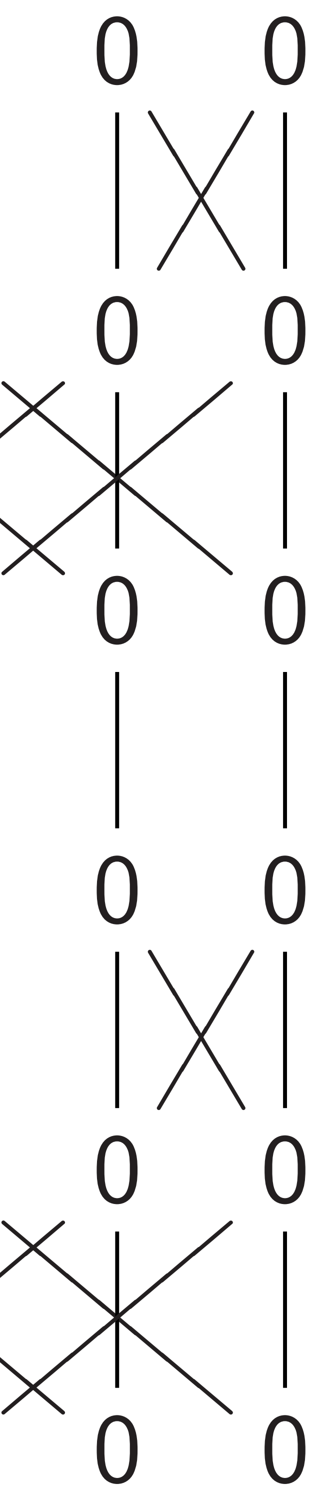
Step 1. Set up pu

1, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0

gate

e.”

.5).



Simon’s algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon’s algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon’s algorith

Step 1. Set up pure zero sta

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 1. Set up pure zero state:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 2. Hadamard$_0$:

1, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 3. Hadamard$_1$:

1, 1, 1, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 4. Hadamard$_2$:

1, 1, 1, 1, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5. C_0NOT_3:

1, 0, 1, 0, 1, 0, 1, 0,
0, 1, 0, 1, 0, 1, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5b. More shuffling:

1, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5c. More shuffling:

1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 1.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5d. More shuffling:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5e. More shuffling:

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5f. More shuffling:

0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5g. More shuffling:

0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5h. More shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5i. More shuffling:

0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 1, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5j. Final shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

Each column is a parallel universe performing its own computations.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 5j. Final shuffling:

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

Each column is a parallel universe performing its own computations.
Surprise: u and $u \oplus 101$ match.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 6. Hadamard$_0$:

$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$0,\ 0,\ 1,\ \overline{1},\ 0,\ 0,\ 1,\ 1,$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$0,\ 0,\ 1,\ 1,\ 0,\ 0,\ 1,\ \overline{1},$
$1,\ \overline{1},\ 0,\ 0,\ 1,\ 1,\ 0,\ 0,$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$1,\ 1,\ 0,\ 0,\ 1,\ \overline{1},\ 0,\ 0.$

Notation: $\overline{1}$ means -1.

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 7. Hadamard$_1$:

$0, 0, 0, 0, 0, 0, 0, 0,$

$1, \overline{1}, \overline{1}, 1, 1, 1, \overline{1}, \overline{1},$

$0, 0, 0, 0, 0, 0, 0, 0,$

$1, 1, \overline{1}, \overline{1}, 1, \overline{1}, \overline{1}, 1,$

$1, \overline{1}, 1, \overline{1}, 1, 1, 1, 1,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$1, 1, 1, 1, 1, \overline{1}, 1, \overline{1}.$

Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

Step 8. Hadamard$_2$:

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Simon's algorithm

Assumptions:

- Given any $u \in \{0,1\}^n$, can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

Traditional algorithm to find s: compute f for many inputs, hope to find collision.

Simon's algorithm finds s with $\approx n$ quantum computations of f.

Example of Simon's algorithm

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Step 9: Measure. Obtain some information about the surprise: a random vector orthogonal to 101.

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Generalize Step 5 to any function $u \mapsto f(u)$ with $f(u) = f(u \oplus s)$. "Usually" algorithm figures out s.

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Start from uniform superposition over all n-bit strings u.

Step 1: Set $a \leftarrow b$ where
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This is fast.

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Negate a around its average.
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Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

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Measure the n qubits. With high probability this finds s.

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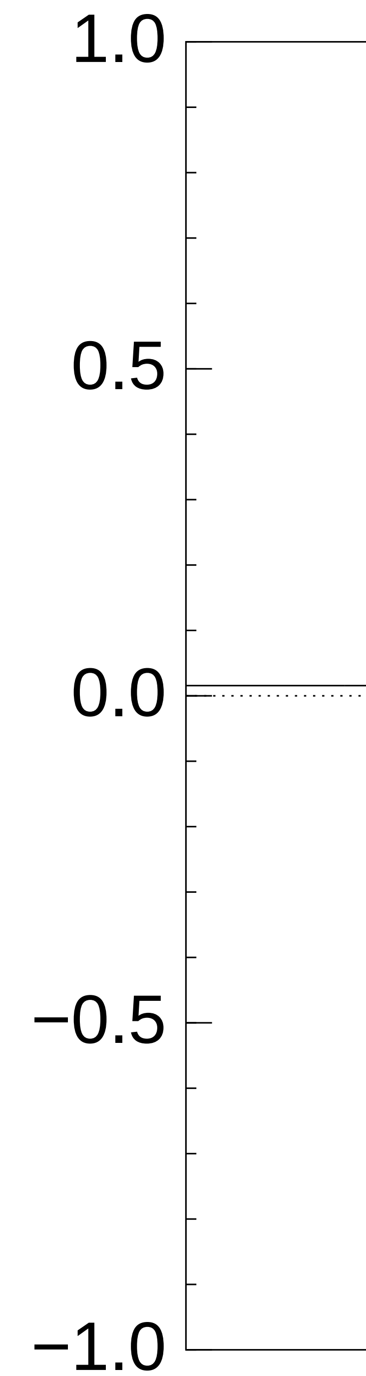

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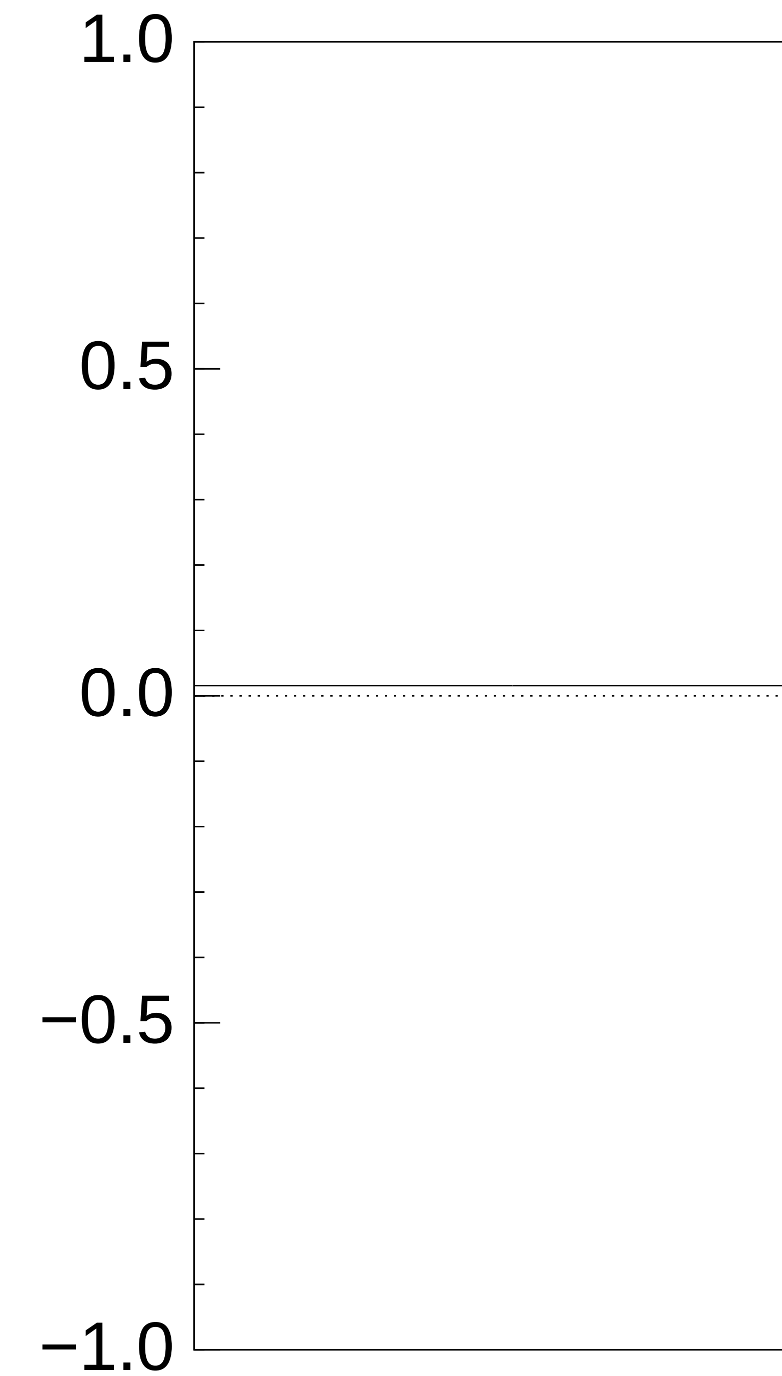

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Normalized graph of $u \mapsto a_u$
for an example with $n = 12$
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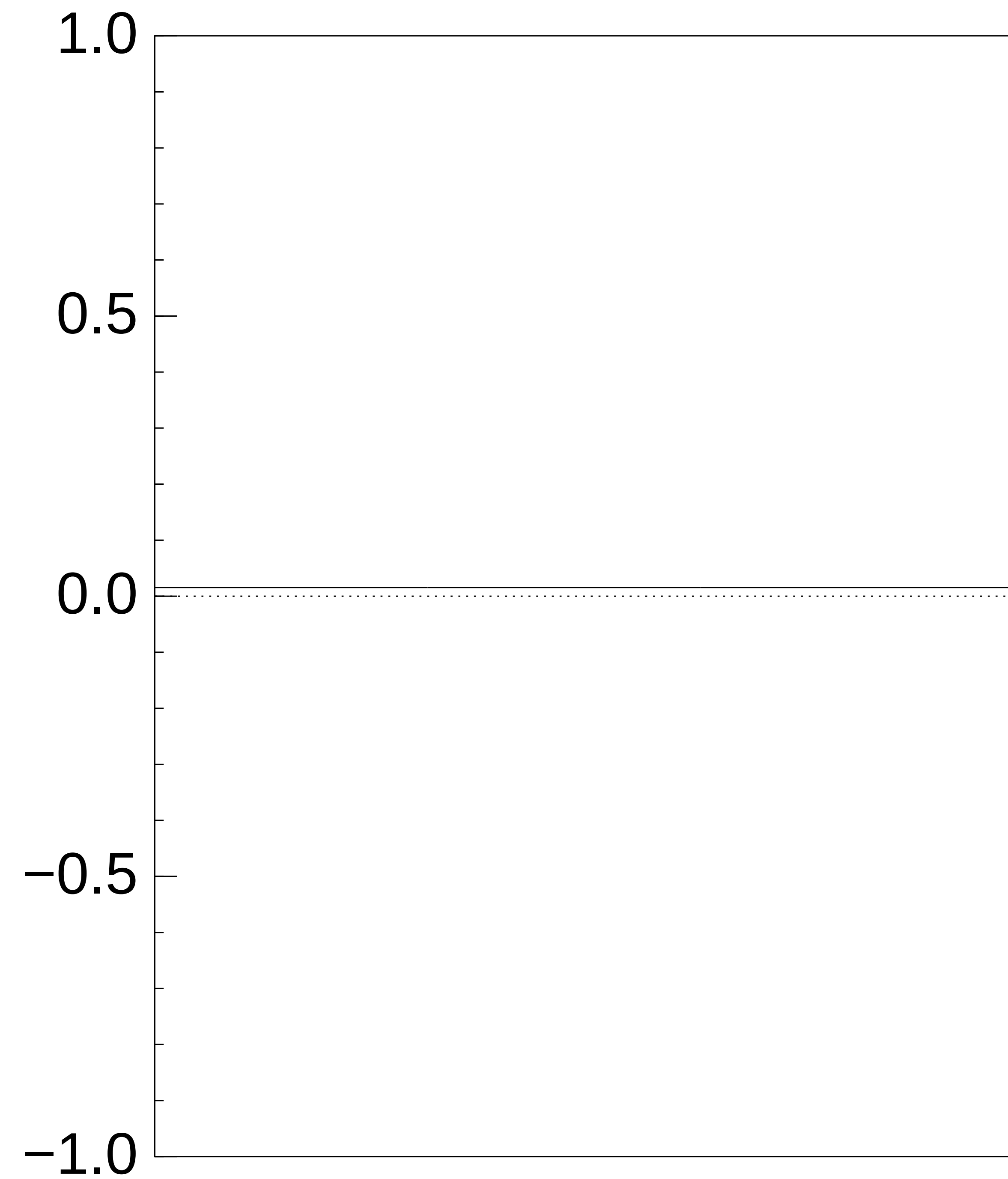

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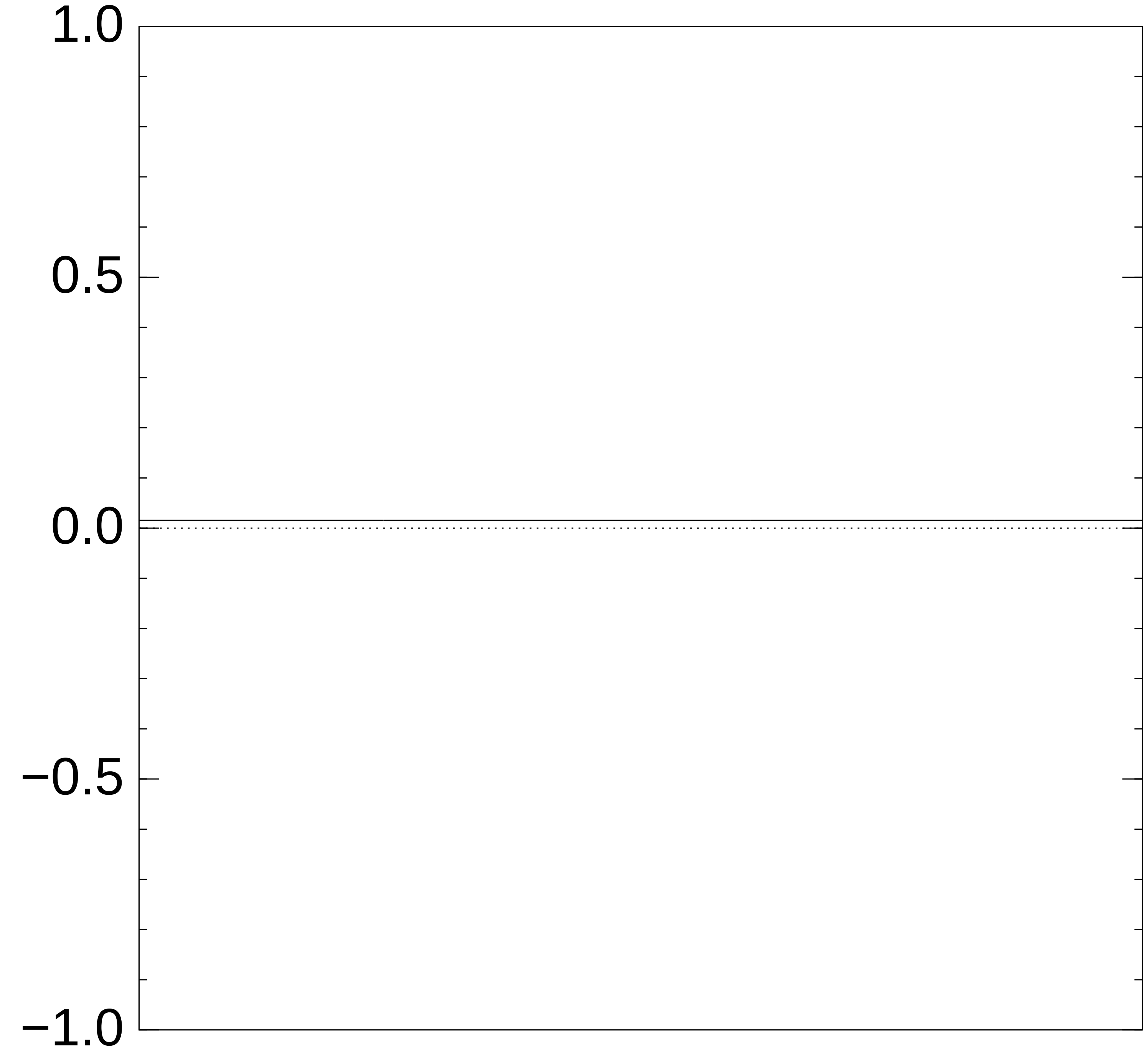

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Measure the n qubits.
With high probability this finds s.

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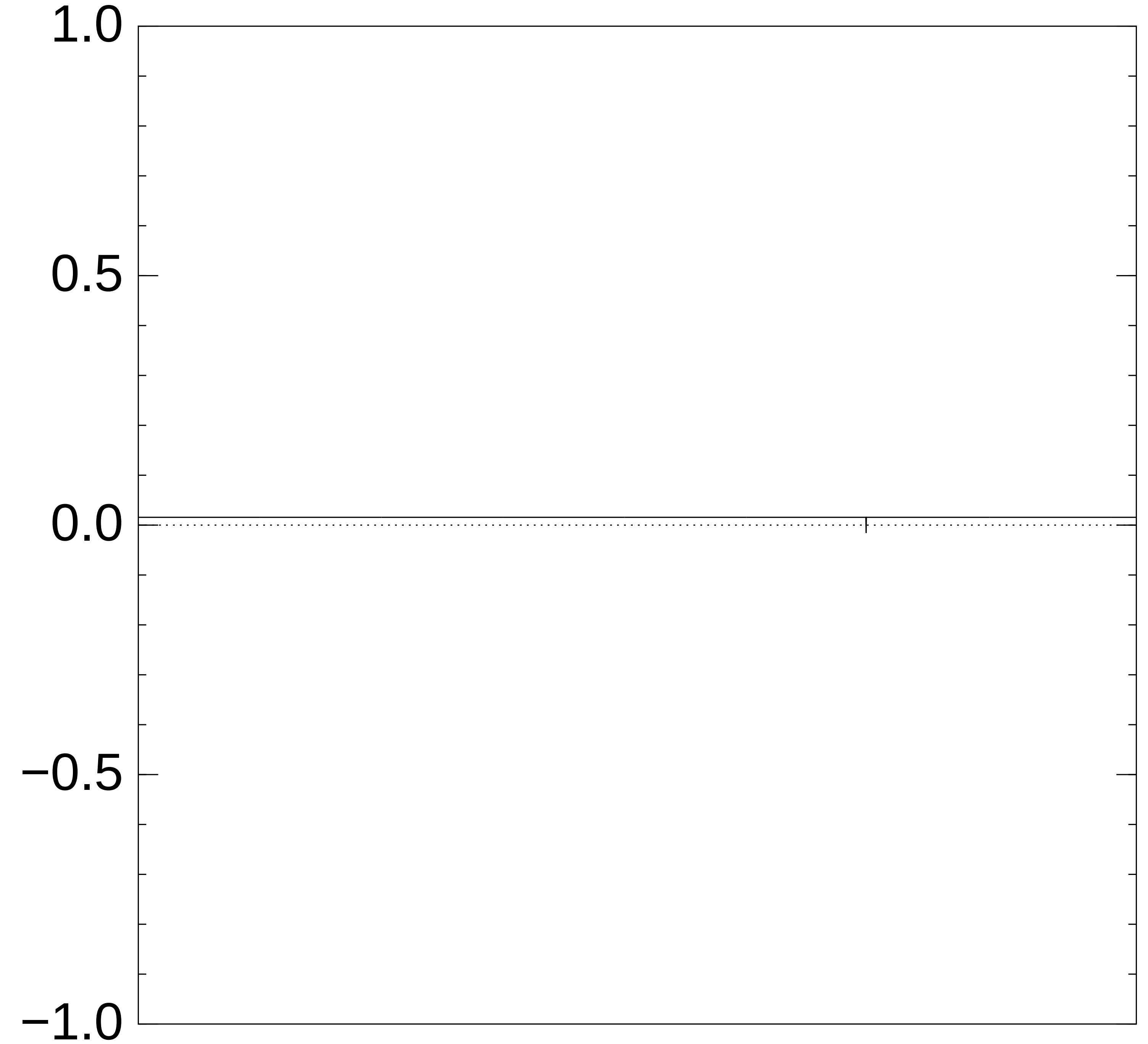

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Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

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With high probability this finds s.

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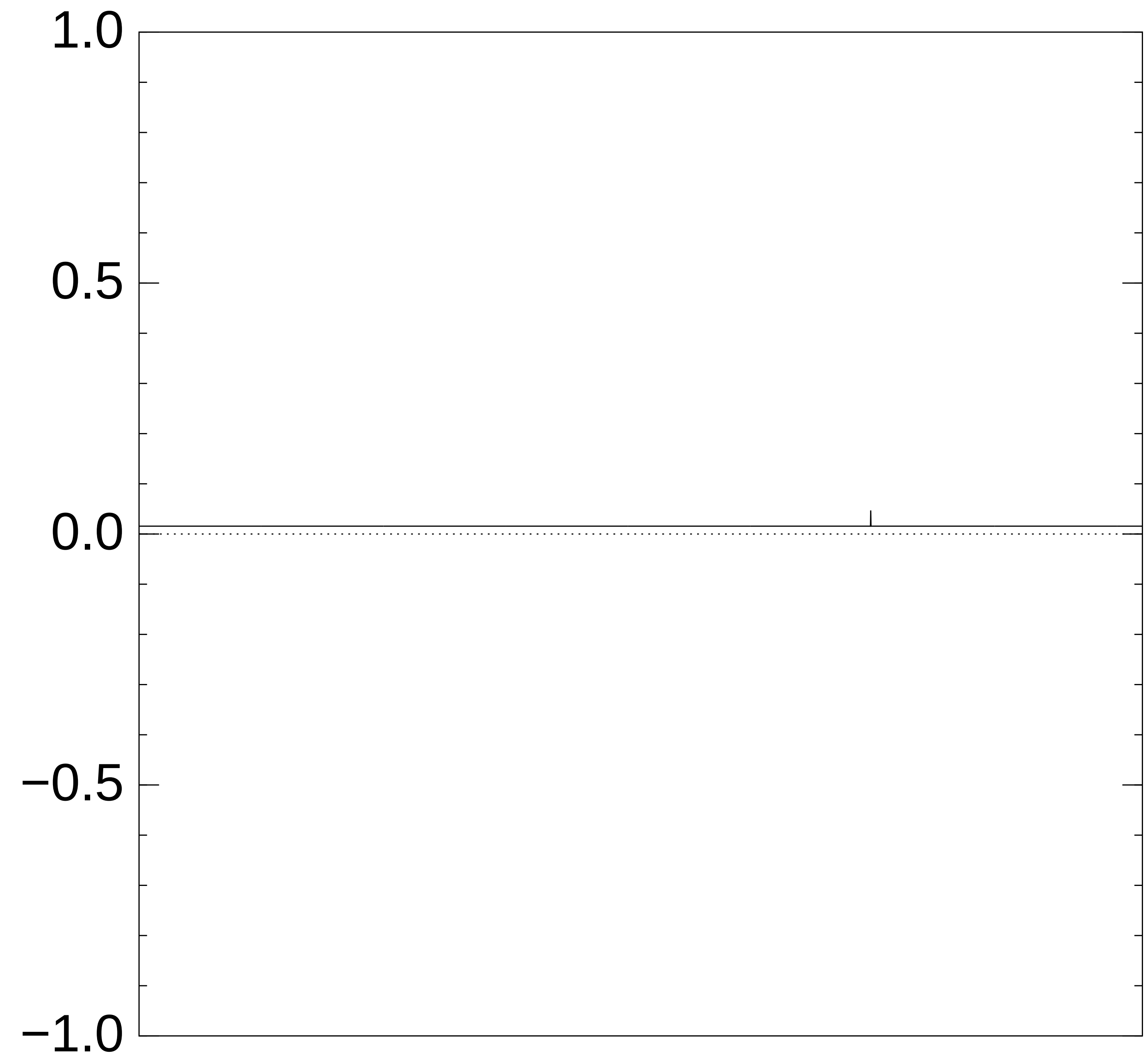

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Measure the n qubits.
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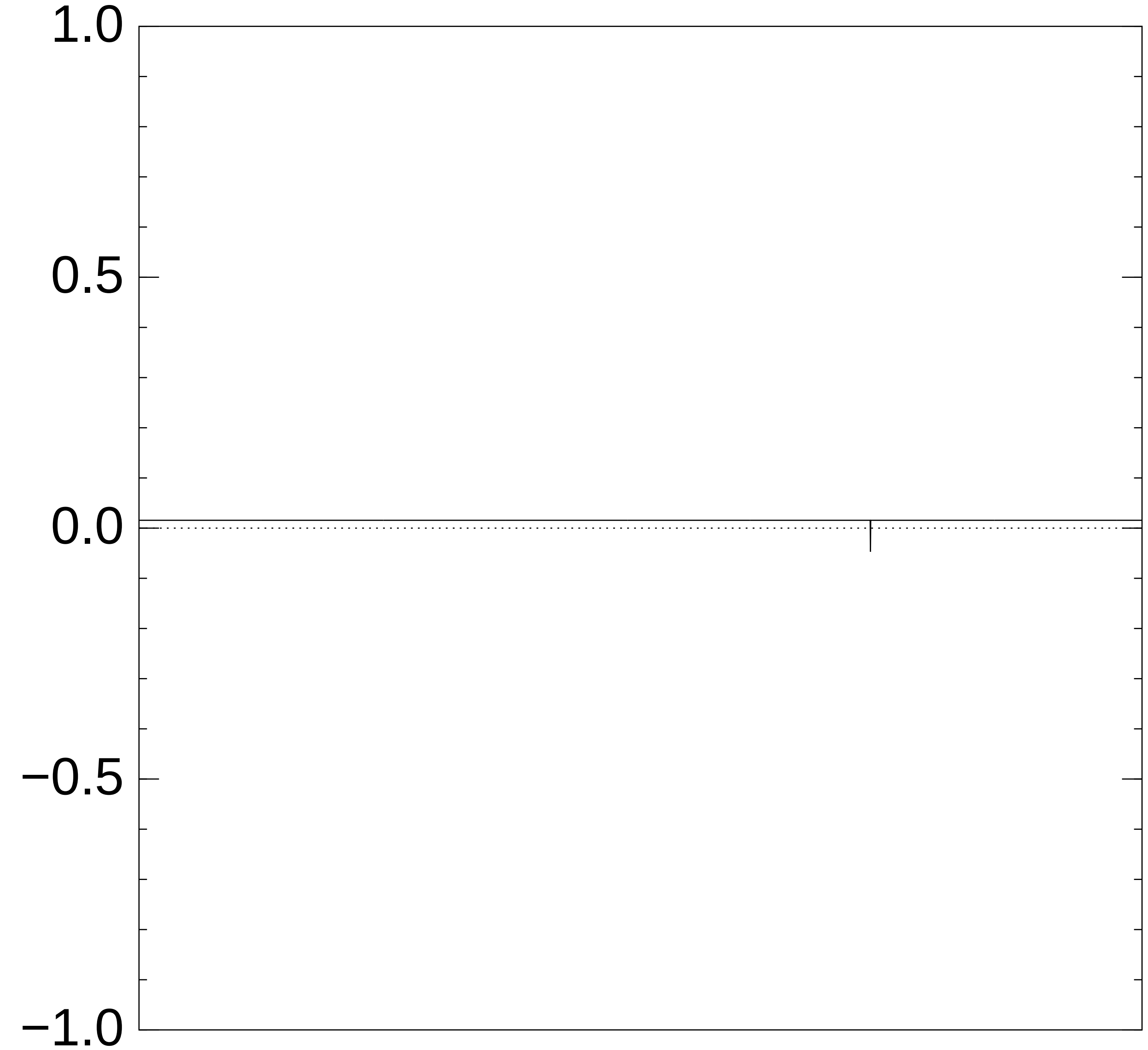

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Measure the n qubits.
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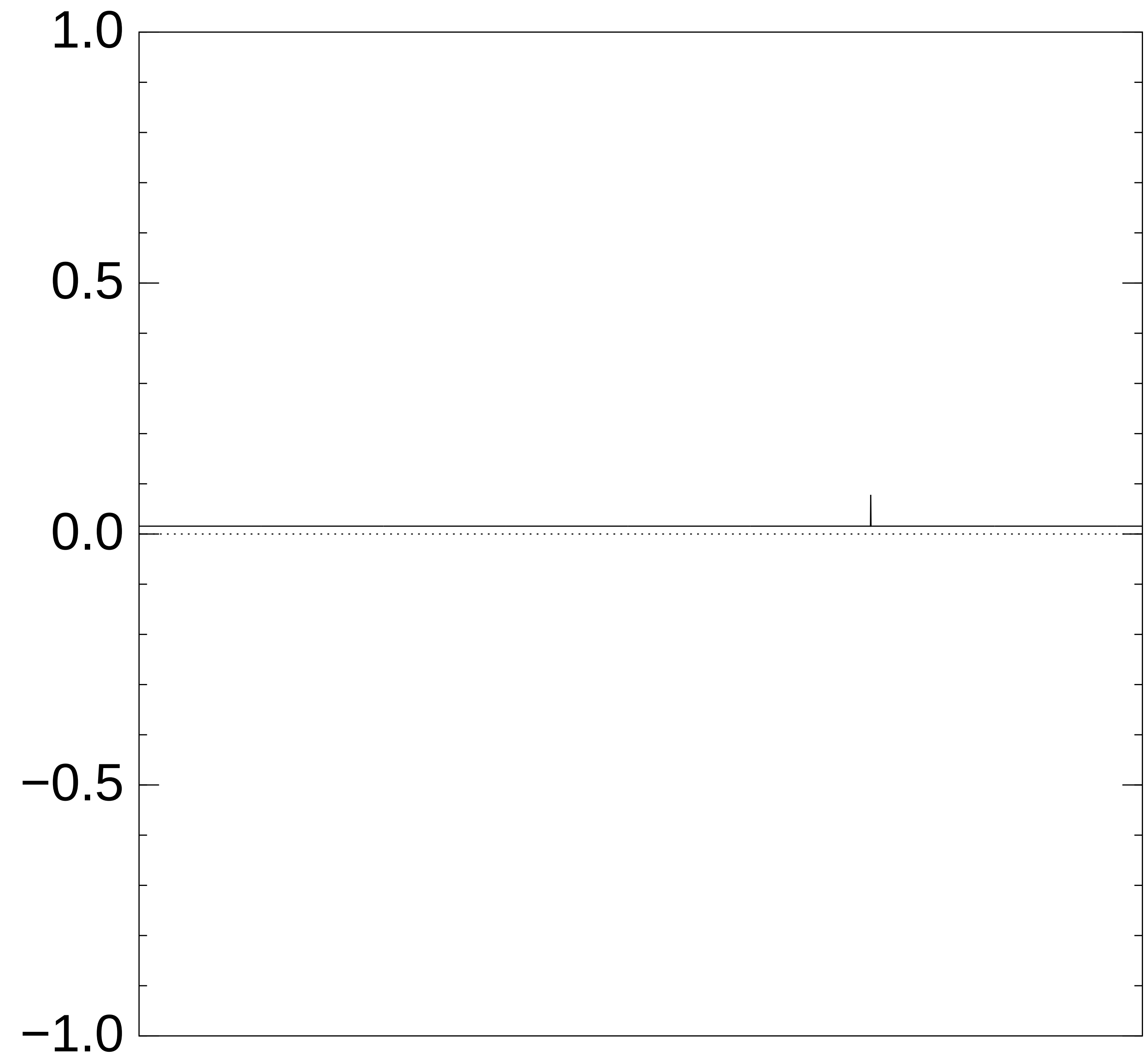
Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $2 \times$ (Step 1 + Step 2):



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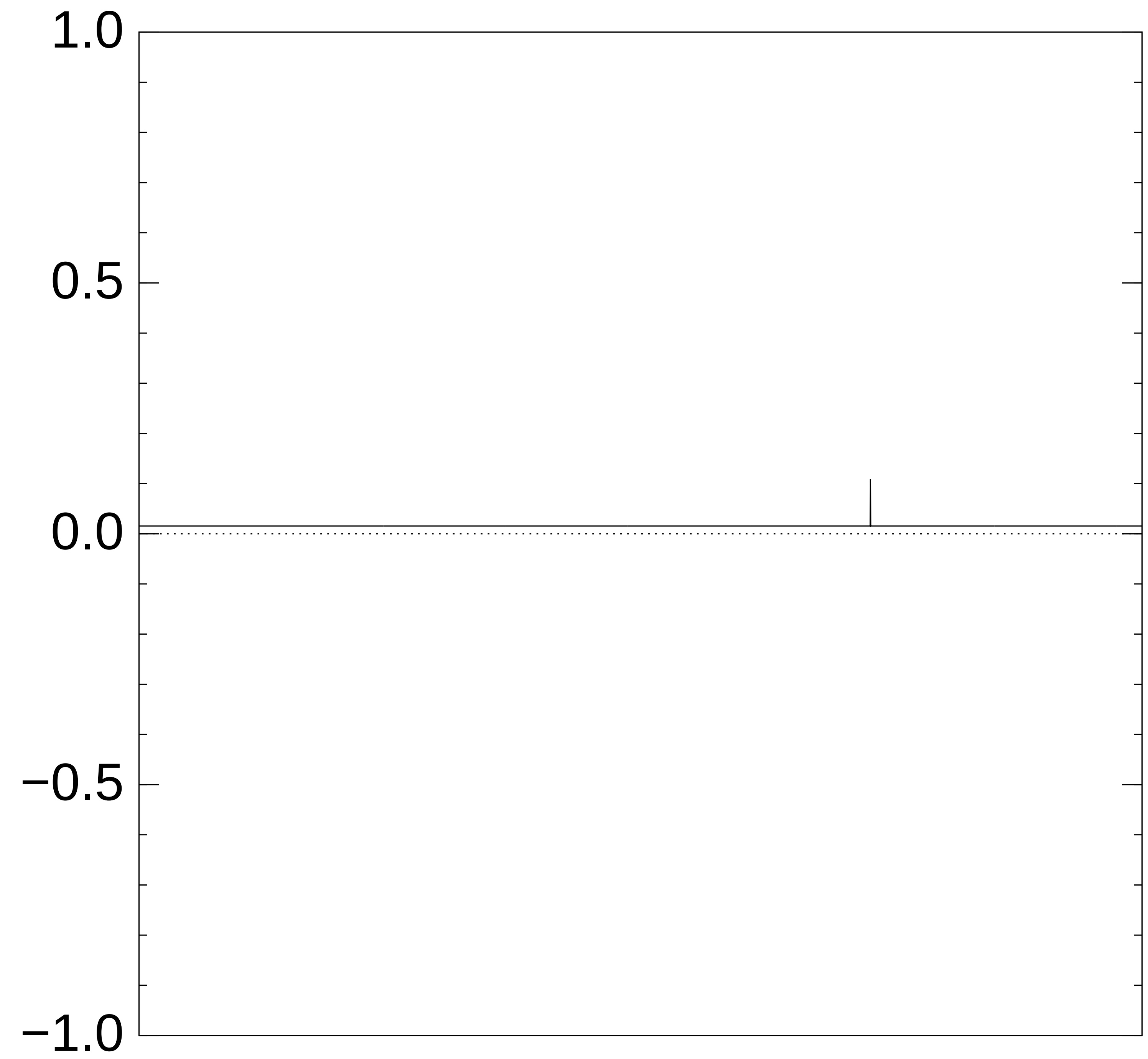
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $3 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

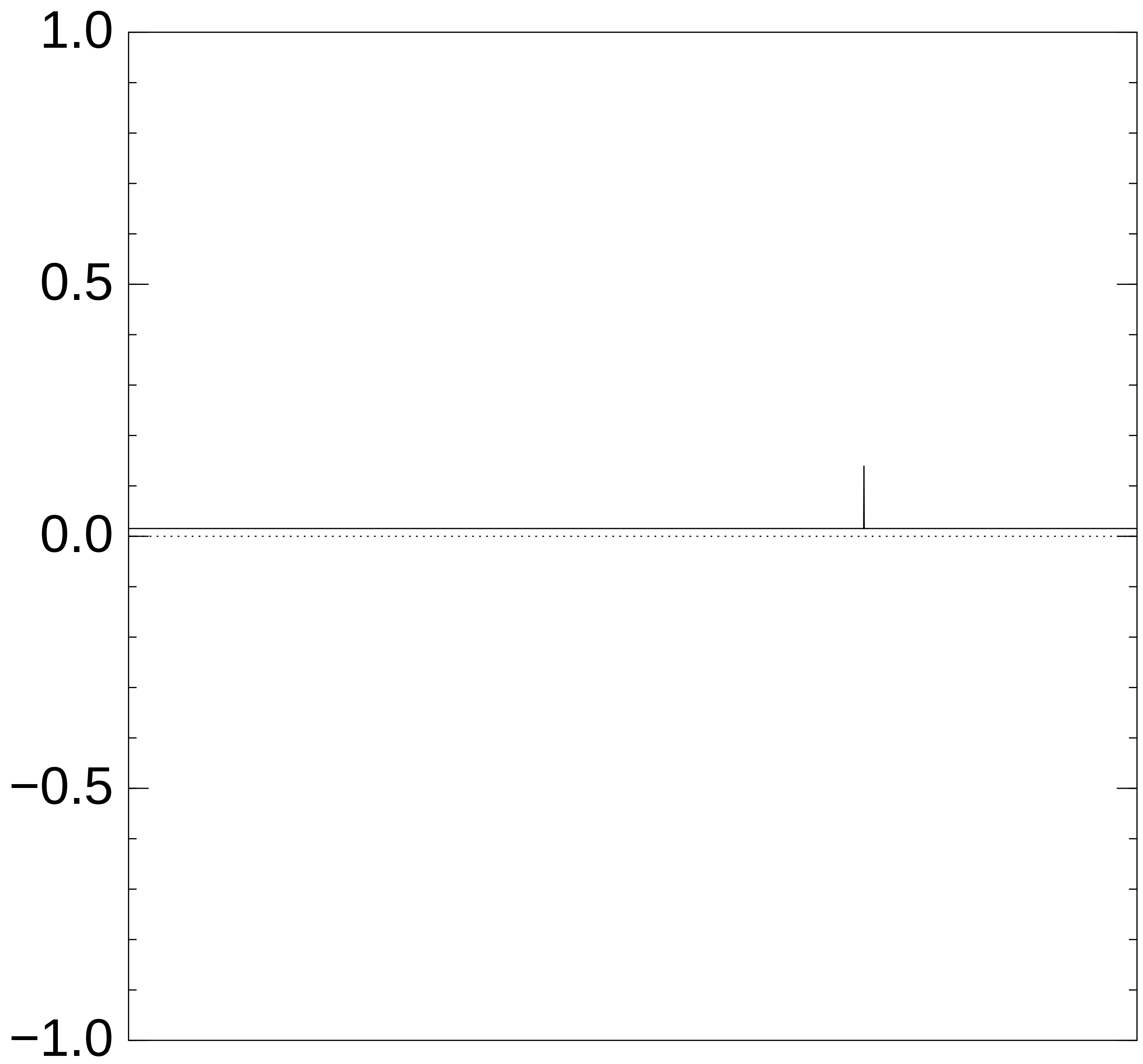
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $4 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

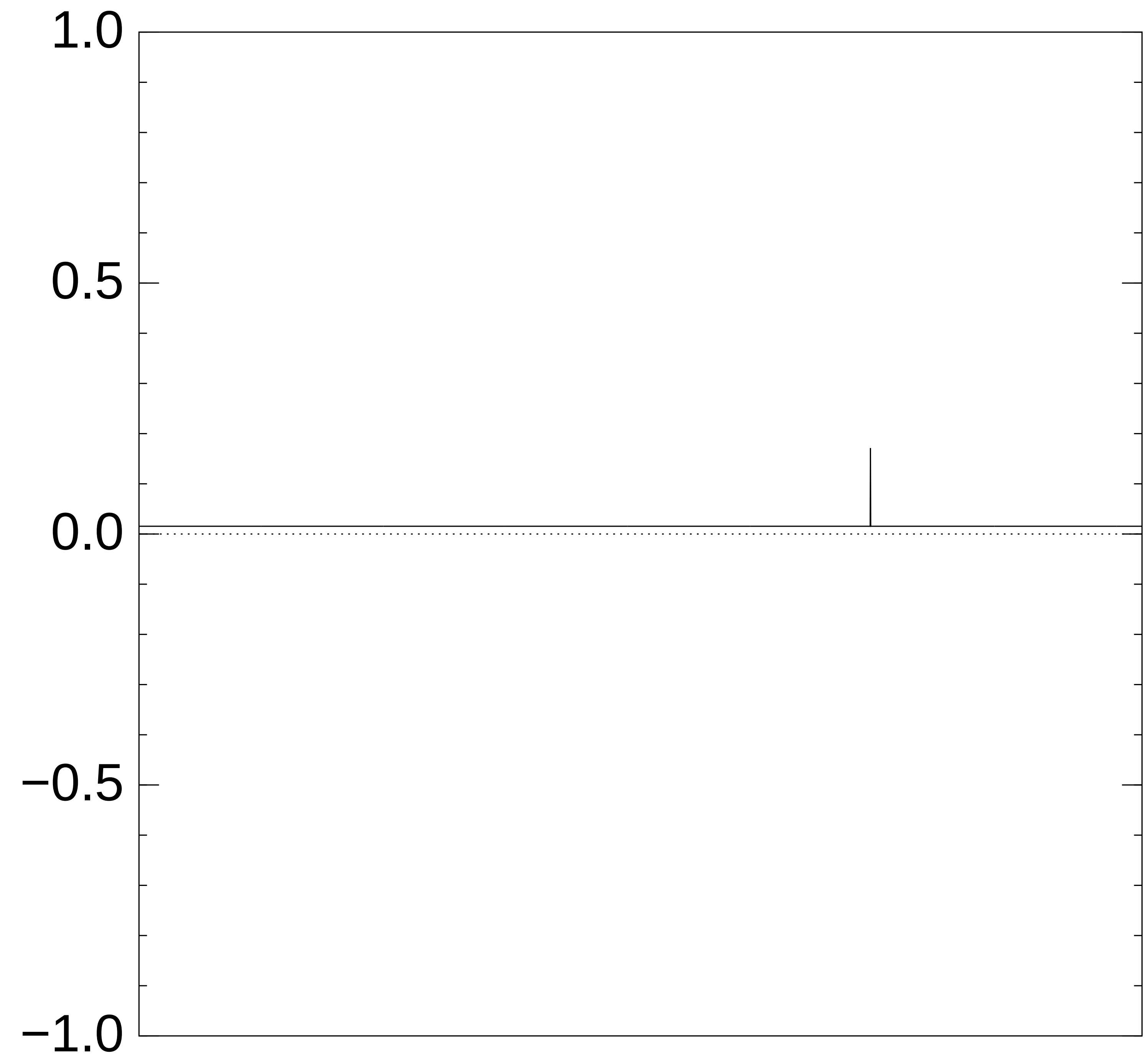
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$
for an example with $n = 12$
after $5 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

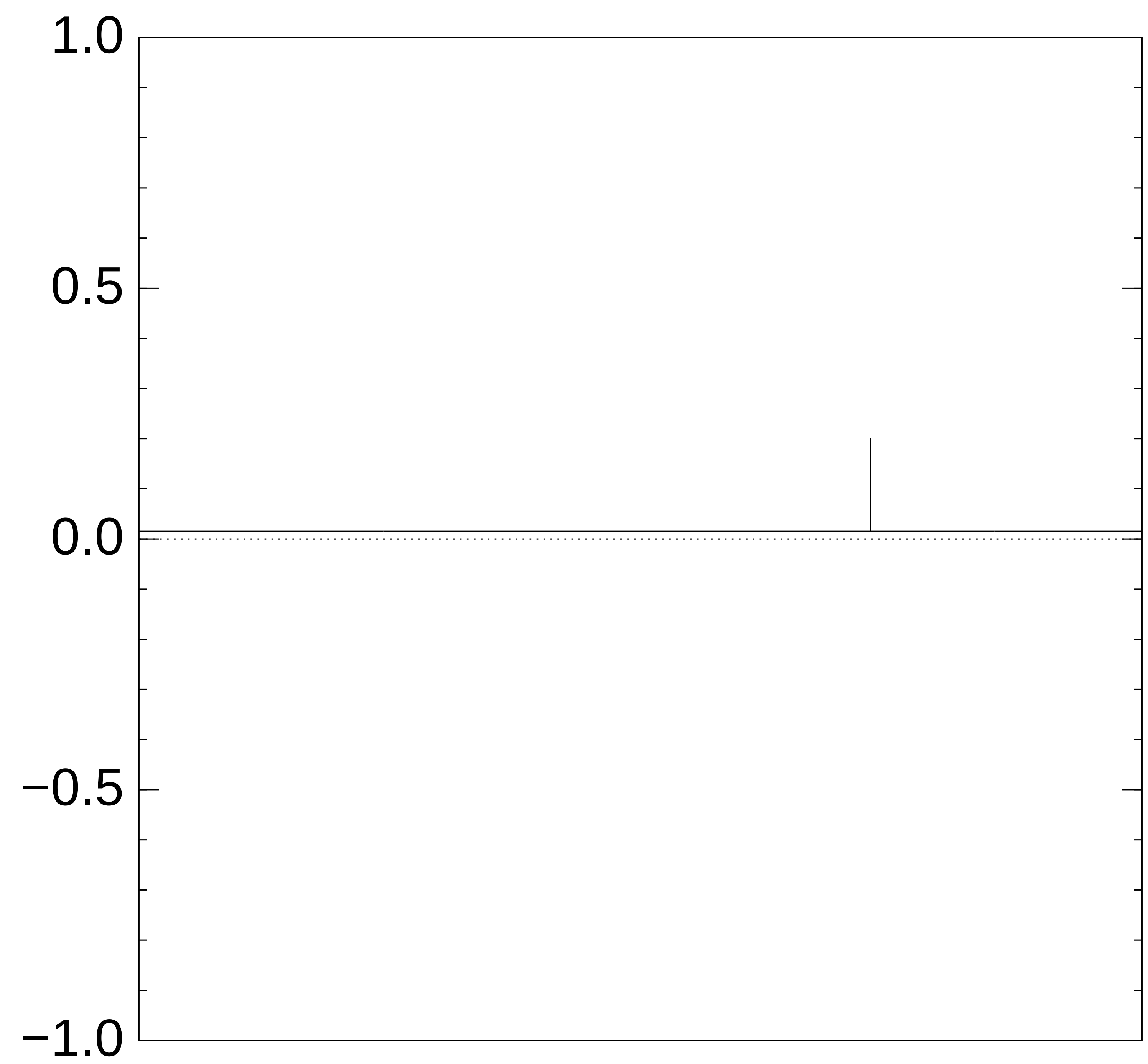
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $6 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over all n-bit strings u.

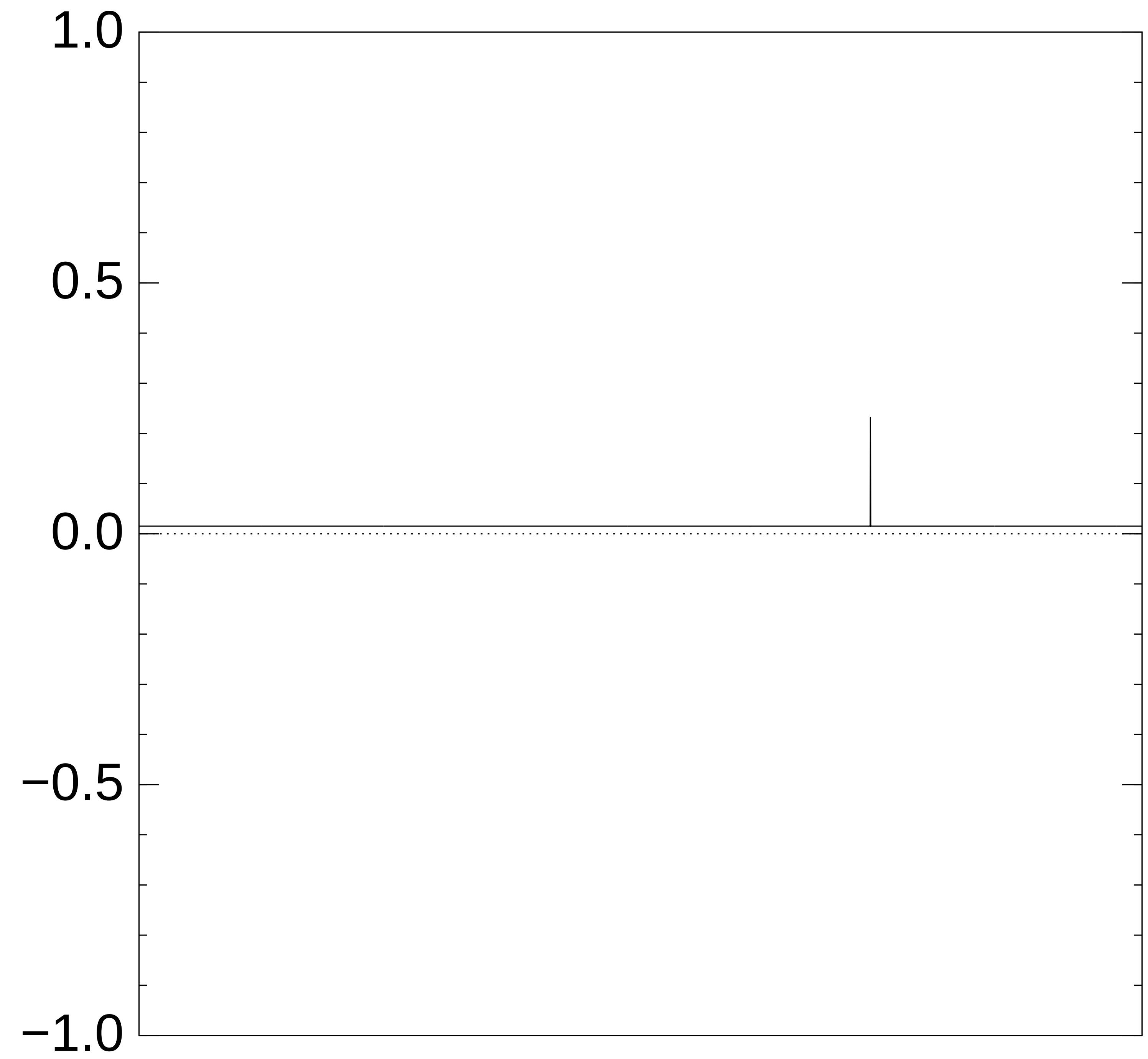
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $7 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

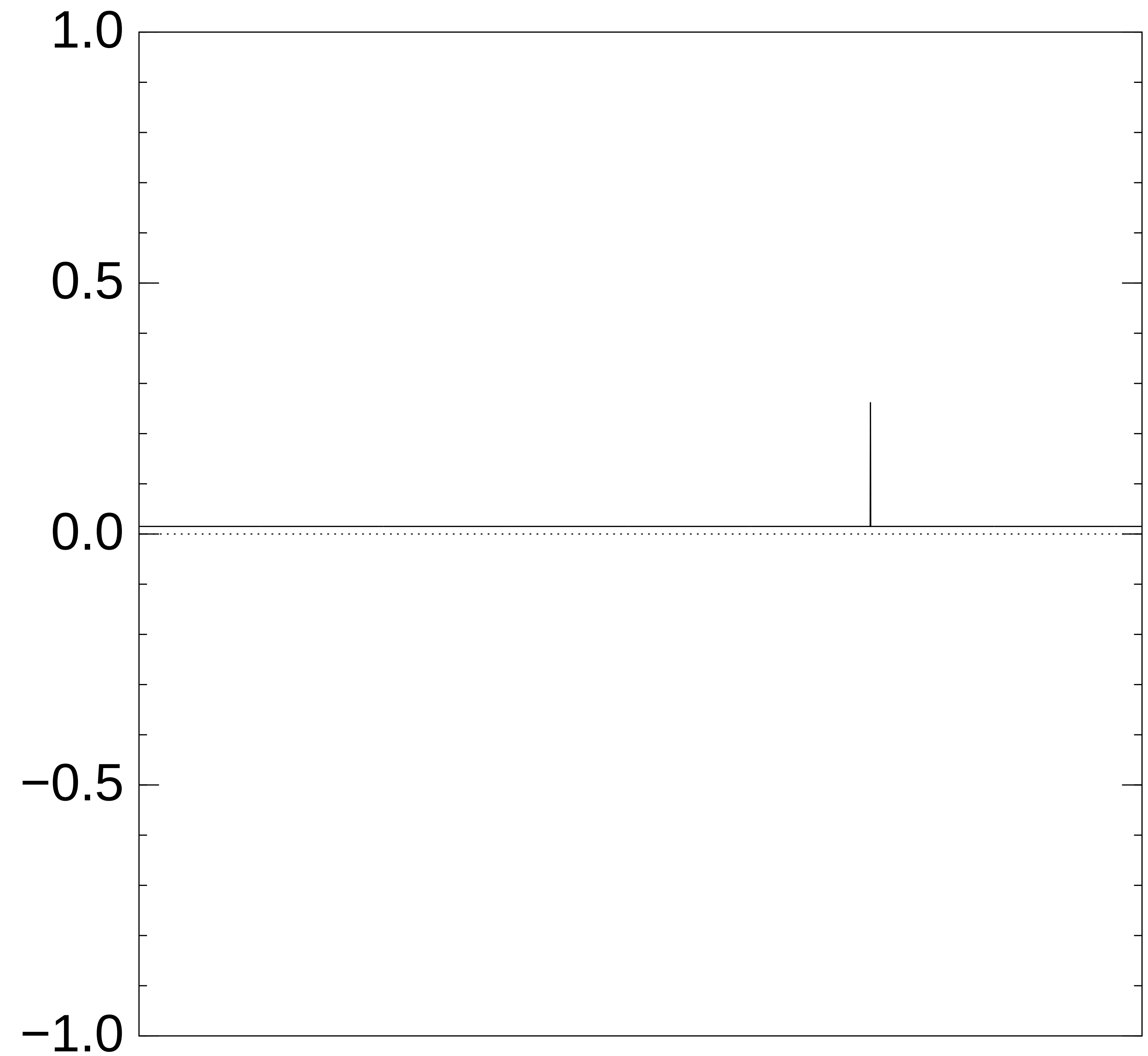
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $8 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

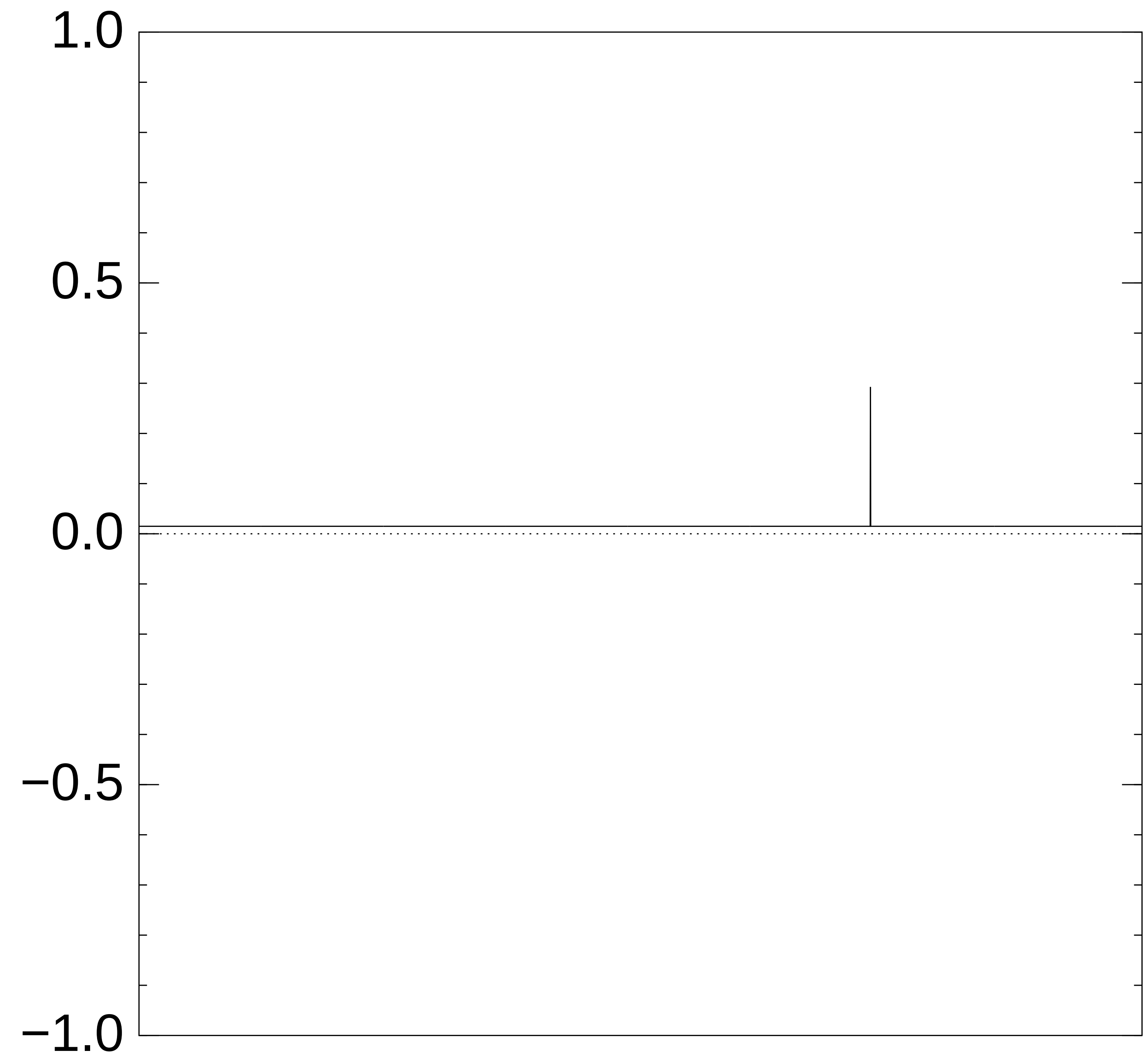
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $9 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

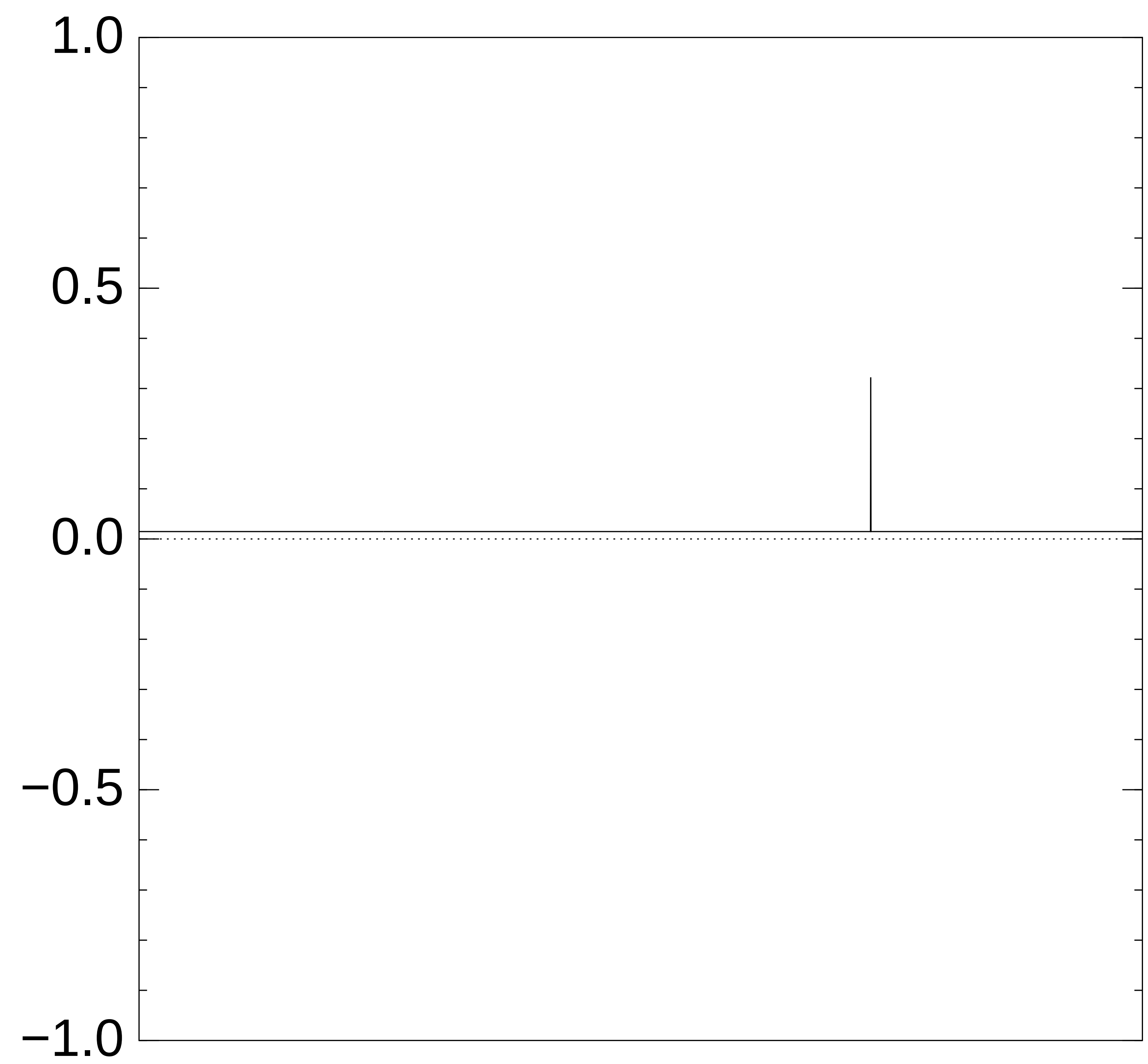
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $10 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

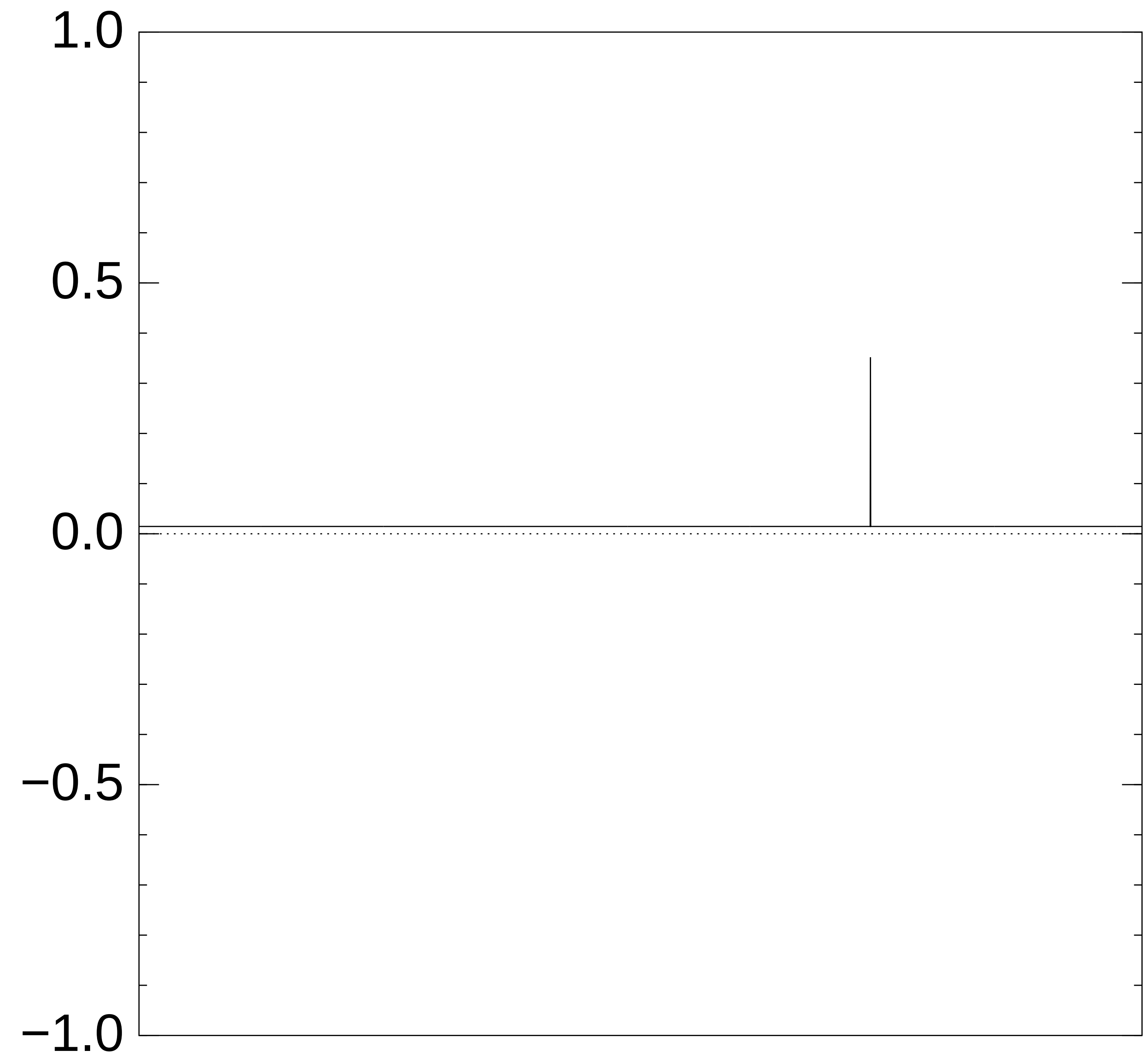
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $11 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

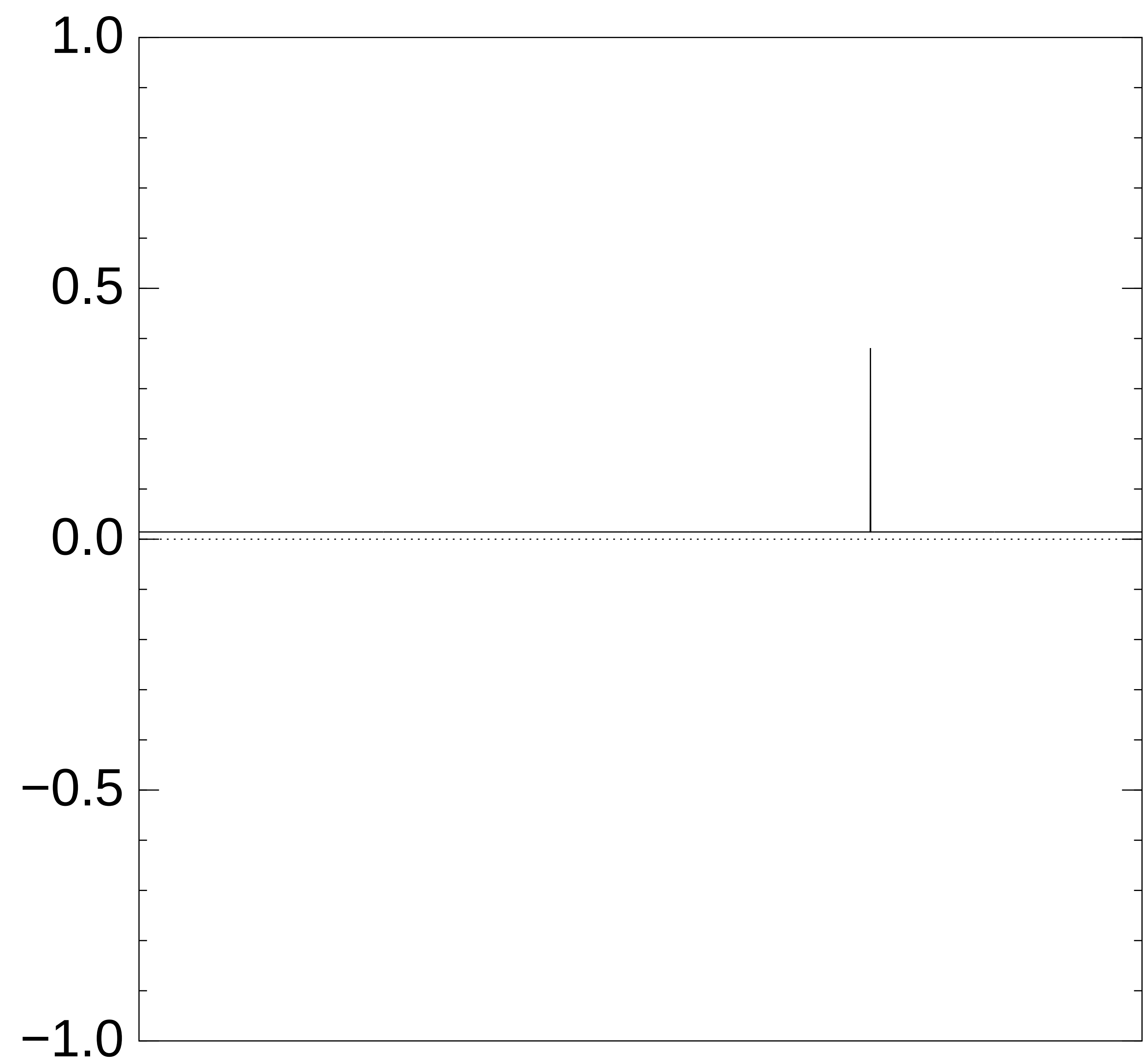
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $12 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

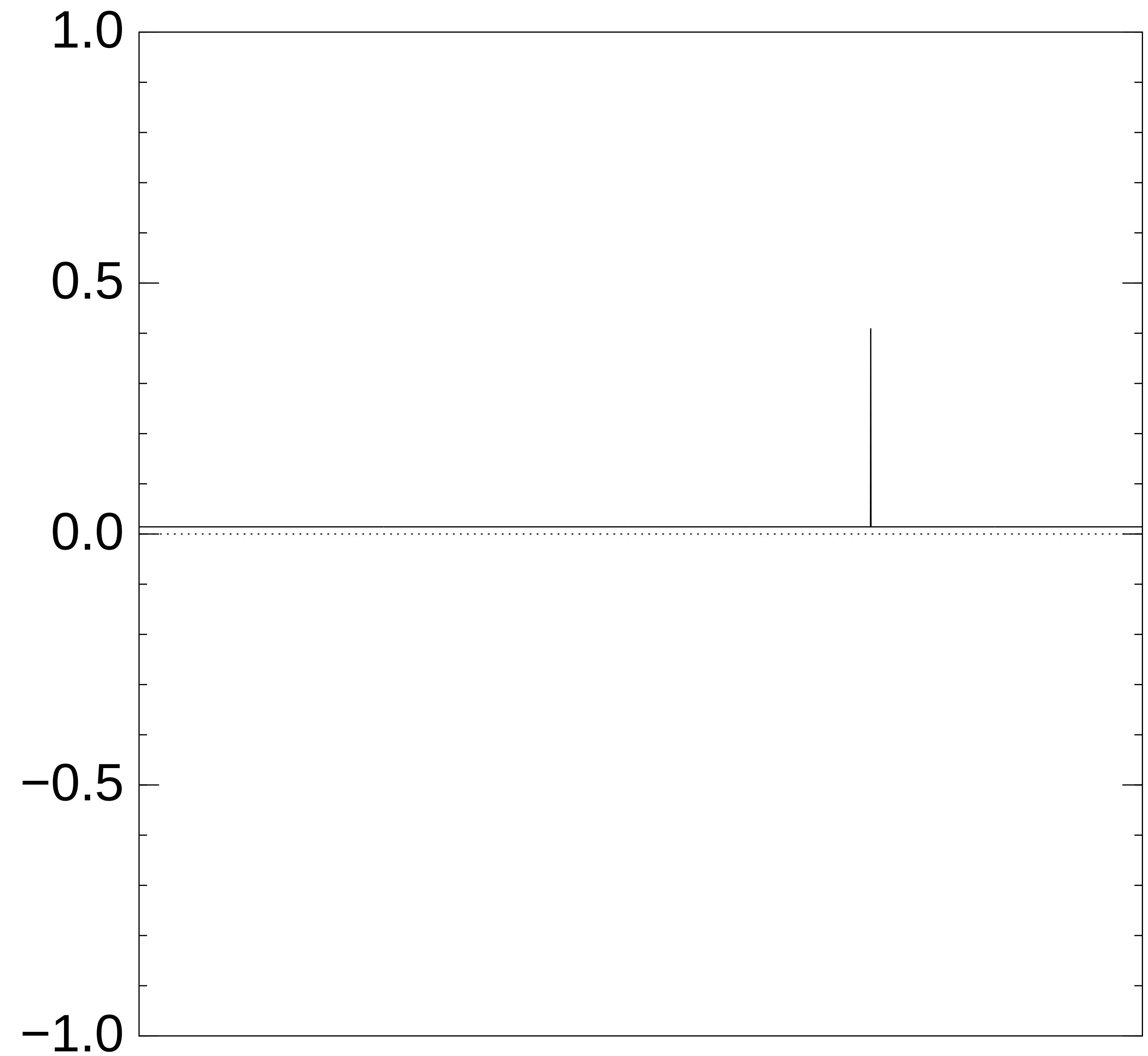
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $13 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

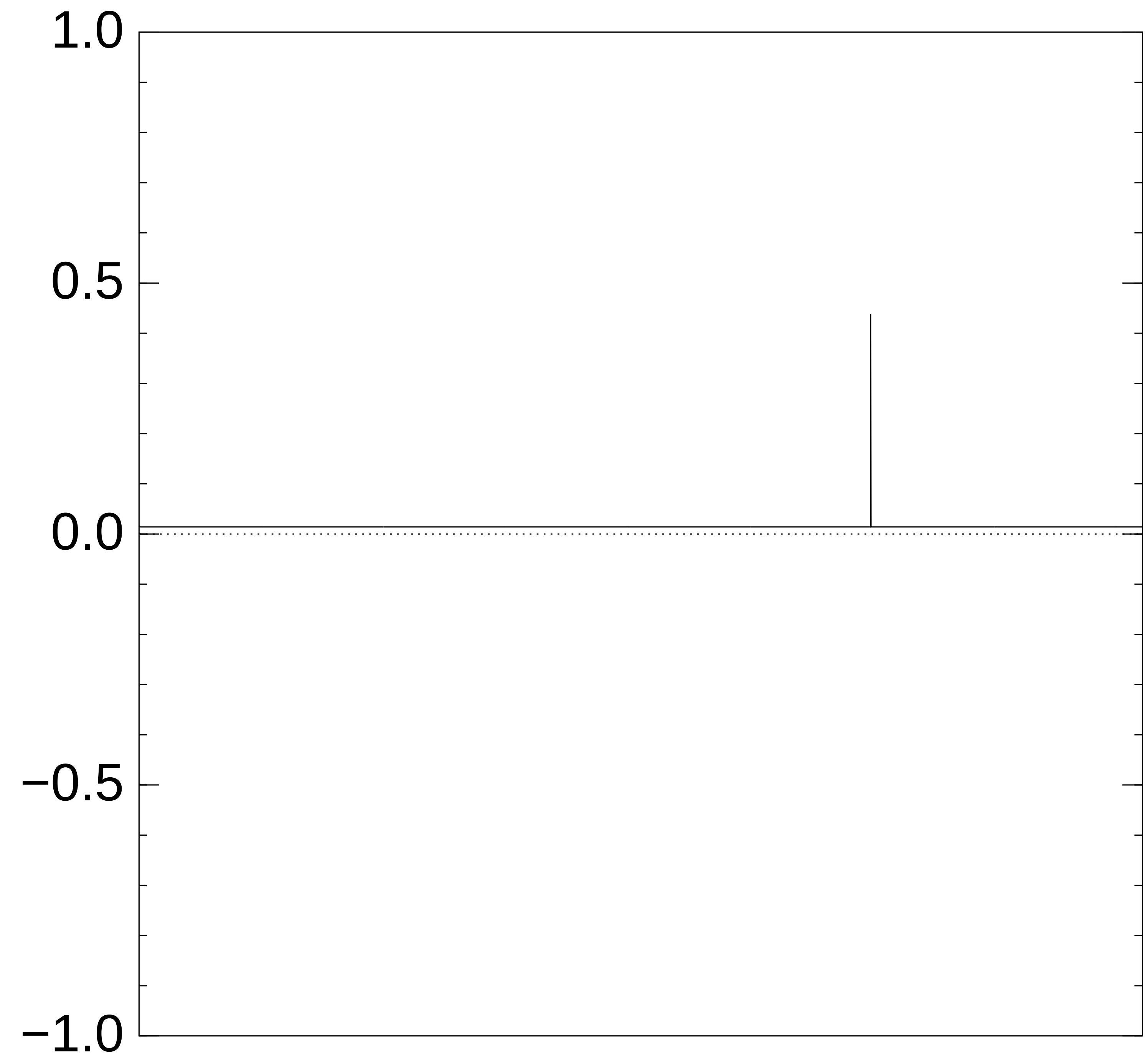
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $14 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over all n-bit strings u.

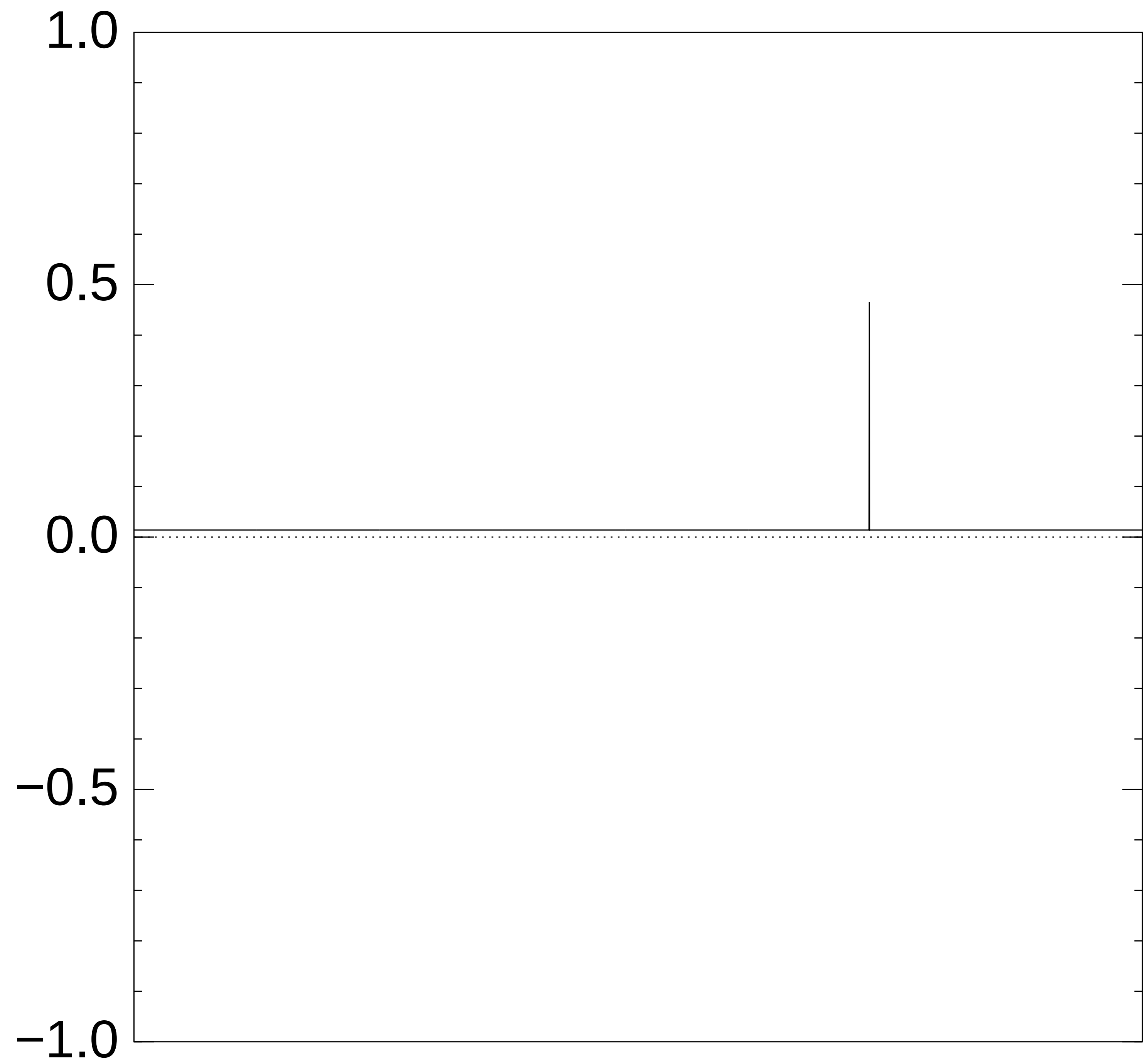
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $15 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over all n-bit strings u.

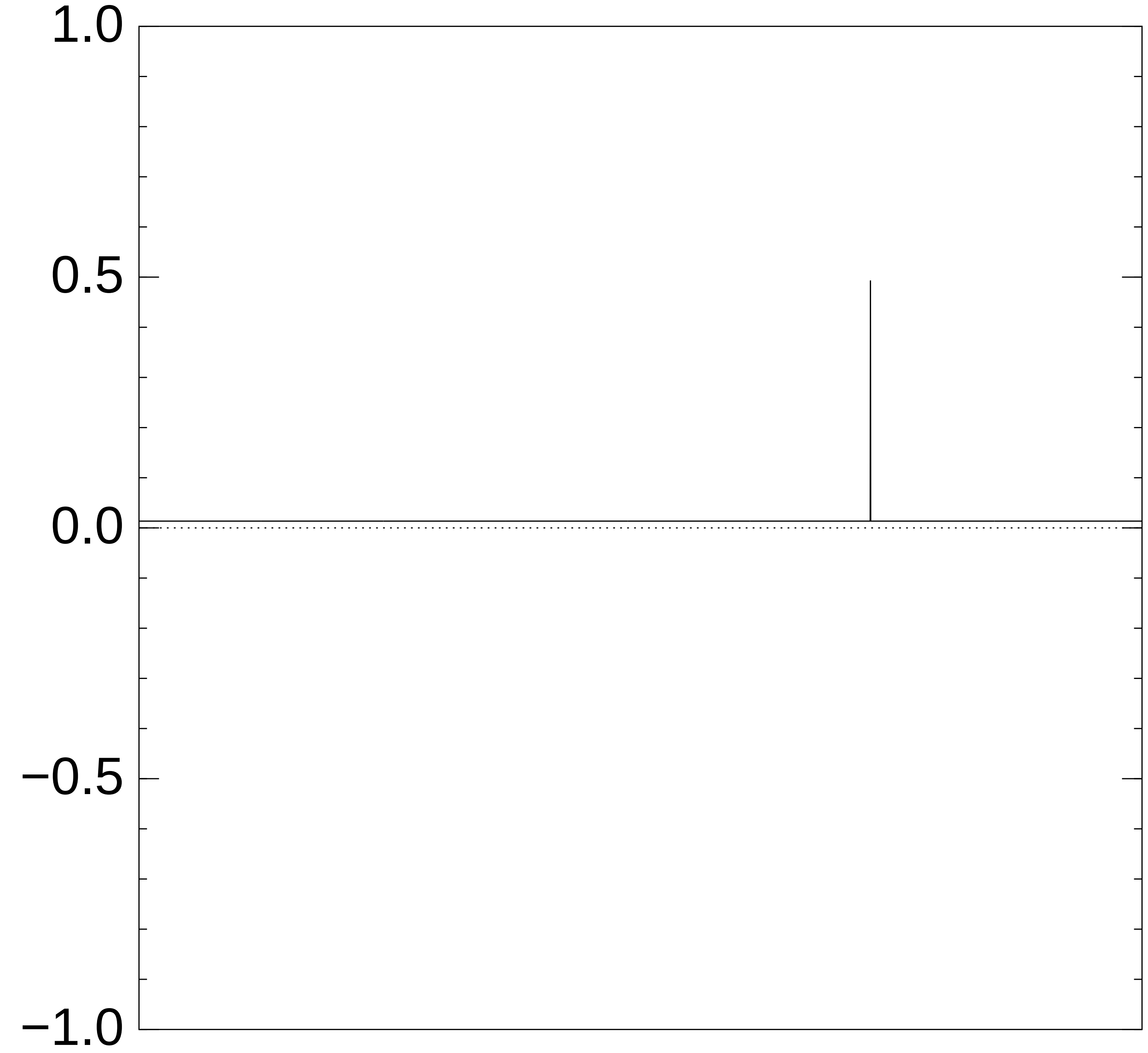
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $16 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

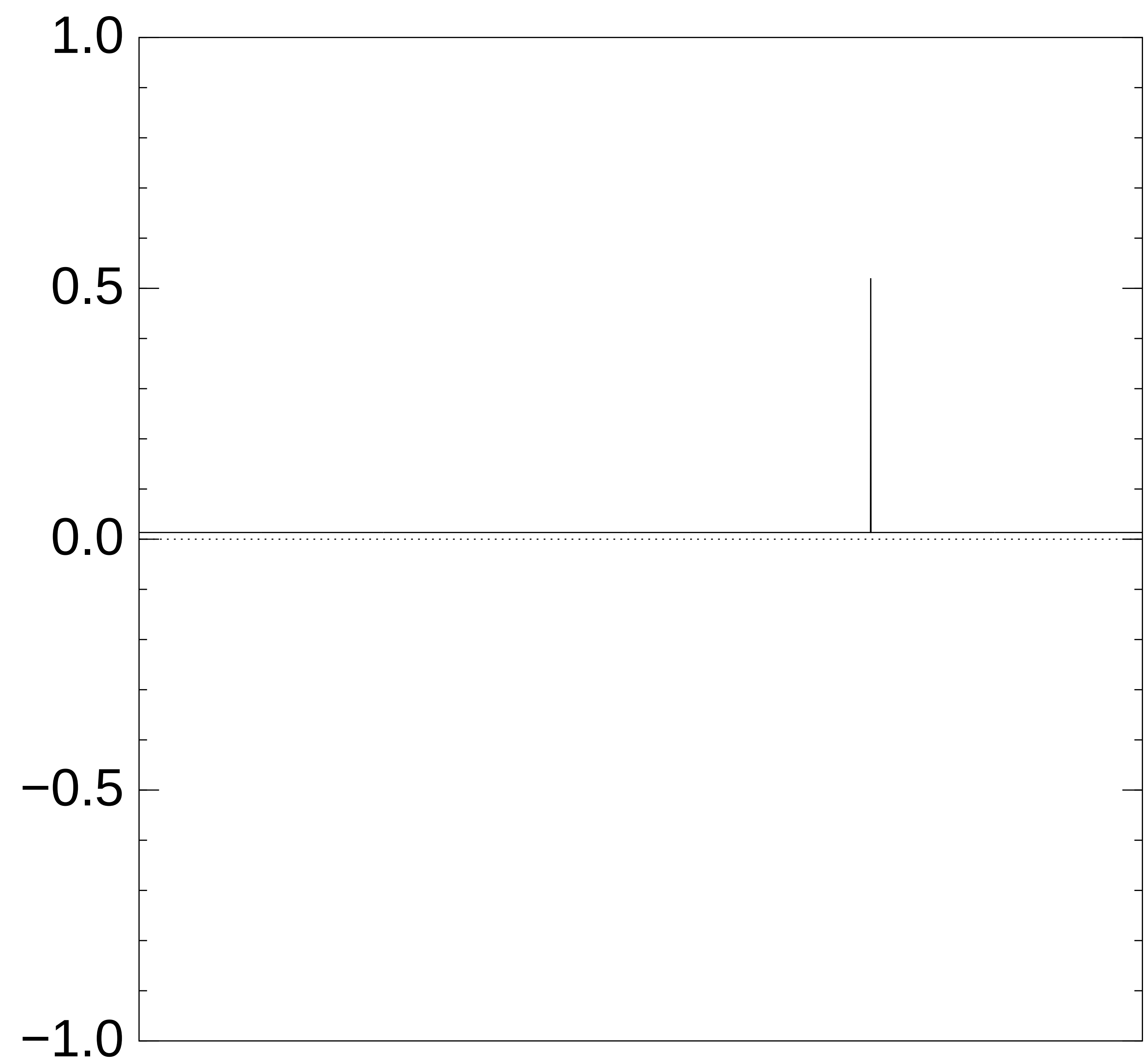
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $17 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

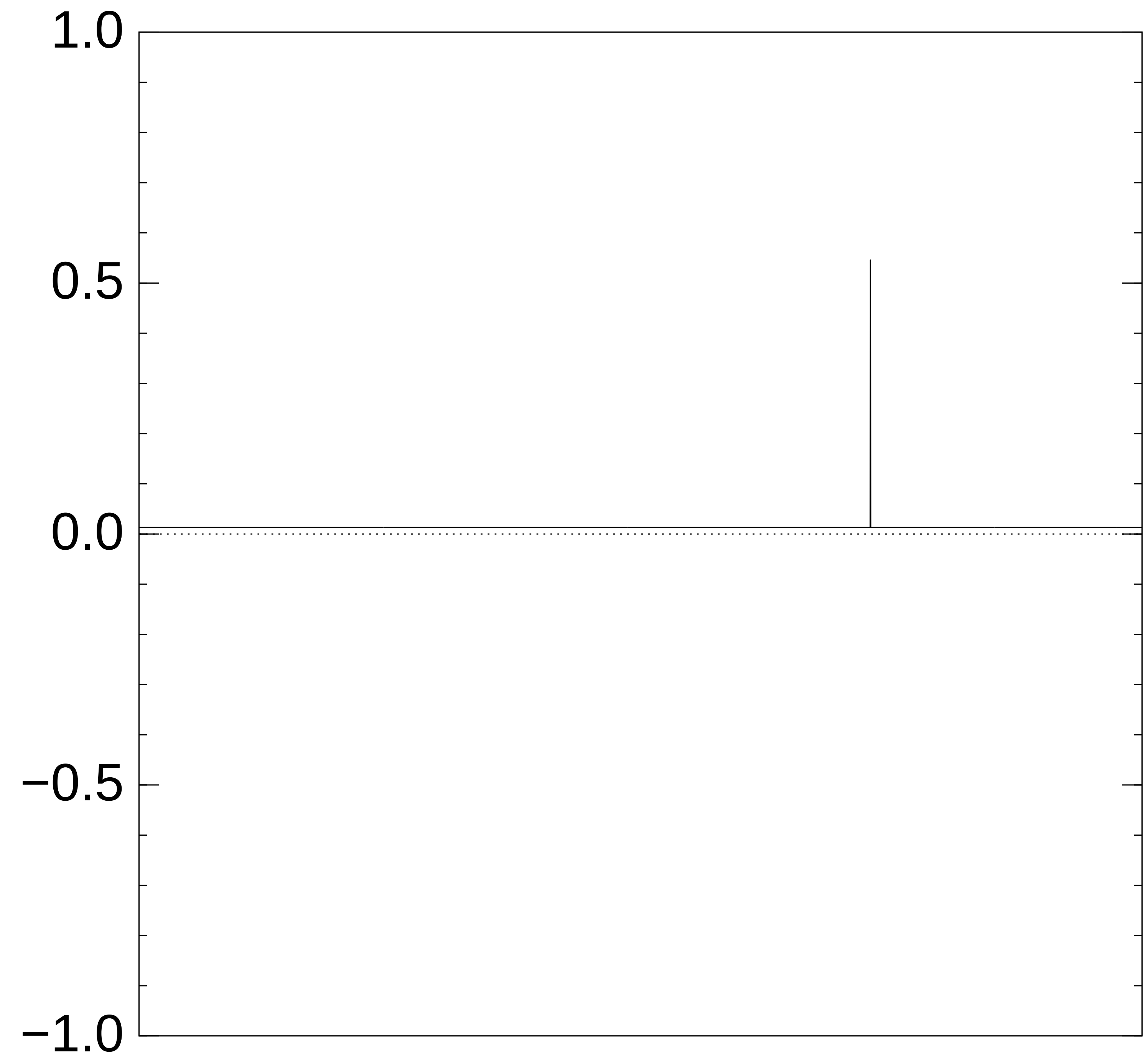
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $18 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

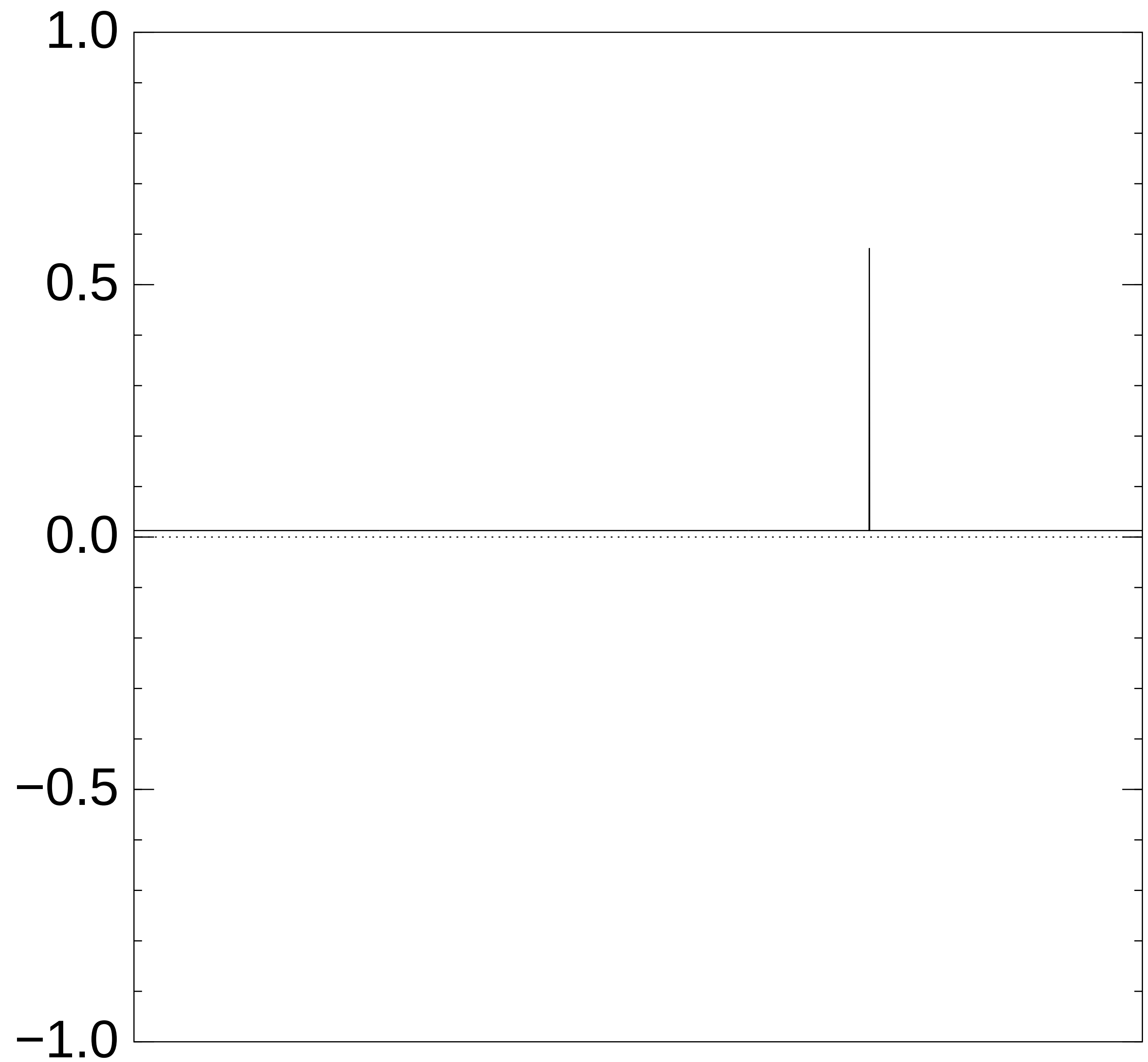
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $19 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

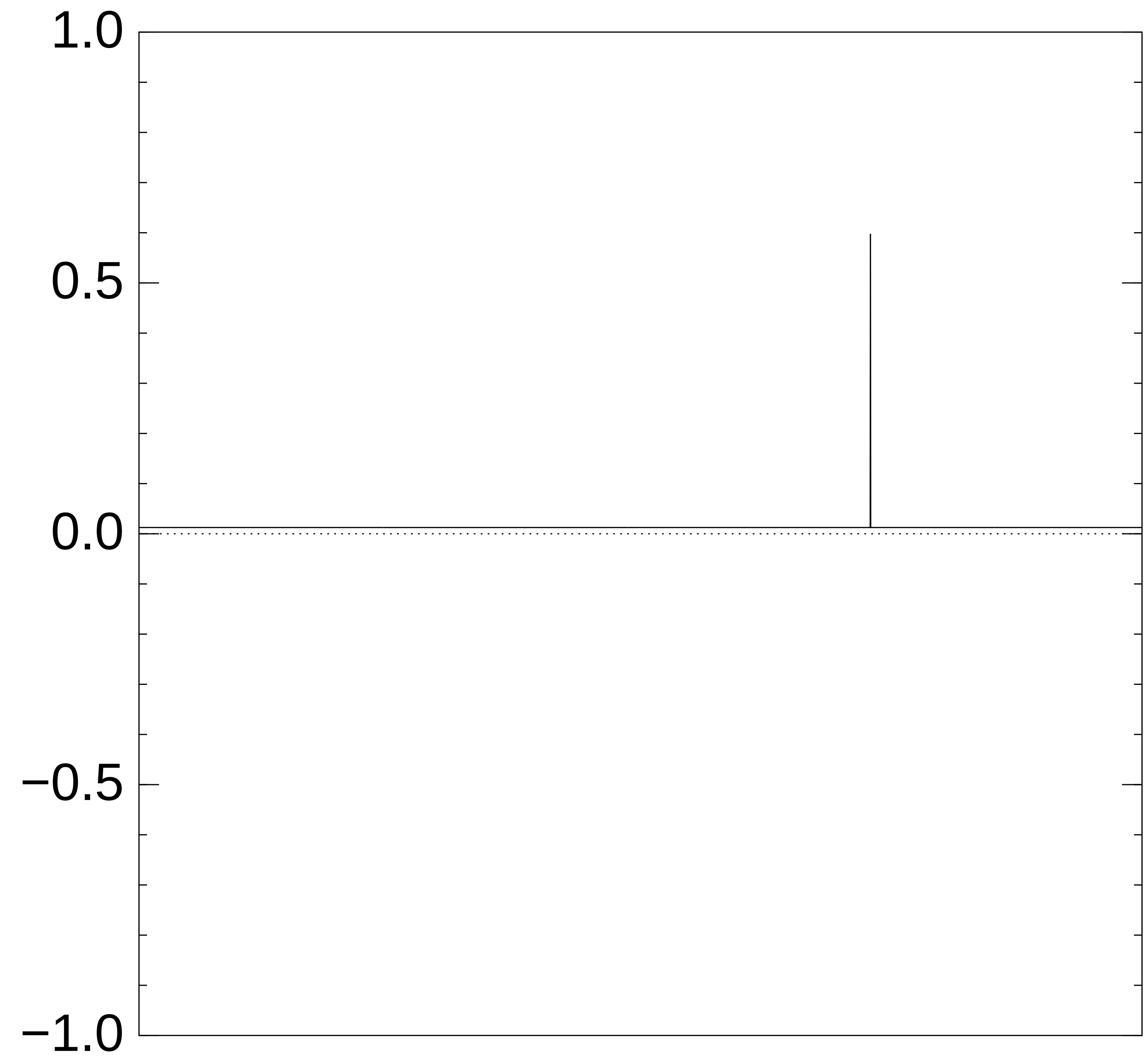
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $20 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

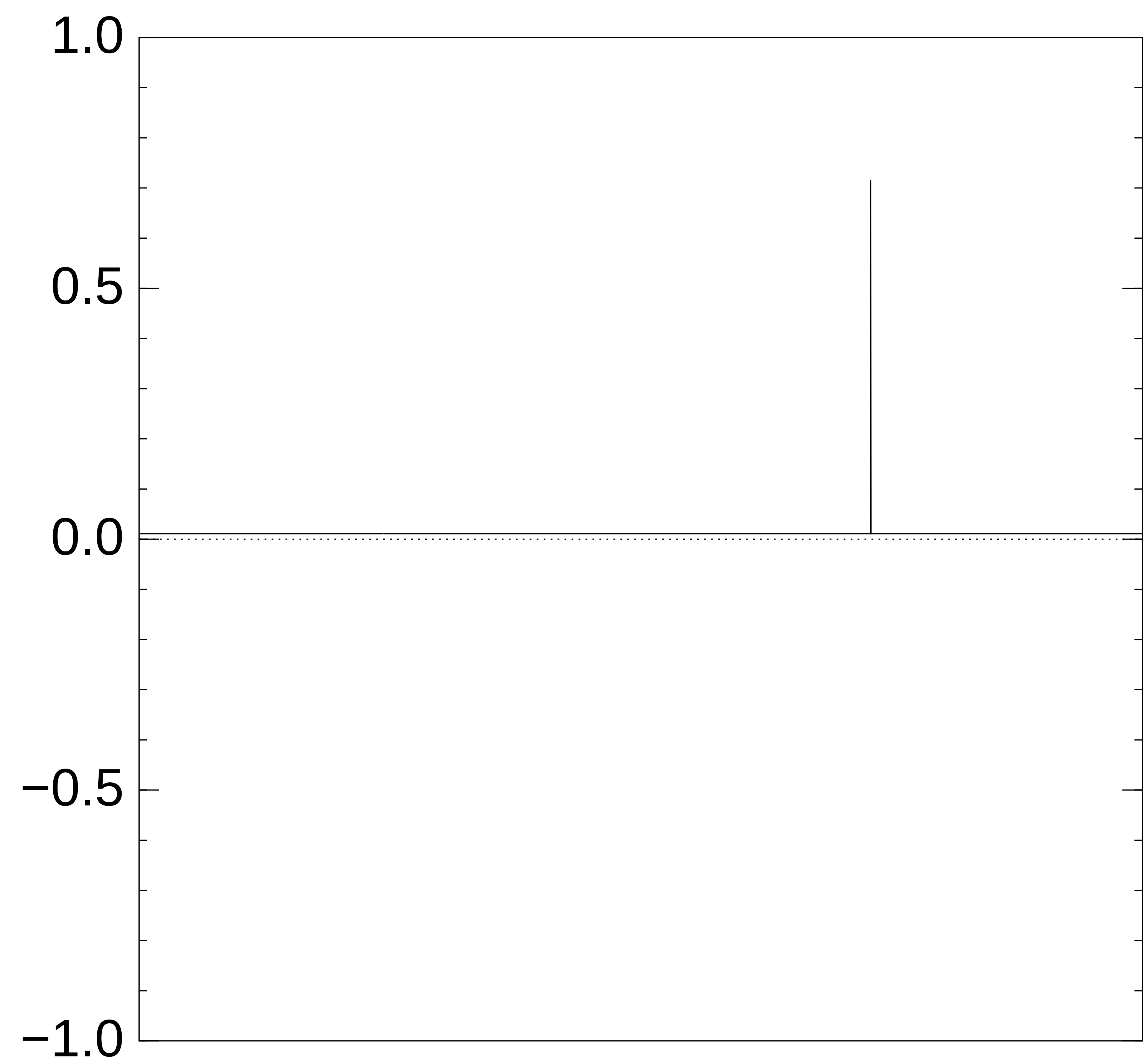
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $25 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

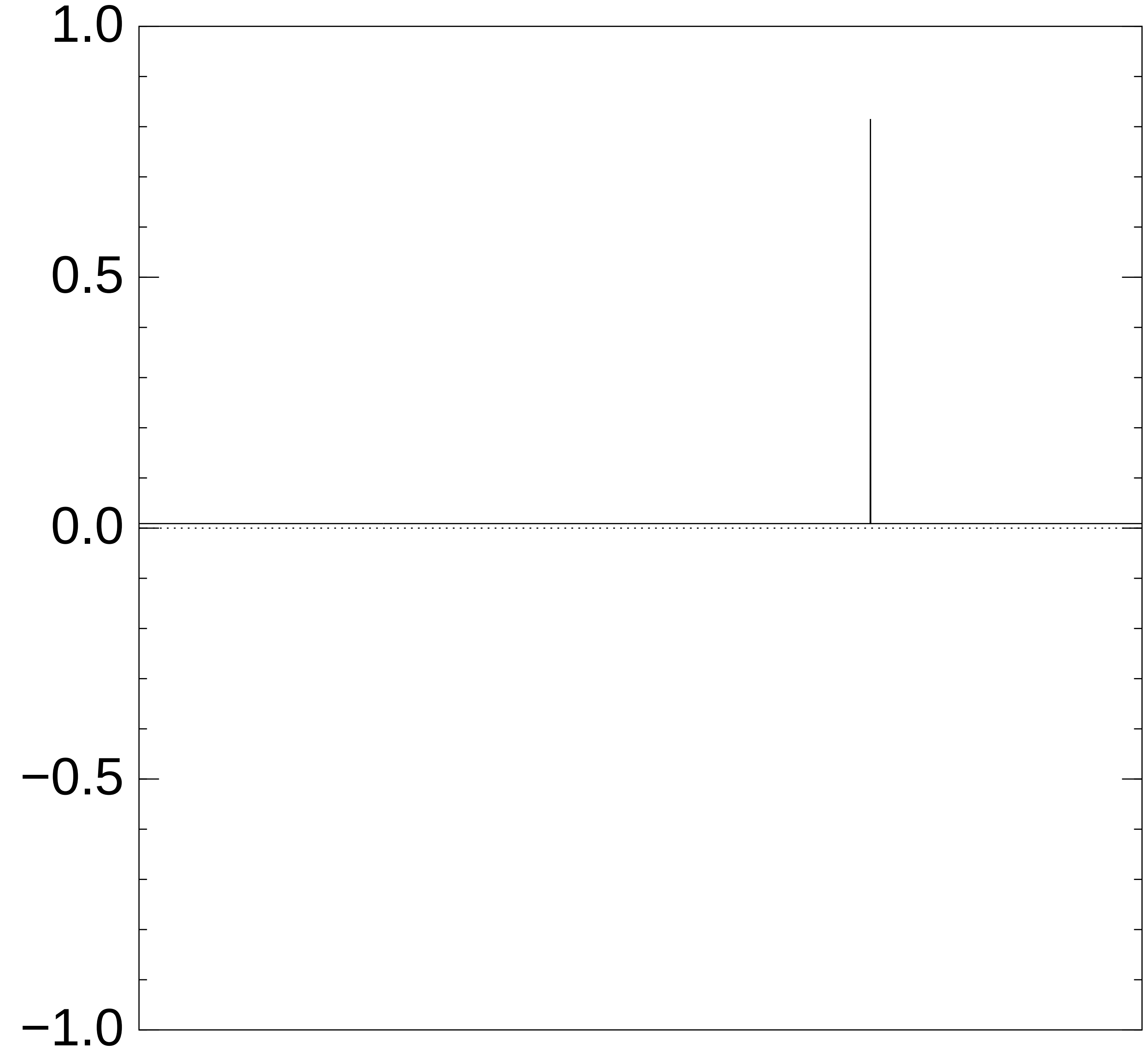
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $30 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

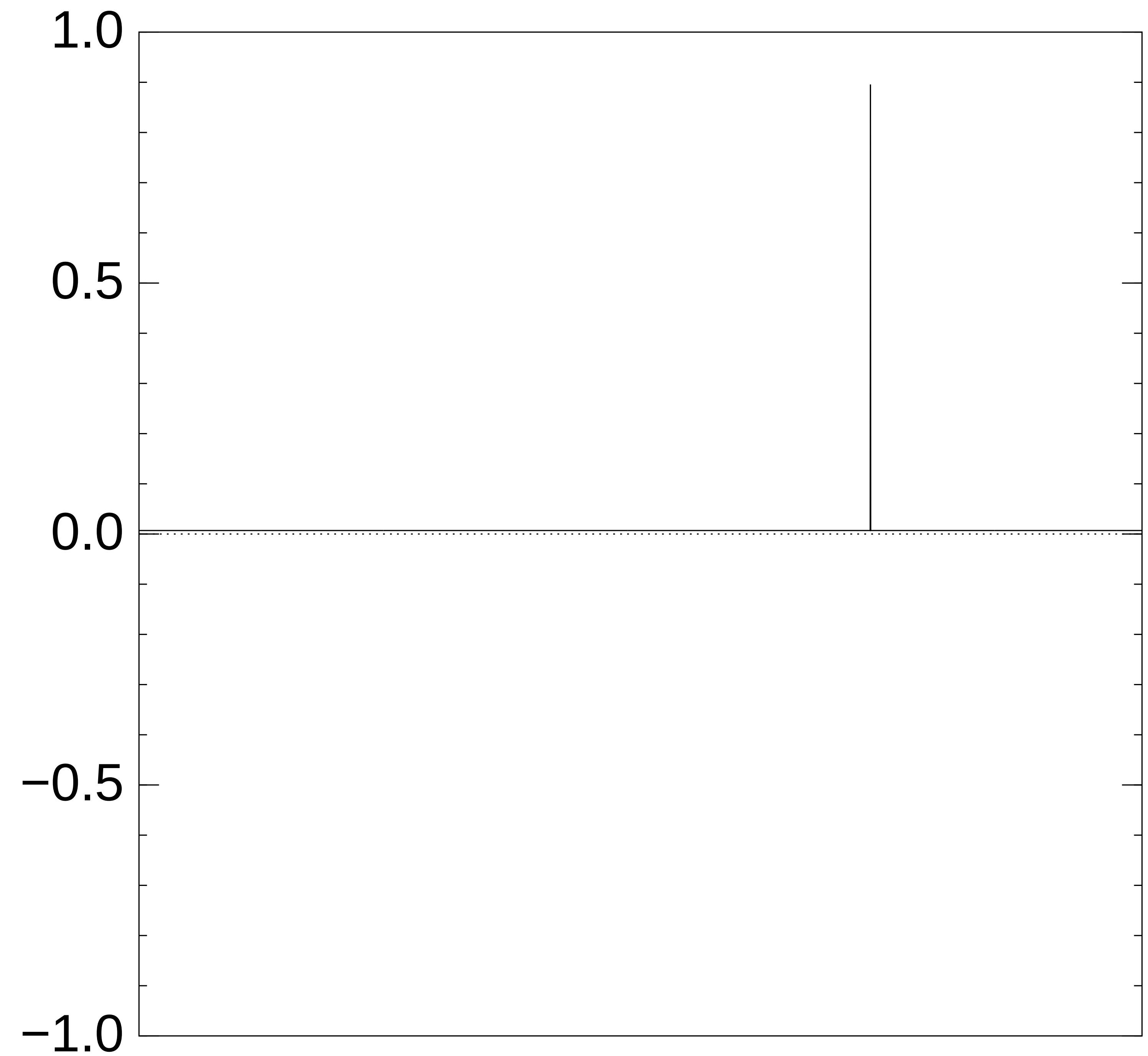
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $35 \times$ (Step 1 + Step 2):



Good moment to stop, measure.

Start from uniform superposition over all n-bit strings u.

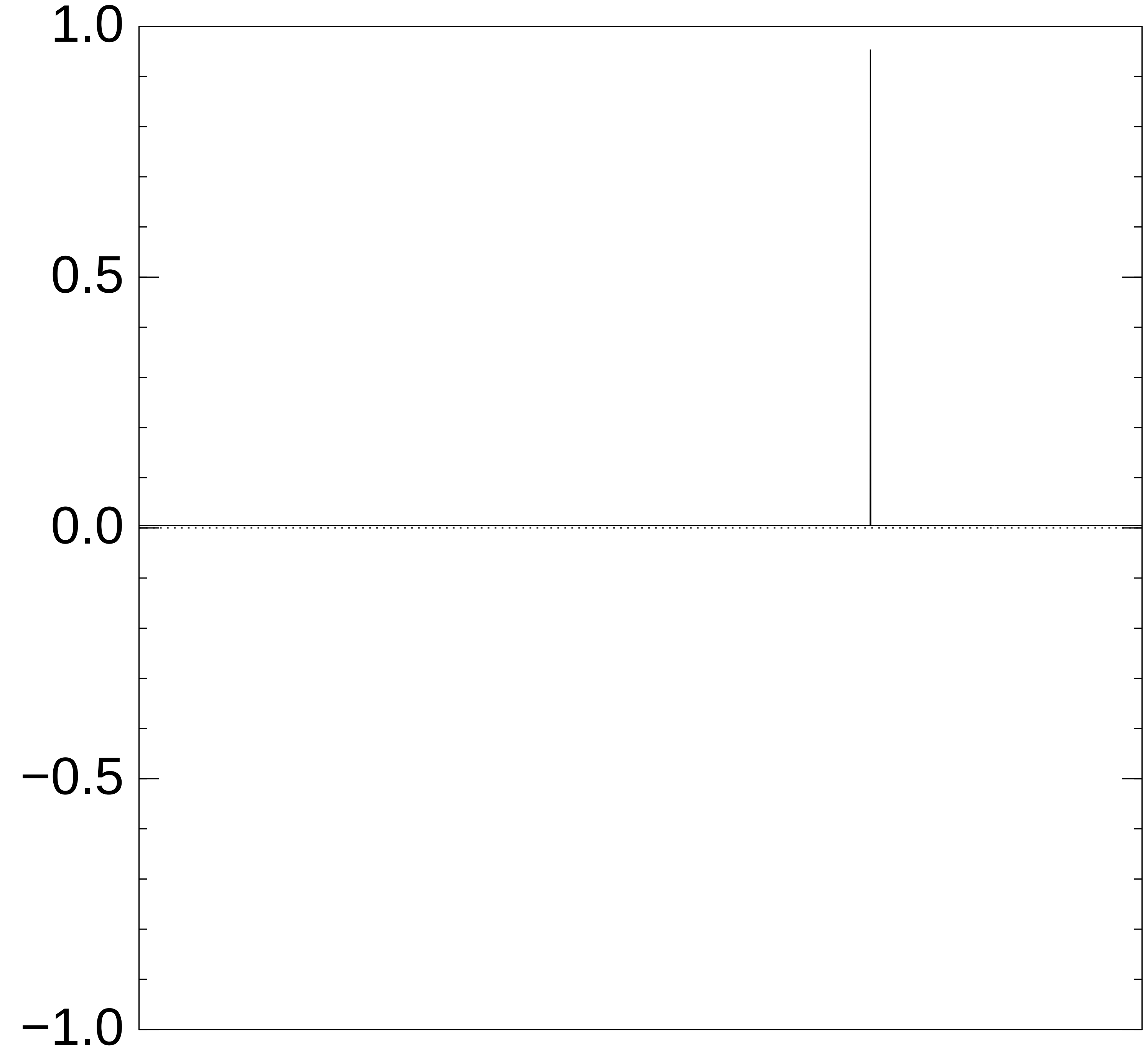
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $40 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

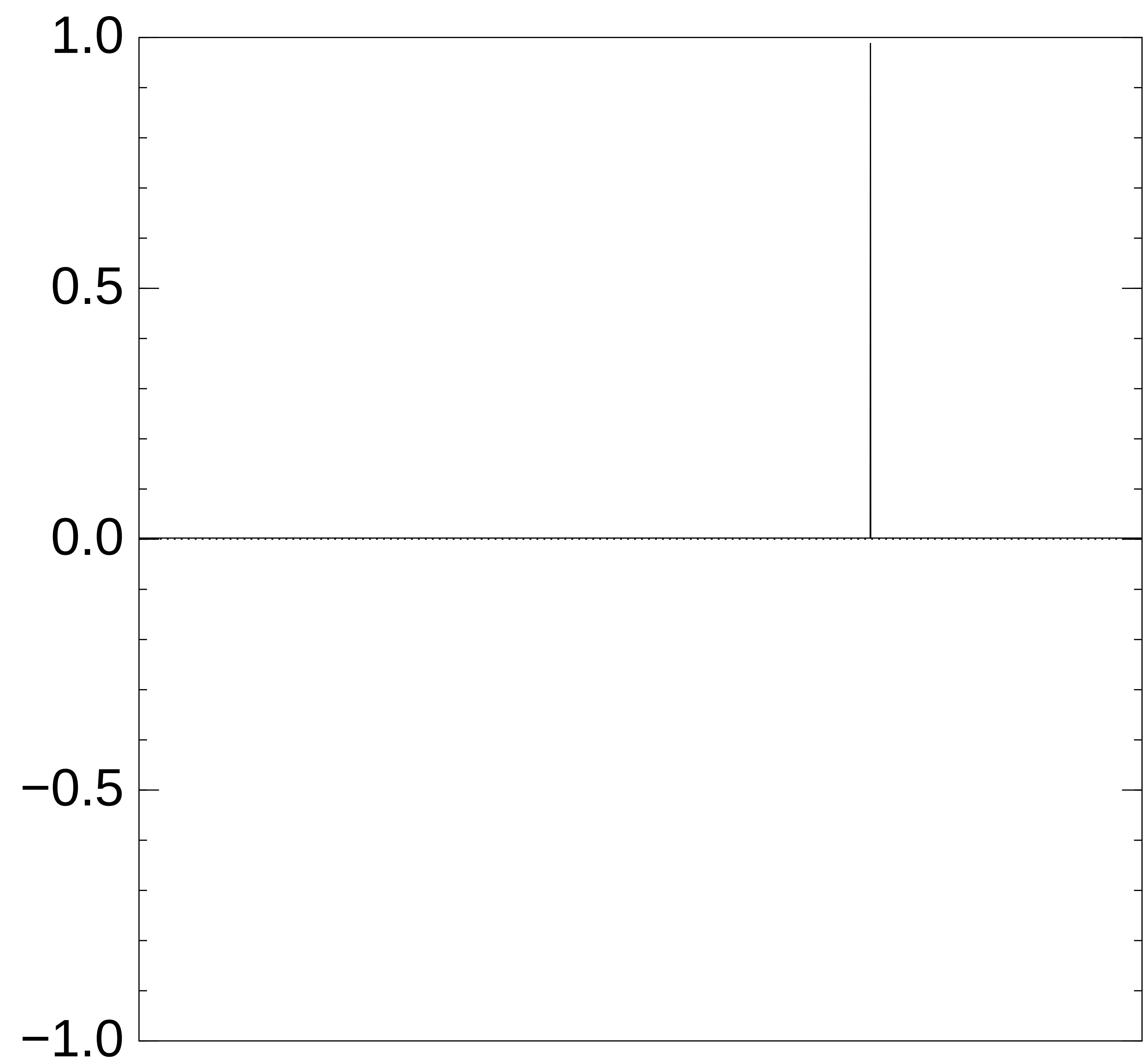
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $45 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

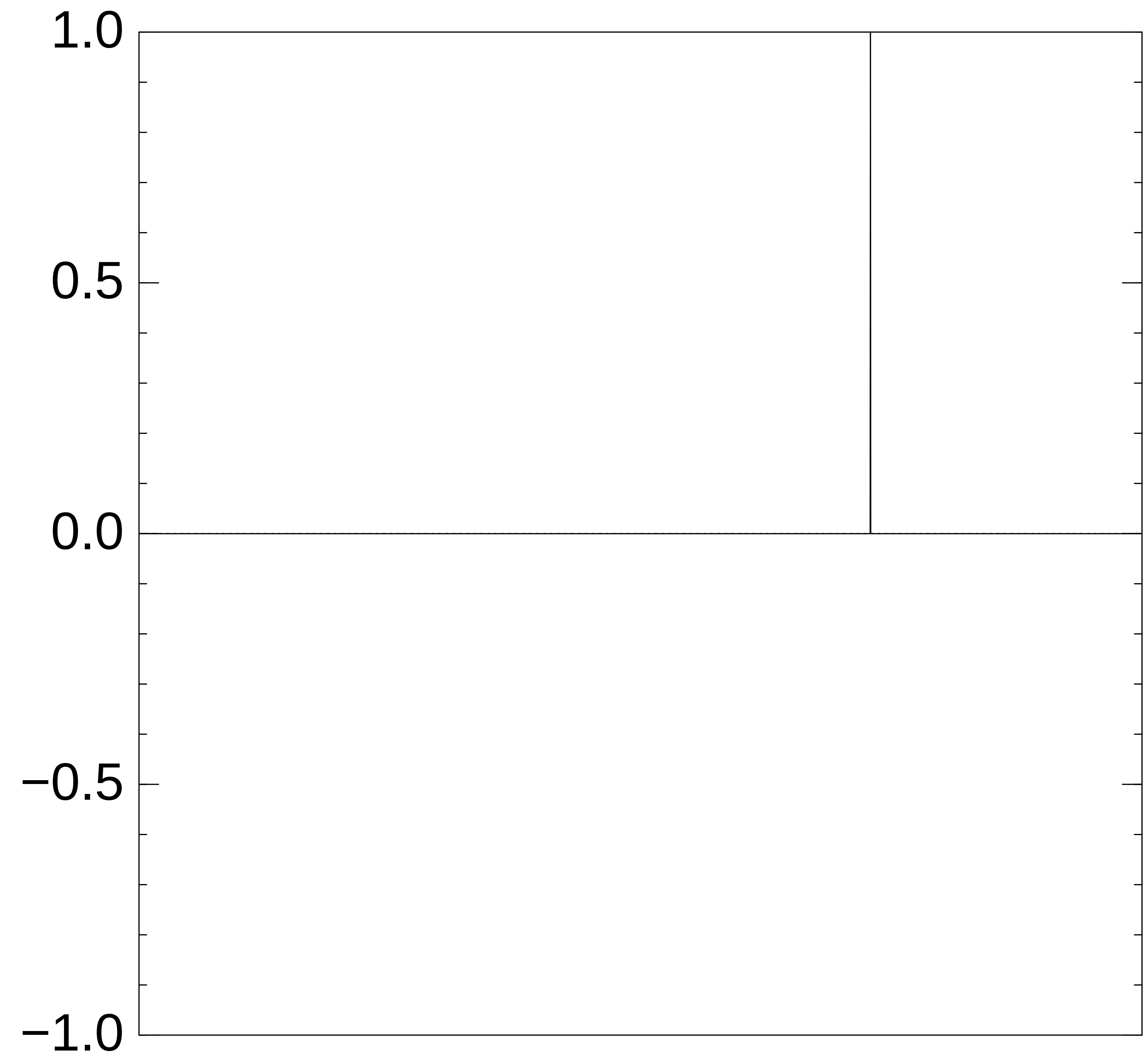
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $50 \times$ (Step 1 + Step 2):



Traditional stopping point.

Start from uniform superposition over all n-bit strings u.

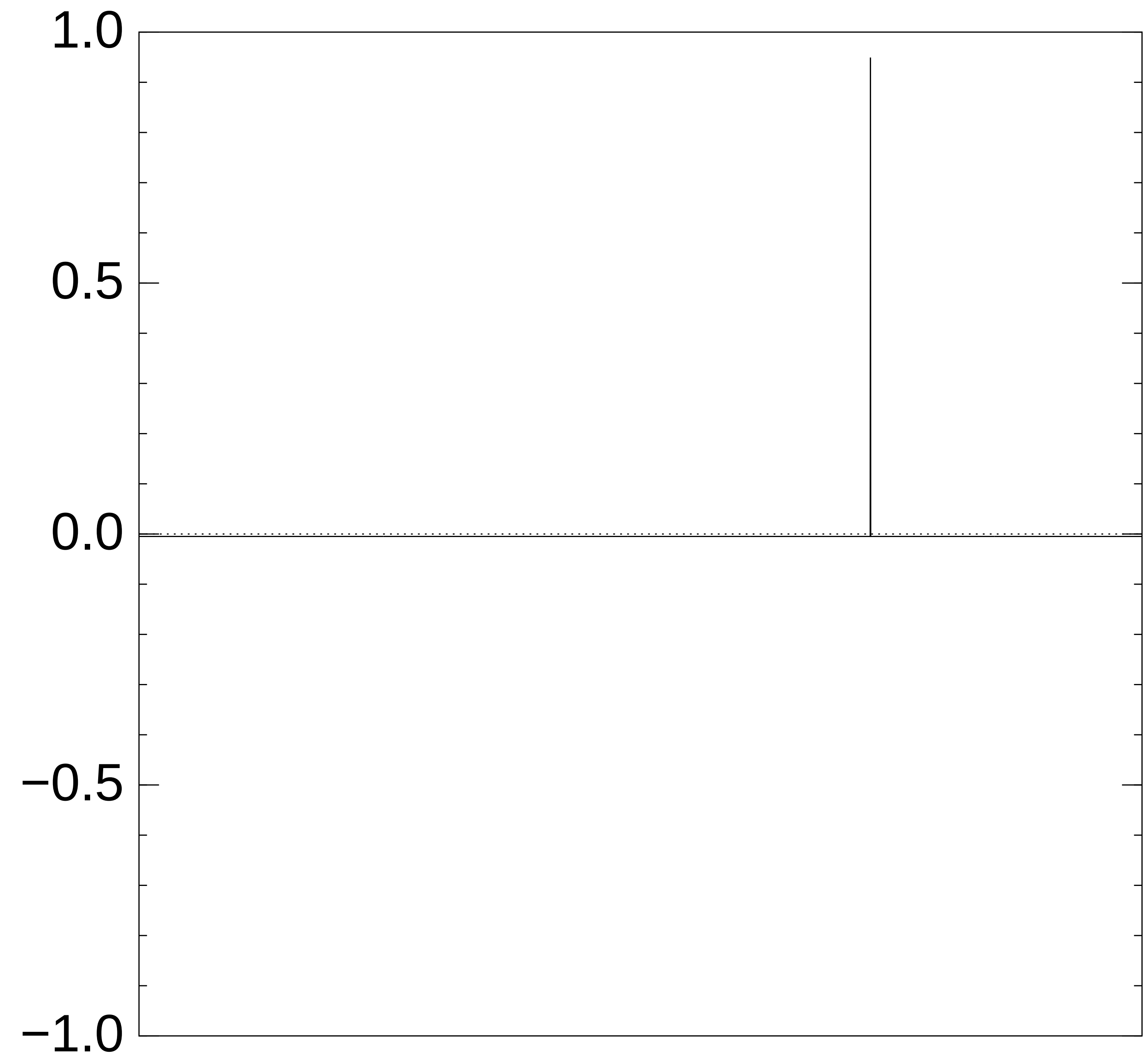
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $60 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over all n-bit strings u.

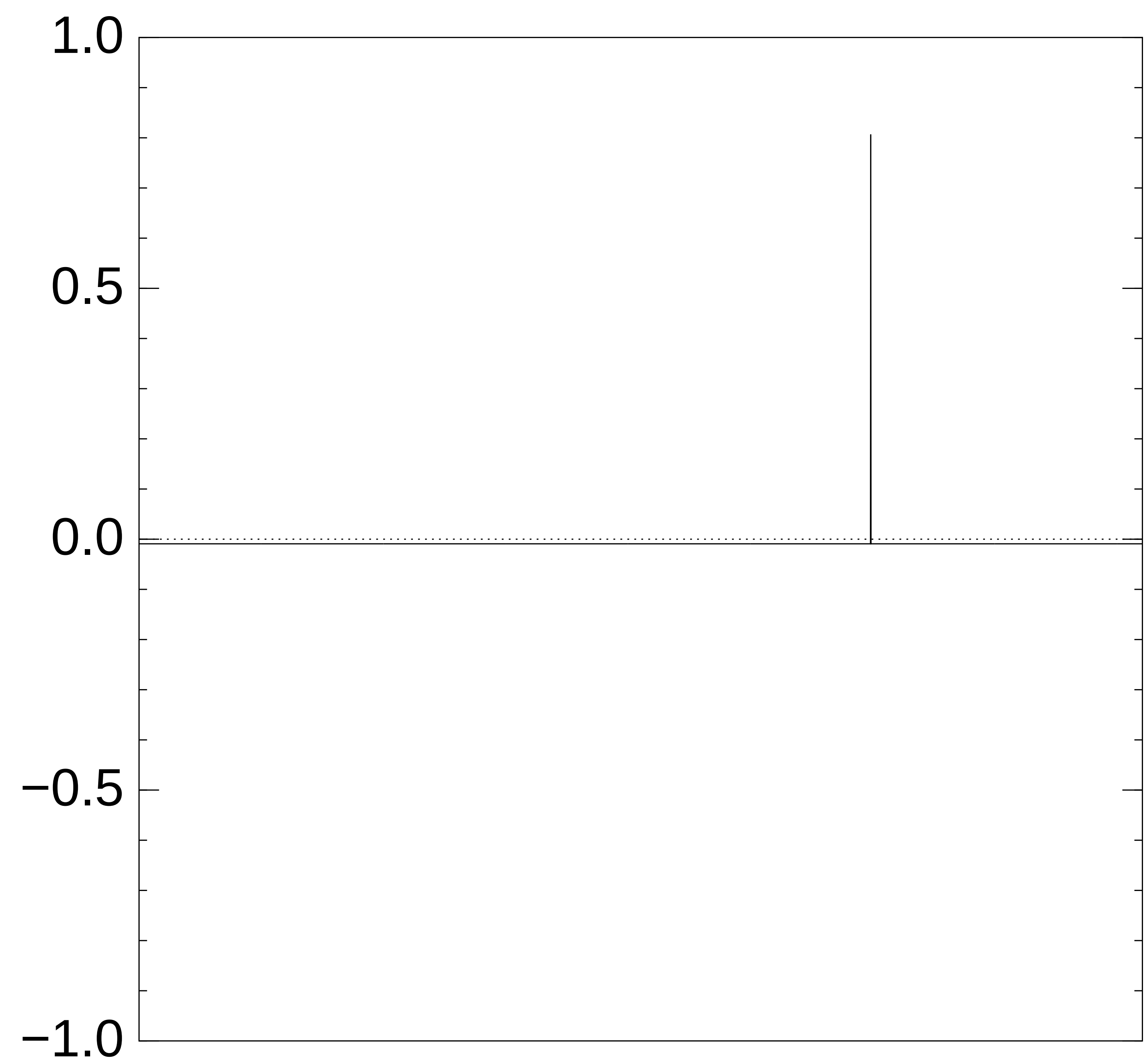
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $70 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

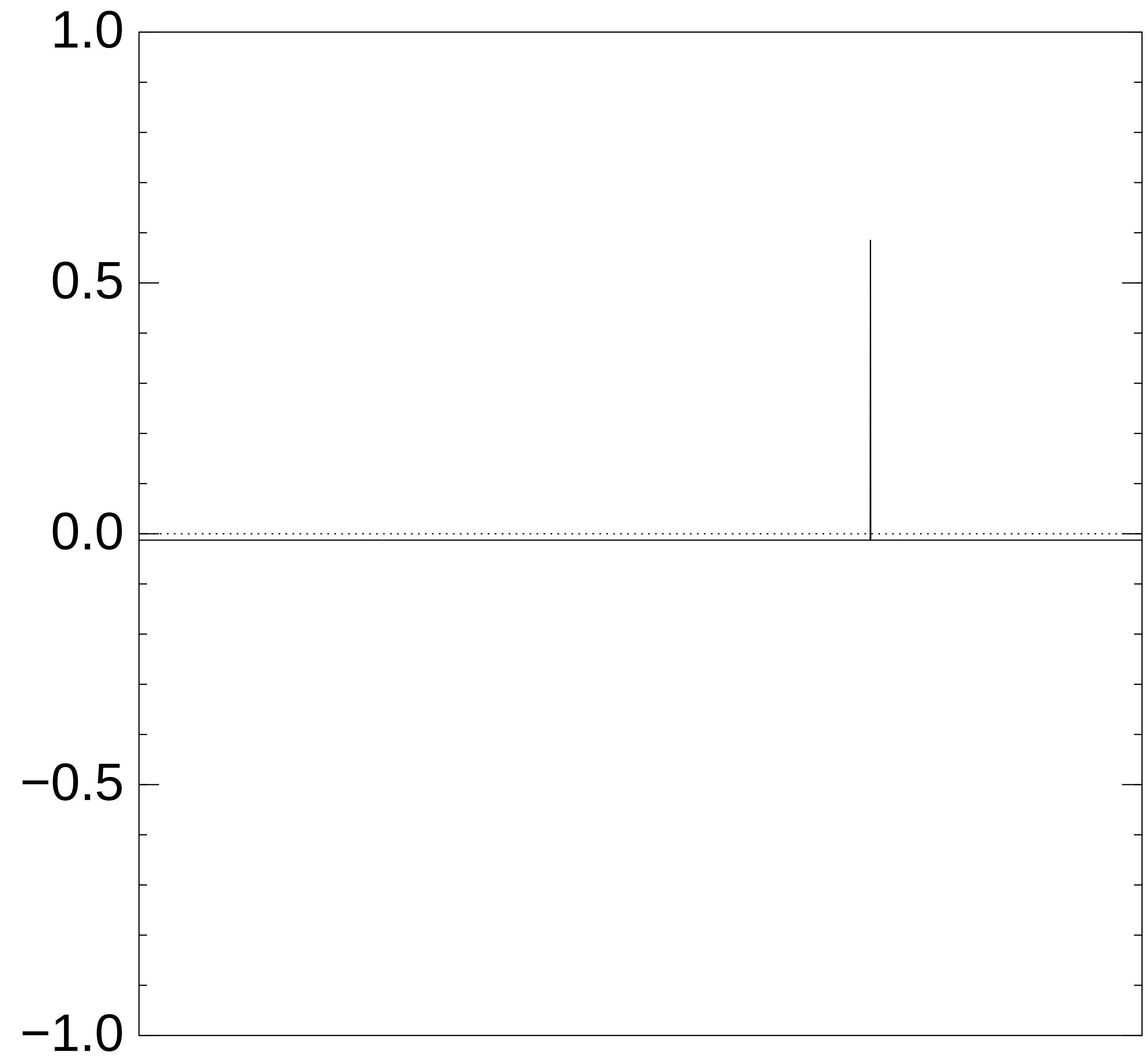
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $80 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

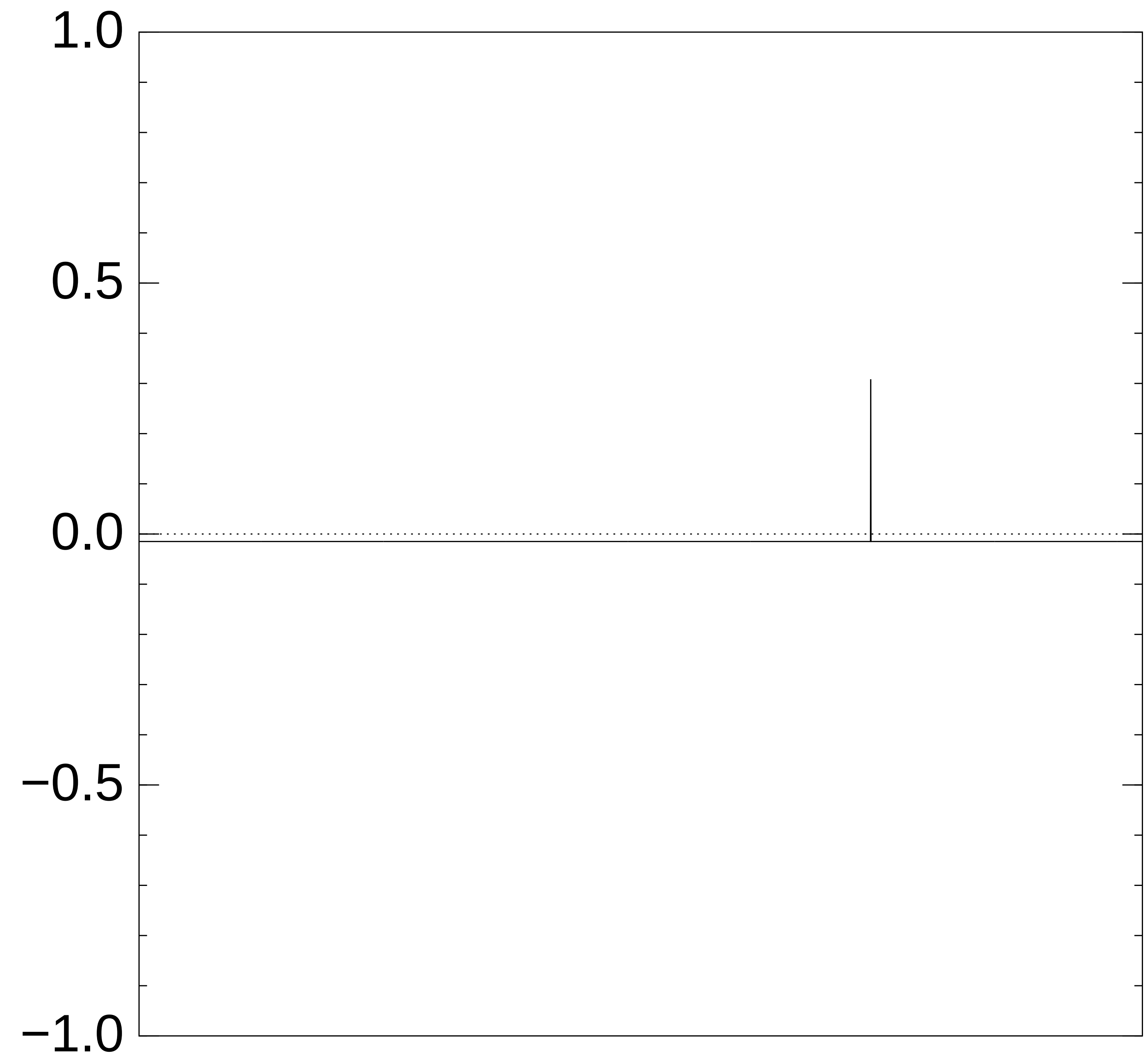
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $90 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings u.

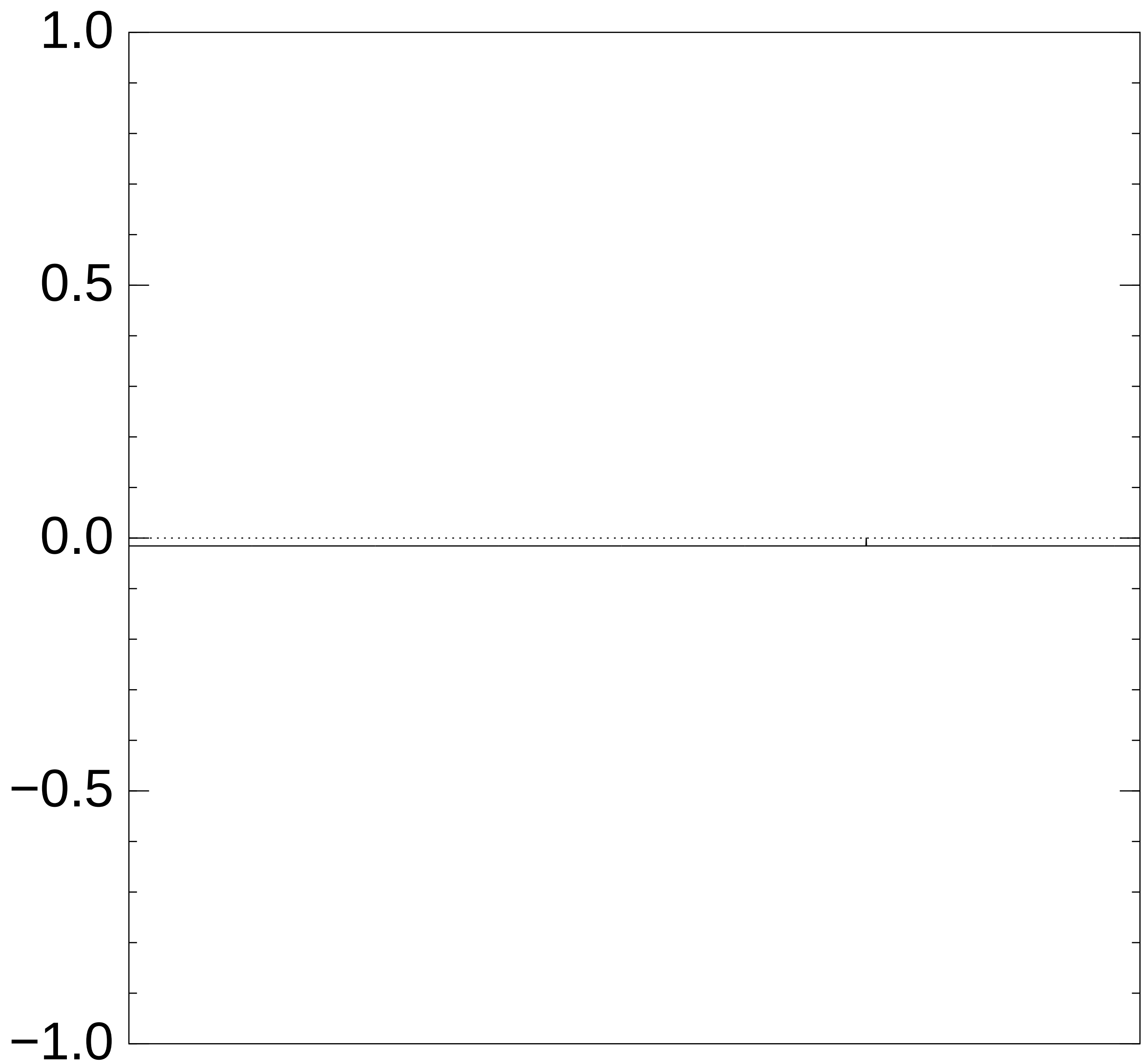
Step 1: Set $a \leftarrow b$ where
$b_u = -a_u$ if $f(u) = 0$,
$b_u = a_u$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $u \mapsto a_u$ for an example with $n = 12$ after $100 \times$ (Step 1 + Step 2):



Very bad stopping point.

om uniform superposition
n-bit strings u.

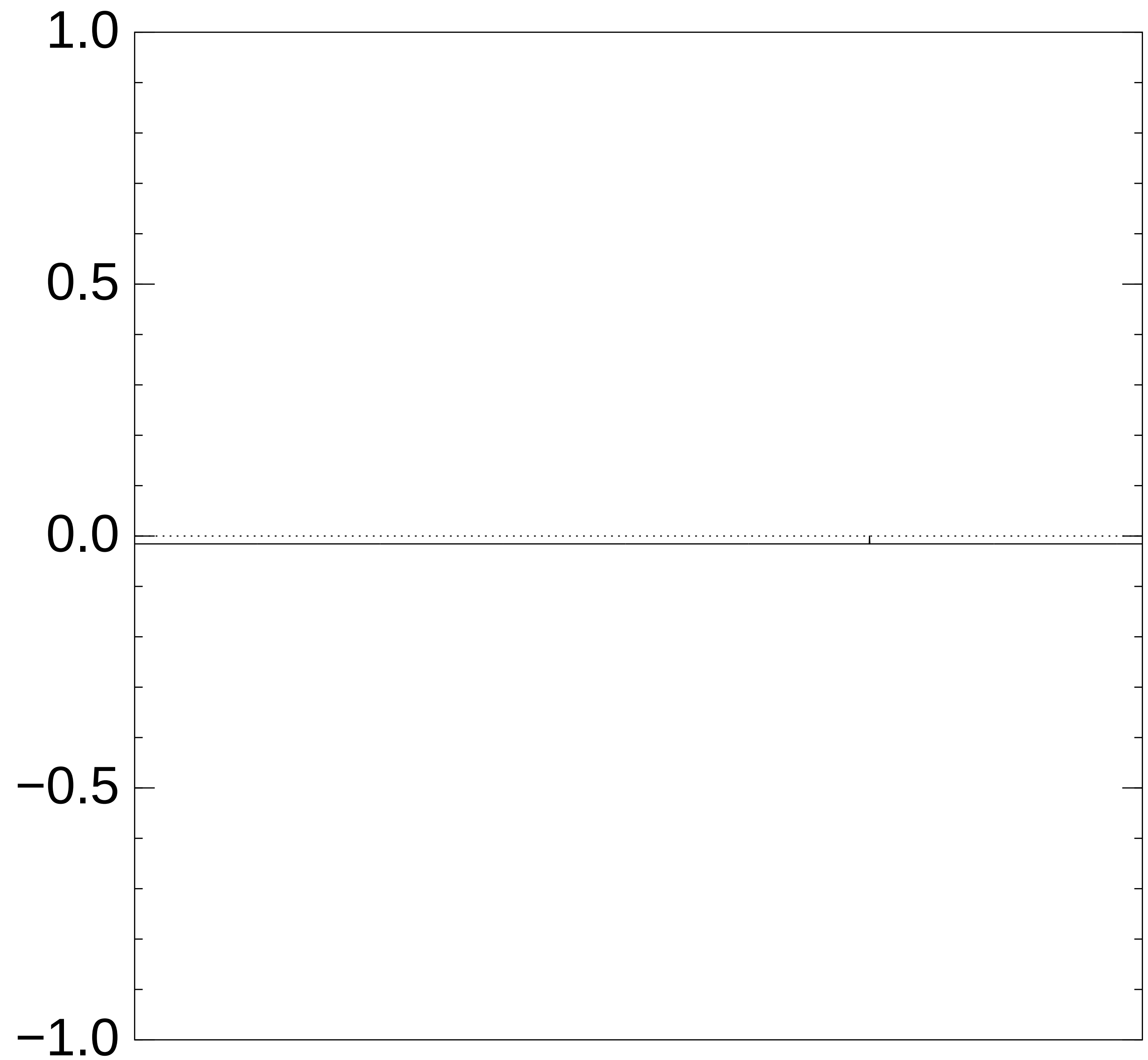
Set $a \leftarrow b$ where
a_u if $f(u) = 0$,
otherwise.
ast.

“Grover diffusion”.
a around its average.
lso fast.

Step 1 + Step 2
$58 \cdot 2^{0.5n}$ times.

the n qubits.
gh probability this finds s.

Normalized graph of $u \mapsto a_u$
for an example with $n = 12$
after $100 \times$ (Step 1 + Step 2):



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Normalized graph of $u \mapsto a_u$
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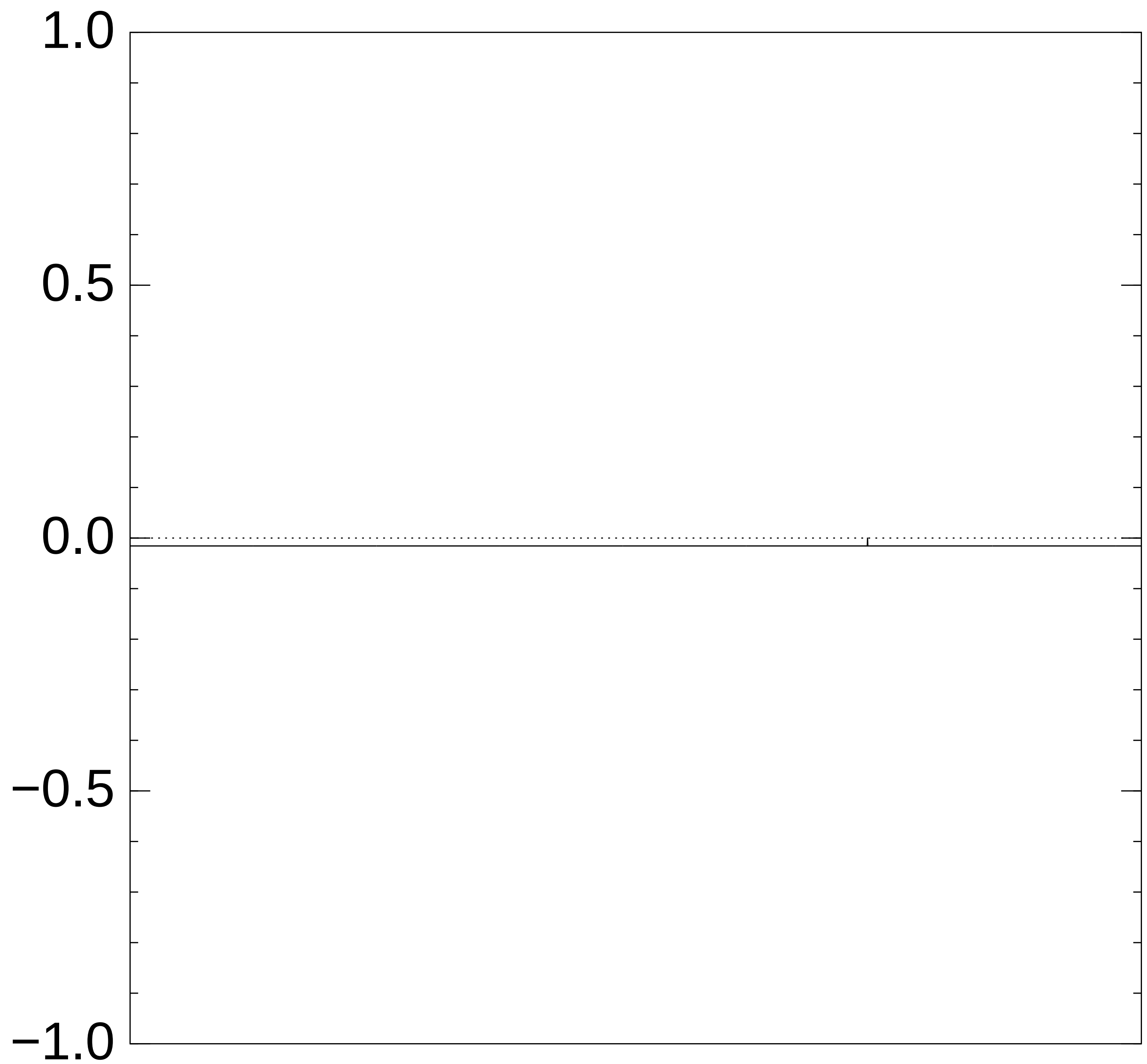

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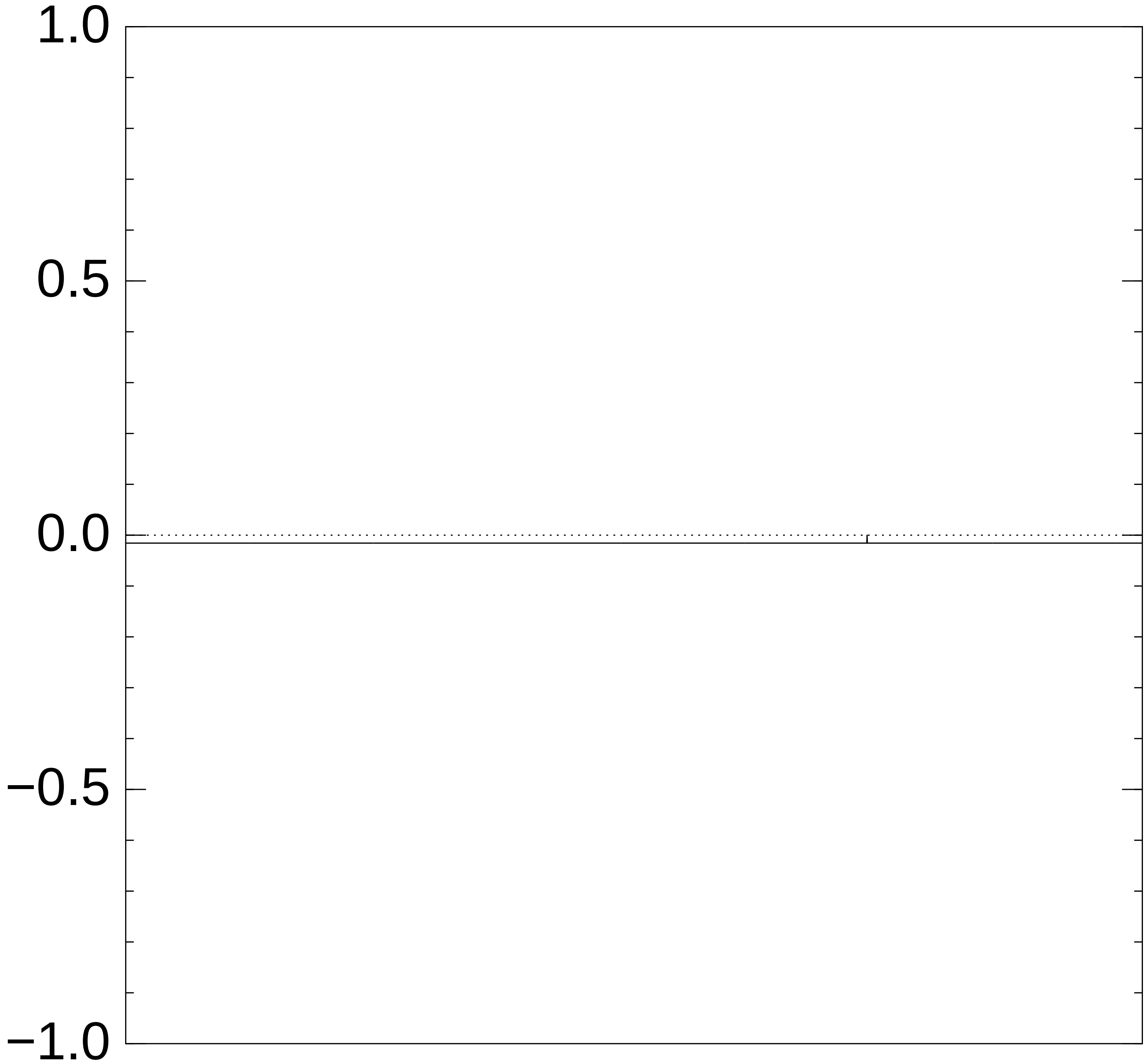

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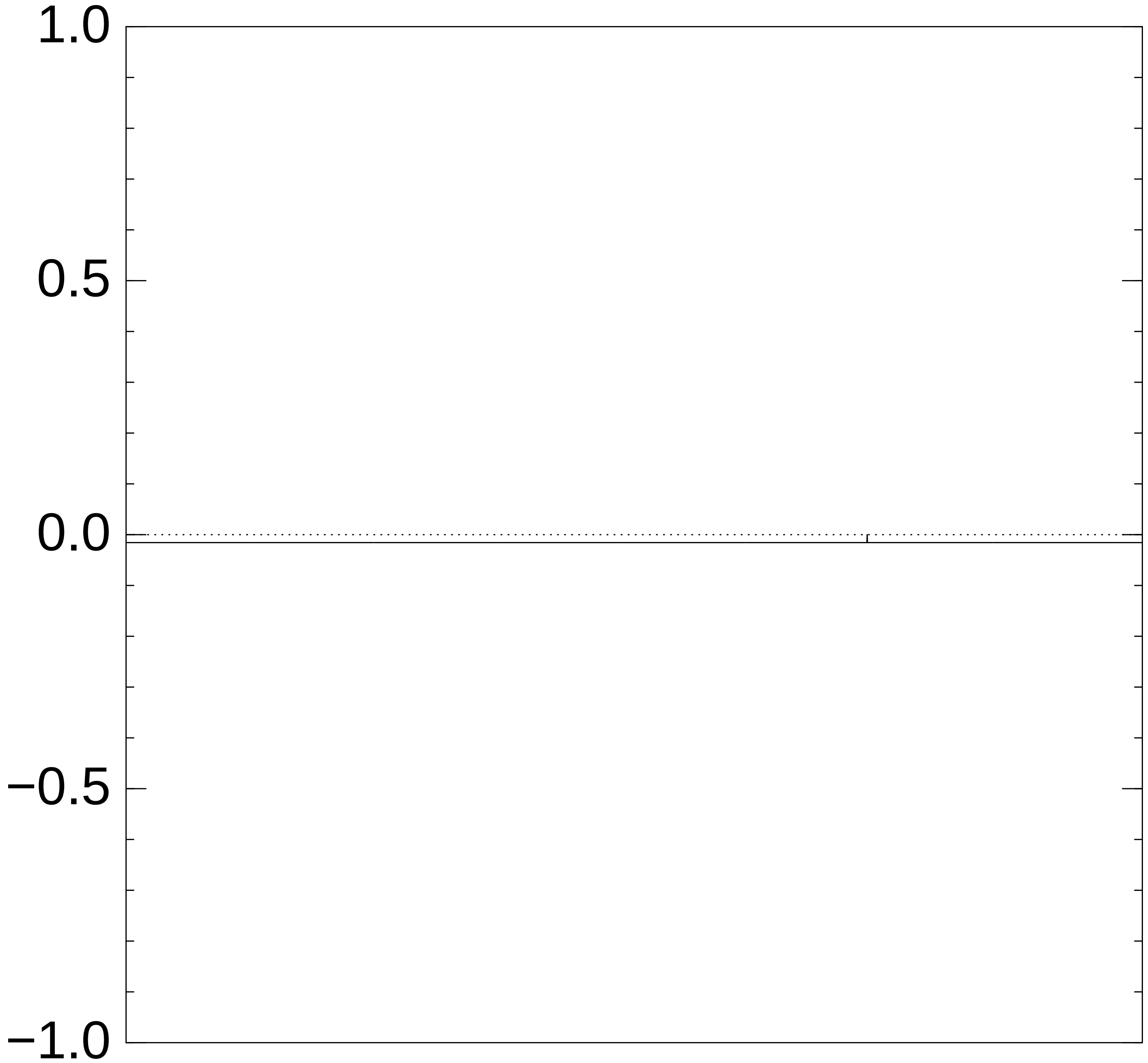

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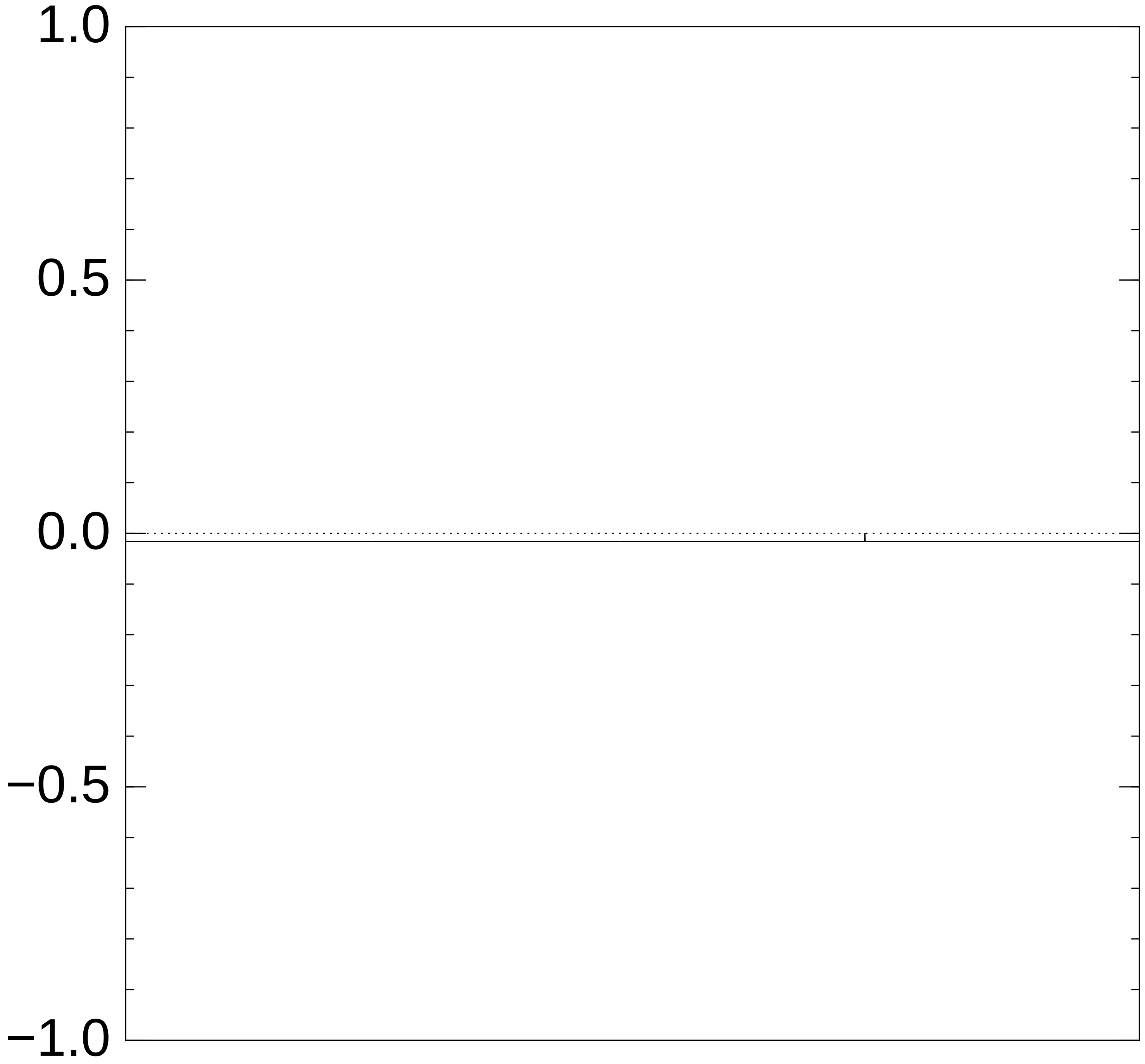

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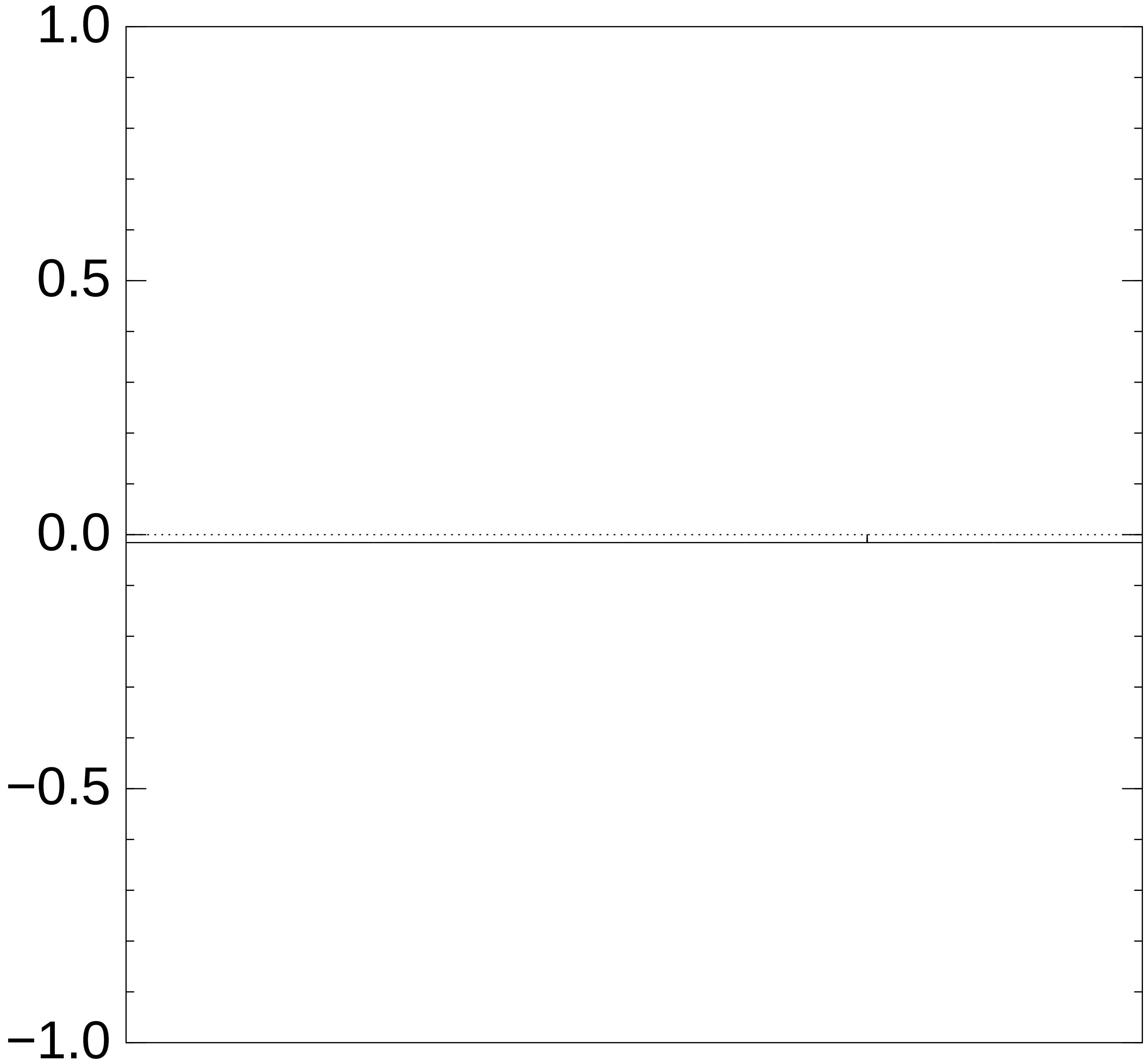

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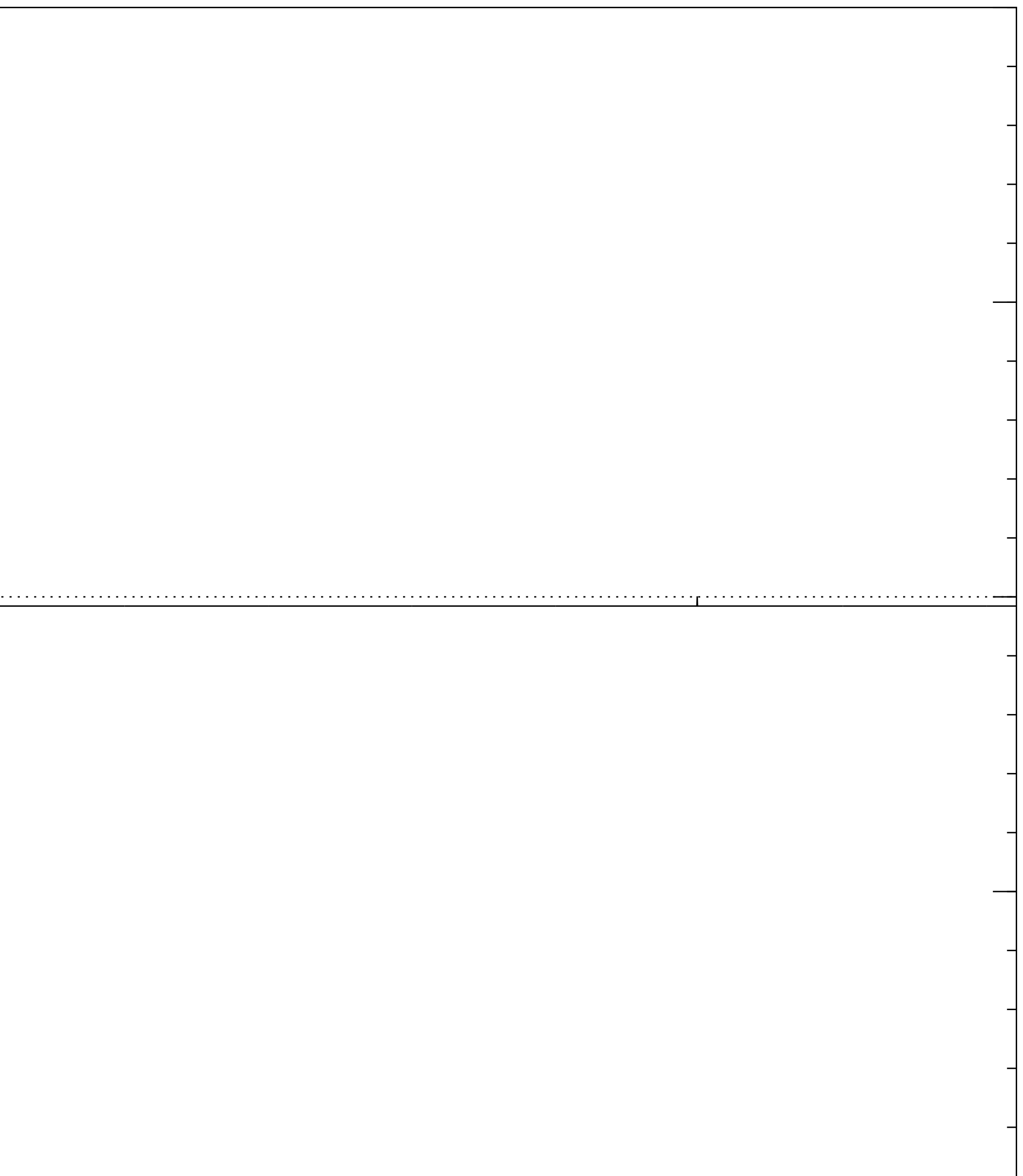

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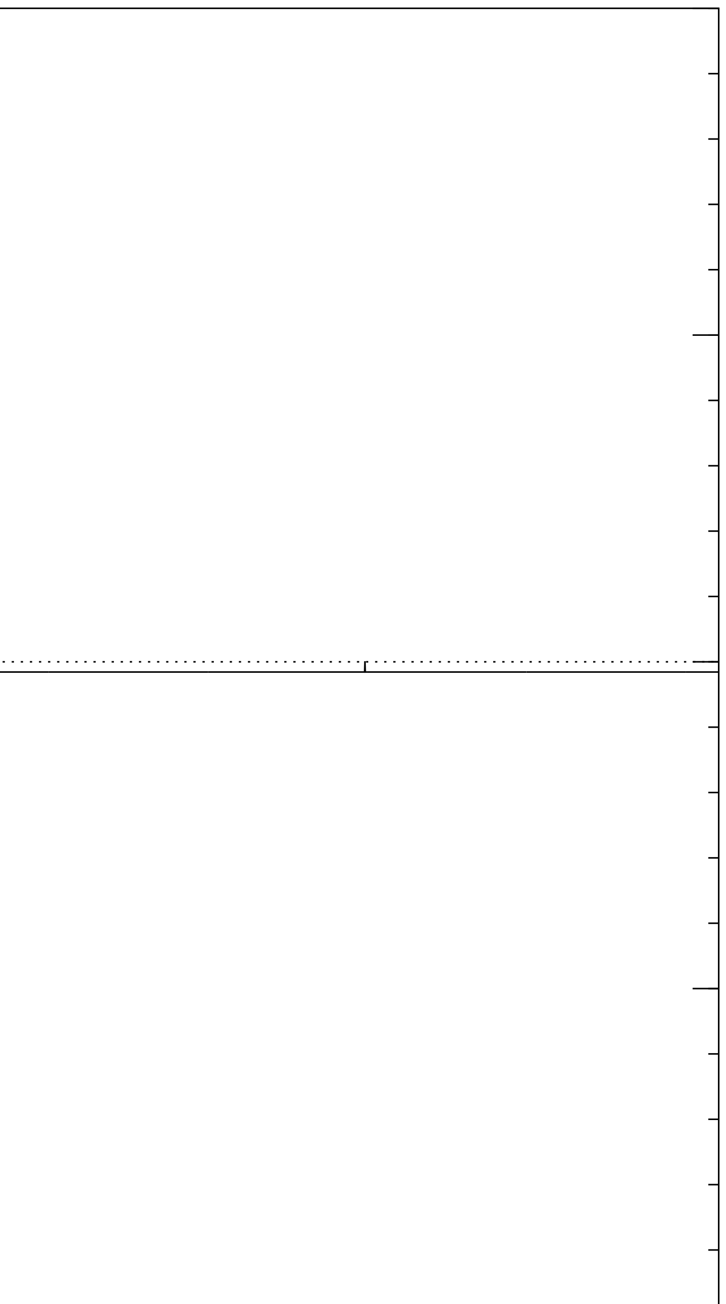

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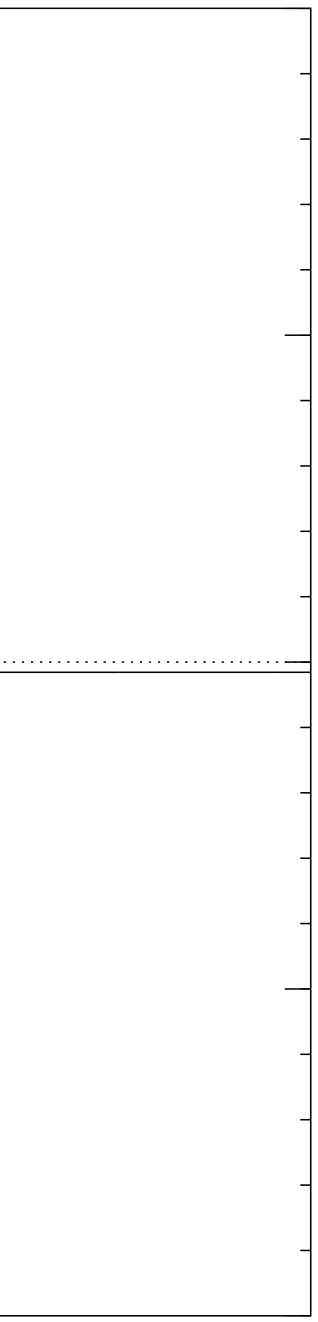

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