Cryptographic software engineering, part 2

Daniel J. Bernstein

Previous part:

- General software engineering.
- Using const-time instructions.

Software optimization

Almost all software is much slower than it could be.

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$\Rightarrow$ fewer users
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  int re
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Target CPU: TI LI
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Reference impleme

```
int sum(int *x)
{
  int result = 0
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  for (i = 0;i <
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A simplified example

Target CPU: TI LM4F120H

microcontroller containing one ARM Cortex-M4F core.

Reference implementation:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;++i
    result += x[i];
  return result;
}
```

Typical situation:

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A simplified example

Target CPU: TI LM4F120H5QR microcontroller containing one ARM Cortex-M4F core.

Reference implementation:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;++i)
    result += x[i];
  return result;
}
```

situation:

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e implementation of X.

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```
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  return result;
}
```

Counting

```
static v
  *const
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Now what?

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  int i;
  for (i = 0;i < 1000;++i)
    result += x[i];
  return result;
}
```


Counting cycles:

```
static volatile 
  *const DWT_CYC
  = (void *) 0xE
...

int beforesum = 
int result = sum
int aftersum = *
UARTprintf("sum 
  result,aftersu
```

Output shows 801
Change 1000 to 50

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A simplified example

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Reference implementation:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;++i)
    result += x[i];
  return result;
}
```


Counting cycles:

```
static volatile unsigned
  *const DWT_CYCCNT
  = (void *) 0xE0001004;
...

int beforesum = *DWT_CYCC
int result = sum(x);
int aftersum = *DWT_CYCCN
UARTprintf("sum %d %d\n",
  result,aftersum-befores
```

Output shows 8012 cycles.

Change 1000 to 500: 4012.

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Bad practice:
Apply random "optimizations"
(and tweak compiler options)
until you get bored.
Keep the fastest results.

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Try `-O1`: 8012 cycles.

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...

int beforesum = *DWT_CYCCNT;
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UARTprintf("sum %d %d\n",
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Try `-Os`: 8012 cycles.
Try `-O1`: 8012 cycles.
Try `-O2`: 8012 cycles.

Counting cycles:

```
static volatile unsigned int
  *const DWT_CYCCNT
  = (void *) 0xE0001004;
...

int beforesum = *DWT_CYCCNT;
int result = sum(x);
int aftersum = *DWT_CYCCNT;
UARTprintf("sum %d %d\n",
  result,aftersum-beforesum);
```

Output shows 8012 cycles.
Change 1000 to 500: 4012.

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Try `-Os`: 8012 cycles.
Try `-O1`: 8012 cycles.
Try `-O2`: 8012 cycles.
Try `-O3`: 8012 cycles.

g cycles:

```
volatile unsigned int
t DWT_CYCCNT
id *) 0xE0001004;


oresum = *DWT_CYCCNT;
ult = sum(x);
ersum = *DWT_CYCCNT;
ntf("sum %d %d\n",
t,aftersum-beforesum);
```

shows 8012 cycles.

1000 to 500: 4012.

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Try `-O1`: 8012 cycles.
Try `-O2`: 8012 cycles.
Try `-O3`: 8012 cycles.

Try mov

```
int sum
{
  int re
  int i
  for (
    resu
  retur
}
```

```
unsigned int
CNT
0001004;

*DWT_CYCCNT;
(x);
DWT_CYCCNT;
%d %d\n",
m-beforesum);
```

2 cycles.

00: 4012.

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Try `-Os`: 8012 cycles.

Try `-O1`: 8012 cycles.

Try `-O2`: 8012 cycles.

Try `-O3`: 8012 cycles.

Try moving the po

```
int sum(int *x)
{
  int result = 0
  int i;
  for (i = 0;i <
    result += *x
  return result;
}
```

```
int

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```


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Try moving the pointer:

```
int sum(int *x)
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}
```

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  int re
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  for (i
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}
```

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8010 cycles.

Try counting dow

```
int sum(int *x)
{
  int result = 0
  int i;
  for (i = 1000;
    result += *x
  return result;
}
```

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Try moving the pointer:

```
int sum(int *x)
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  int result = 0;
  int i;
  for (i = 0;i < 1000;++i)
    result += *x++;
  return result;
}
```

8010 cycles.

Try counting down:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 1000;i > 0;--i
    result += *x++;
  return result;
}
```

Try moving the pointer:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;++i)
    result += *x++;
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```
(int *x)

esult = 0;
;
i = 0;i < 1000;++i)
ult += *x++;
n result;
```

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Try usin

```
int sum
{
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  int *
  while
    resu
  retur
}
```

ointer:

```
;
 1000;++i)
++;
```


Try counting down:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 1000;i > 0;--i)
    result += *x++;
  return result;
}
```

8010 cycles.

Try using an end p

```
int sum(int *x)
{
  int result = 0
  int *y = x + 1
  while (x != y)
    result += *x
  return result;
}
```

Try counting down:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 1000;i > 0;--i)
    result += *x++;
  return result;
}
```

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  int i;
  for (i = 1000;i > 0;--i)
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  return result;
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Back to

```
int sum
{
  int re
  int i
  for (i
    resu
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  }
  return
}
```

n:

```
;

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++;
```


Try using an end pointer:

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```

8010 cycles.

Back to original. T

```
int sum(int *x)
{
  int result = 0
  int i;
  for (i = 0;i <
    result += x[
    result += x[
  }
  return result;
}
```

)

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Try using an end pointer:

```
int sum(int *x)
{
  int result = 0;
  int *y = x + 1000;
  while (x != y)
    result += *x++;
  return result;
}
```

8010 cycles.

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Back to original. Try unrolli

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i +
    result += x[i];
    result += x[i + 1];
  }
  return result;
}
```

Try using an end pointer:

```
int sum(int *x)
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  int *y = x + 1000;
  while (x != y)
    result += *x++;
  return result;
}
```

8010 cycles.

Back to original. Try unrolling:

```
int sum(int *x)
{
  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 2) {
    result += x[i];
    result += x[i + 1];
  }
  return result;
}
```

Try using an end pointer:

```
int sum(int *x)
{
  int result = 0;
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  return result;
}
```

8010 cycles.

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  int result = 0;
  int i;
  for (i = 0;i < 1000;i += 2) {
    result += x[i];
    result += x[i + 1];
  }
  return result;
}
```

5016 cycles.

```
int sum
{
  int re
  int i
  for (i
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    resu
  }
  returr
}
```

pointer:

```
;
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++;
```


Back to original. Try unrolling:

```
int sum(int *x)
{
  int result = 0;
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  for (i = 0;i < 1000;i += 2) {
    result += x[i];
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  }
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int sum(int *x)
{
  int result = 0
  int i;
  for (i = 0;i <
    result += x[
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  }
  return result;
}
```

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int sum(int *x)
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4016 cycles. “Are we done yet?”

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Why observed time is highe non-consecutive LDRs; costs of manipulating `i`.

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Lower bound for n LDR + n ADD: $2n+1$ cycles, including n cycles of arithmetic.

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```
int sum
{
  int r
  int *
  int x
      x
  while
    x0 =
    x1 =
    x2 =
    x3 =
    x4 =
    x5 =
    x6 =
```

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```
int sum(int *x)
{
  int result = 0
  int *y = x + 1
  int x0,x1,x2,x
      x5,x6,x7,x

  while (x != y)
    x0 = 0[(vola
    x1 = 1[(vola
    x2 = 2[(vola
    x3 = 3[(vola
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int sum(int *x)
{
  int result = 0;
  int *y = x + 1000;
  int x0,x1,x2,x3,x4,
      x5,x6,x7,x8,x9;

  while (x != y) {
    x0 = 0[(volatile int
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```
x7 = 7[(vola
x8 = 8[(vola
x9 = 9[(vola
result += x0
result += x1
result += x2
result += x3
result += x4
result += x5
result += x6
result += x7
result += x8
result += x9
x0 = 10[(vol
x1 = 11[(vol
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```

```
x2 =
x3 =
x4 =
x5 =
x6 =
x7 =
x8 =
x9 =
x +=
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resu
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```

```
;
000;
3,x4,
8,x9;

 {
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result += x4;
result += x5;
result += x6;
result += x7;
result += x8;
result += x9;
x0 = 10[(volatile int *)x];
x1 = 11[(volatile int *)x];
```

```
x2 = 12[(vol
x3 = 13[(vol
x4 = 14[(vol
x5 = 15[(vol
x6 = 16[(vol
x7 = 17[(vol
x8 = 18[(vol
x9 = 19[(vol
x += 20;
result += x0
result += x1
result += x2
result += x3
result += x4
result += x5
```

```
*)x];
*)x];
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*)x];
*)x];
*)x];
```

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```
x7 = 7[(volatile int *)x];
x8 = 8[(volatile int *)x];
x9 = 9[(volatile int *)x];
result += x0;
result += x1;
result += x2;
result += x3;
result += x4;
result += x5;
result += x6;
result += x7;
result += x8;
result += x9;
x0 = 10[(volatile int *)x];
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x2 = 12[(volatile int
x3 = 13[(volatile int
x4 = 14[(volatile int
x5 = 15[(volatile int
x6 = 16[(volatile int
x7 = 17[(volatile int
x8 = 18[(volatile int
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x += 20;
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result += x2;
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```

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x7 = 7[(volatile int *)x];
x8 = 8[(volatile int *)x];
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result += x0;
result += x1;
result += x2;
result += x3;
result += x4;
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x += 20;
result += x0;
result += x1;
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```

```
= 7[(volatile int *)x];
= 8[(volatile int *)x];
= 9[(volatile int *)x];
ult += x0;
ult += x1;
ult += x2;
ult += x3;
ult += x4;
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ult += x6;
ult += x7;
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ult += x9;
= 10[(volatile int *)x];
= 11[(volatile int *)x];
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result += x3;
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2526 cycles. Even better in asm.

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NTRU encryption example: Randomly order 761 trits $(\pm 1, \ldots, \pm 1, 0, \ldots, 0)$, wt 286.

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Any output is produced in $\le 119!(n-119)!\binom{2^{31}+n-1}{n}$ ways; i.e., $< 1.02 \cdot 2^{31n}/\binom{n}{119}$ ways.
Factor $<$1.02 increase in attacker's chance of winning.

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Sorting network on next slide: Batcher's merge-exchange sort. $\Theta(n(\log n)^2)$ `minmax` operations; $(1/4)(e^2-e+4)n-1$ for $n=2^e$.

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```
void sort(int32
{ long long t,p,
  t = 1; if (n <
  while (t < n-t
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    for (i = 0;i
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```
void sort(int32 *x,long l
{ long long t,p,q,i;
  t = 1; if (n < 2) retur
  while (t < n-t) t += t;
  for (p = t;p > 0;p >>=
    for (i = 0;i < n-p;++
      if (!(i & p))
        minmax(x+i,x+i+p)
    for (q = t;q > p;q >>
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$\geq$3008 cycles for $n = 1024$.

Current software: 7328 cycles.

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  t = 1; if (n < 2) return;
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    for (i = 0;i < n-p;++i)
      if (!(i & p))
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  for (p = t;p > 0;p >>= 1) {
    for (i = 0;i < n-p;++i)
      if (!(i & p))
        minmax(x+i,x+i+p);
    for (q = t;q > p;q >>= 1)
      for (i = 0;i < n-q;++i)
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```
rt(int32 *x,long long n)
long t,p,q,i;
; if (n < 2) return;
 (t < n-t) t += t;
p = t;p > 0;p >>= 1) {
 (i = 0;i < n-p;++i)
f (!(i & p))
 minmax(x+i,x+i+p);
 (q = t;q > p;q >>= 1)
or (i = 0;i < n-q;++i)
 if (!(i & p))
   minmax(x+i+p,x+i+q);
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 0;p >>= 1) {
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Constant-time code faster than "optimized" non-constant-time code? How is this possible?

People optimize algorithms for a naive model of CPUs:

- Branches are fast.
- Random access is fast.

How many cycles on, e.g., Intel Haswell CPU core?

Every cycle: a vector of 8 32-bit "min" operations and a vector of 8 32-bit "max" operations.

$\geq$3008 cycles for $n = 1024$.
Current software: 7328 cycles.
(Can gap be narrowed?)

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`uint32` string $(f_0,$
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$2^{179} f_7 + 2^{204} f_8 +$

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Usually faster representation

`uint32` string $(f_0, f_1, \ldots, f_9)$

represents $f_0 + 2^{26} f_1 + 2^{51} f$

$2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^1$

$2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9$.

Constant bound on each f_i.

More limbs than before,

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After multiplication,

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```
int32 f
int32 g
...
int64 f
int64 f
  f7_2
...
int64 h

...
c4 = (h
h5 += c
```

int library:
`uint32` string
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$2^{32(\ell-1)} f_{\ell-1}$.

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```
int32 f7_2 = 2 *
int32 g7_19 = 19
...
int64 f0g4 = f0
int64 f7g7_38 =
  f7_2 * (int64)
...
int64 h4 = f0g4
         + f2g2
         + f4g0
         + f6g8_
         + f8g6_
...
c4 = (h4 + (int6
h5 += c4; h4 -=
```

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```
int32 f7_2 = 2 * f7;
int32 g7_19 = 19 * g7;
...
int64 f0g4 = f0 * (int64)
int64 f7g7_38 =
  f7_2 * (int64) g7_19;
...
int64 h4 = f0g4 + f1g3_2
         + f2g2 + f3g1_2
         + f4g0 + f5g9_38
         + f6g8_19 + f7g7
         + f8g6_19 + f9g5
...
c4 = (h4 + (int64)(1<<25)
h5 += c4; h4 -= c4 << 26;
```

Usually faster representation:
`uint32` string $(f_0, f_1, \ldots, f_9)$
represents $f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9$.

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         + f6g8_19 + f7g7_38
         + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```

faster representation:
string $(f_0, f_1, \ldots, f_9)$
ts $f_0 + 2^{26}f_1 + 2^{51}f_2 +$
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int32 g7_19 = 19 * g7;
...
int64 f0g4 = f0 * (int64) g4;
int64 f7g7_38 =
  f7_2 * (int64) g7_19;
...
int64 h4 = f0g4 + f1g3_2
         + f2g2 + f3g1_2
         + f4g0 + f5g9_38
         + f6g8_19 + f7g7_38
         + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```

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$f_1, \ldots, f_9)$

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$^{128} f_5 + 2^{153} f_6 +$

$2^{230} f_9$.

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$1, \ldots, f_9)$.


```
int32 f7_2 = 2 * f7;
int32 g7_19 = 19 * g7;
...
int64 f0g4 = f0 * (int64) g4;
int64 f7g7_38 =
  f7_2 * (int64) g7_19;
...
int64 h4 = f0g4 + f1g3_2
         + f2g2 + f3g1_2
         + f4g0 + f5g9_38
         + f6g8_19 + f7g7_38
         + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```

Initial computatio

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modulo $x^{10} - 19$.

Exercise: Which p

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```
int32 f7_2 = 2 * f7;
int32 g7_19 = 19 * g7;
...
int64 f0g4 = f0 * (int64) g4;
int64 f7g7_38 =
  f7_2 * (int64) g7_19;
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         + f6g8_19 + f7g7_38
         + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```

Initial computation of h0, . .
is polynomial multiplication
modulo $x^{10} - 19$.
Exercise: Which polynomials
are being multiplied?

```
int32 f7_2 = 2 * f7;
int32 g7_19 = 19 * g7;
...
int64 f0g4 = f0 * (int64) g4;
int64 f7g7_38 =
  f7_2 * (int64) g7_19;
...
int64 h4 = f0g4 + f1g3_2
         + f2g2 + f3g1_2
         + f4g0 + f5g9_38
         + f6g8_19 + f7g7_38
         + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```


Initial computation of `h0`, . . . , `h9` is polynomial multiplication modulo $x^{10} - 19$.

Exercise: Which polynomials are being multiplied?

```
int32 f7_2 = 2 * f7;
int32 g7_19 = 19 * g7;
...
int64 f0g4 = f0 * (int64) g4;
int64 f7g7_38 =
  f7_2 * (int64) g7_19;
...
int64 h4 = f0g4 + f1g3_2
         + f2g2 + f3g1_2
         + f4g0 + f5g9_38
         + f6g8_19 + f7g7_38
         + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```


Initial computation of `h0`, . . . , `h9` is polynomial multiplication modulo $x^{10} - 19$.

Exercise: Which polynomials are being multiplied?

Reduction modulo $x^{10} - 19$ and carries such as `h4`→`h5` **squeeze** the product into limited-size representation suitable for next multiplication.

```
int32 f7_2 = 2 * f7;
int32 g7_19 = 19 * g7;
...
int64 f0g4 = f0 * (int64) g4;
int64 f7g7_38 =
  f7_2 * (int64) g7_19;
...
int64 h4 = f0g4 + f1g3_2
         + f2g2 + f3g1_2
         + f4g0 + f5g9_38
         + f6g8_19 + f7g7_38
         + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```


Initial computation of h0, ..., h9 is polynomial multiplication modulo $x^{10} - 19$.

Exercise: Which polynomials are being multiplied?

Reduction modulo $x^{10} - 19$ and carries such as h4→h5 **squeeze** the product into limited-size representation suitable for next multiplication.

At end of computation: **freeze** representation into unique representation suitable for network transmission.

```
7_2 = 2 * f7;
7_19 = 19 * g7;

0g4 = f0 * (int64) g4;
7g7_38 =
* (int64) g7_19;

4 = f0g4 + f1g3_2
  + f2g2 + f3g1_2
  + f4g0 + f5g9_38
  + f6g8_19 + f7g7_38
  + f8g6_19 + f9g5_38;

4 + (int64)(1<<25)) >> 26;
4; h4 -= c4 << 26;
```


Initial computation of `h0`, . . . , `h9` is polynomial multiplication modulo $x^{10} - 19$.
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Reduction modulo $x^{10} - 19$ and carries such as `h4`→`h5` **squeeze** the product into limited-size representation suitable for next multiplication.

At end of computation: **freeze** representation into unique representation suitable for network transmission.

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see, e.g.

```
 f7;
 * g7;

* (int64) g4;

 g7_19;

+ f1g3_2
+ f3g1_2
+ f5g9_38
19 + f7g7_38
19 + f9g5_38;

4)(1<<25)) >> 26;
c4 << 26;
```


Initial computation of `h0`, . . . , `h9` is polynomial multiplication modulo $x^{10} - 19$.

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Much more about
see, e.g., 2015 Ch

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Initial computation of `h0`, . . . , `h9` is polynomial multiplication modulo $x^{10} - 19$.

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Verifying constant time: increasingly automated.

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Verifying constant time: increasingly automated.

Testing can miss rare bugs that attacker might trigger.
Fix: prove that software matches mathematical spec; have computer check proofs.

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```
p = 2**2
A = 4866
x2,z2,x3
for i in
  ni = b
  x2,x3
  z2,z3
  x3,z3
   4*x1*
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```

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gfverif has verified
implementation of
plus occasional an
against the followi

```
p = 2**255-19
A = 486662
x2,z2,x3,z3 = 1,
for i in reverse
  ni = bit(n,i)
  x2,x3 = cswap(
  z2,z3 = cswap(
  x3,z3 = (4*(x2
   4*x1*(x2*z3-z
  x2,z2 = ((x2**
   4*x2*z2*(x2**
```

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gfverif has verified ref10 implementation of X25519, plus occasional annotations, against the following specifi

```
p = 2**255-19
A = 486662
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(2
  ni = bit(n,i)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
  x3,z3 = (4*(x2*x3-z2*z3
   4*x1*(x2*z3-z2*x3)**2)
  x2,z2 = ((x2**2-z2**2)*
   4*x2*z2*(x2**2+A*x2*z2
```

Much more about ECC speed:
see, e.g., 2015 Chou.

Verifying constant time:
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2017 HACL* X25519 in Firefox.

gfverif has verified `ref10`
implementation of X25519,
plus occasional annotations,
against the following specification:

```
p = 2**255-19
A = 486662
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  ni = bit(n,i)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
  x3,z3 = (4*(x2*x3-z2*z3)**2,
   4*x1*(x2*z3-z2*x3)**2)
  x2,z2 = ((x2**2-z2**2)**2,
   4*x2*z2*(x2**2+A*x2*z2+z2**2))
```

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```
p = 2**255-19
A = 486662
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  ni = bit(n,i)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
  x3,z3 = (4*(x2*x3-z2*z3)**2,
   4*x1*(x2*z3-z2*x3)**2)
  x2,z2 = ((x2**2-z2**2)**2,
   4*x2*z2*(x2**2+A*x2*z2+z2**2))
```

```
  x3,z3
  x2,z2
  cut(x
  cut(x
  cut(z
  cut(z
  x2,x3
  z2,z3
cut(x2)
cut(z2)
return 
```

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implementation of X25519,
plus occasional annotations,
against the following specification:

```
p = 2**255-19
A = 486662
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  ni = bit(n,i)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
  x3,z3 = (4*(x2*x3-z2*z3)**2,
   4*x1*(x2*z3-z2*x3)**2)
  x2,z2 = ((x2**2-z2**2)**2,
   4*x2*z2*(x2**2+A*x2*z2+z2**2))
```

```
  x3,z3 = (x3%p,
  x2,z2 = (x2%p,
  cut(x2)
  cut(x3)
  cut(z2)
  cut(z3)
  x2,x3 = cswap(
  z2,z3 = cswap(
cut(x2)
cut(z2)
return x2*pow(z2
```

What's verified: o
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and is between 0 a

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gfverif has verified `ref10` implementation of X25519, plus occasional annotations, against the following specification:

```
p = 2**255-19
A = 486662
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  ni = bit(n,i)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
  x3,z3 = (4*(x2*x3-z2*z3)**2,
   4*x1*(x2*z3-z2*x3)**2)
  x2,z2 = ((x2**2-z2**2)**2,
   4*x2*z2*(x2**2+A*x2*z2+z2**2))
  x3,z3 = (x3%p,z3%p)
  x2,z2 = (x2%p,z2%p)
  cut(x2)
  cut(x3)
  cut(z2)
  cut(z3)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
cut(x2)
cut(z2)
return x2*pow(z2,p-2,p)
```

What's verified: output of r

is the same as spec mod p,

and is between 0 and $p-1$.

gfverif has verified `ref10` implementation of X25519, plus occasional annotations, against the following specification:

```
p = 2**255-19
A = 486662
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  ni = bit(n,i)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
  x3,z3 = (4*(x2*x3-z2*z3)**2,
   4*x1*(x2*z3-z2*x3)**2)
  x2,z2 = ((x2**2-z2**2)**2,
   4*x2*z2*(x2**2+A*x2*z2+z2**2))
```



```
  x3,z3 = (x3%p,z3%p)
  x2,z2 = (x2%p,z2%p)
  cut(x2)
  cut(x3)
  cut(z2)
  cut(z3)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
cut(x2)
cut(z2)
return x2*pow(z2,p-2,p)
```

What's verified: output of `ref10` is the same as spec mod p, and is between 0 and $p - 1$.

as verified ref10
ntation of X25519,
asional annotations,
the following specification:

```
255-19
662
3,z3 = 1,0,x1,1
n reversed(range(255)):
bit(n,i)
 = cswap(x2,x3,ni)
 = cswap(z2,z3,ni)
 = (4*(x2*x3-z2*z3)**2,
*(x2*z3-z2*x3)**2)
 = ((x2**2-z2**2)**2,
*z2*(x2**2+A*x2*z2+z2**2))
```



```
  x3,z3 = (x3%p,z3%p)
  x2,z2 = (x2%p,z2%p)
  cut(x2)
  cut(x3)
  cut(z2)
  cut(z3)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
cut(x2)
cut(z2)
return x2*pow(z2,p-2,p)
```

What's verified: output of ref10 is the same as spec mod p, and is between 0 and $p-1$.

"What a

NIST P-
$2^{256} - 2$

ECDSA
reductio
an integ

Write A
$(A_{15}, A_1$
$A_8, A_7,$
meaning

Define
$T; S_1; S_2$
as

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ng specification:

```
0,x1,1
d(range(255)):

x2,x3,ni)
z2,z3,ni)
*x3-z2*z3)**2,
2*x3)**2)
2-z2**2)**2,
2+A*x2*z2+z2**2))
```



```
    x3,z3 = (x3%p,z3%p)
    x2,z2 = (x2%p,z2%p)
    cut(x2)
    cut(x3)
    cut(z2)
    cut(z3)
    x2,x3 = cswap(x2,x3,ni)
    z2,z3 = cswap(z2,z3,ni)
cut(x2)
cut(z2)
return x2*pow(z2,p-2,p)
```

What's verified: output of `ref10` is the same as spec mod p, and is between 0 and $p-1$.

"What a differenc

NIST P-256 prime
$2^{256} - 2^{224} + 2^{192}$

ECDSA standard s
reduction procedu
an integer "A less

Write A as
$(A_{15}, A_{14}, A_{13}, A_{12}$
A_8, A_7, A_6, A_5, A
meaning $\sum_i A_i 2^{32}$

Define
$T; S_1; S_2; S_3; S_4; D$
as

cation:

255)):

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2+z2**2))

```
  x3,z3 = (x3%p,z3%p)
  x2,z2 = (x2%p,z2%p)
  cut(x2)
  cut(x3)
  cut(z2)
  cut(z3)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
cut(x2)
cut(z2)
return x2*pow(z2,p-2,p)
```

What's verified: output of `ref10` is the same as spec mod p, and is between 0 and $p-1$.

"What a difference a prime

NIST P-256 prime p is $2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$

ECDSA standard specifies reduction procedure given an integer "A less than p^2":

Write A as $(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}$
$A_8, A_7, A_6, A_5, A_4, A_3, A_2,$
meaning $\sum_i A_i 2^{32i}$.

Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3$
as

```
  x3,z3 = (x3%p,z3%p)
  x2,z2 = (x2%p,z2%p)
  cut(x2)
  cut(x3)
  cut(z2)
  cut(z3)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
cut(x2)
cut(z2)
return x2*pow(z2,p-2,p)
```

What's verified: output of `ref10` is the same as spec mod p, and is between 0 and $p-1$.

"What a difference a prime makes"

NIST P-256 prime p is $2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

ECDSA standard specifies reduction procedure given an integer "A less than p^2":

Write A as $(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$, meaning $\sum_i A_i 2^{32i}$.

Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$ as

```
 = (x3%p,z3%p)
 = (x2%p,z2%p)
2)
3)
2)
3)
 = cswap(x2,x3,ni)
 = cswap(z2,z3,ni)


x2*pow(z2,p-2,p)
```

verified: output of `ref10`
me as spec mod p,
etween 0 and $p-1$.

"What a difference a prime makes"

NIST P-256 prime p is $2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

ECDSA standard specifies reduction procedure given an integer "A less than p^2":

Write A as
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9,$
$A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$,
meaning $\sum_i A_i 2^{32i}$.

Define
$T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$
as

$(A_7, A_6,$
$(A_{15}, A_1$
$(0, A_{15},$
$(A_{15}, A_1$
$(A_8, A_{13}$
$(A_{10}, A_8$
$(A_{11}, A_9$
$(A_{12}, 0,$
$(A_{13}, 0,$

Comput
$S_4 - D_1$

Reduce
subtract

```
z3%p)
z2%p)
```

```
x2,x3,ni)
z2,z3,ni)
```

```
,p-2,p)
```

utput of `ref10`
c mod p,
and $p - 1$.

“What a difference a prime makes”

NIST P-256 prime p is
$2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

ECDSA standard specifies reduction procedure given an integer “A less than p^2”:

Write A as
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9,$
$A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$,
meaning $\sum_i A_i 2^{32i}$.

Define
$T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$
as

$(A_7, A_6, A_5, A_4, A_3$
$(A_{15}, A_{14}, A_{13}, A_{12}$
$(0, A_{15}, A_{14}, A_{13},$
$(A_{15}, A_{14}, 0, 0, 0,$
$(A_8, A_{13}, A_{15}, A_{14},$
$(A_{10}, A_8, 0, 0, 0, A$
$(A_{11}, A_9, 0, 0, A_{15},$
$(A_{12}, 0, A_{10}, A_9, A$
$(A_{13}, 0, A_{11}, A_{10},$

Compute $T + 2S_1$
$S_4 - D_1 - D_2 -$

Reduce modulo p
subtracting a few

ref10

"What a difference a prime makes"

NIST P-256 prime p is $2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

ECDSA standard specifies reduction procedure given an integer "A less than p^2":

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Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$ as

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A$

$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0$

$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$

$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8$

$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11},$

$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_1$

$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13},$

$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}$

$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15},$

Compute $T + 2S_1 + 2S_2 +$ $S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by addin subtracting a few copies" of

“What a difference a prime makes”

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$2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

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$T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$
as

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p “by adding or subtracting a few copies” of p.

a difference a prime makes"

-256 prime p is
$^{224} + 2^{192} + 2^{96} - 1$.

standard specifies
n procedure given
er "A less than p^2":

as
$_4, A_{13}, A_{12}, A_{11}, A_{10}, A_9,$
$A_6, A_5, A_4, A_3, A_2, A_1, A_0),$
$\sum_i A_i 2^{32i}$.

$_2; S_3; S_4; D_1; D_2; D_3; D_4$

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

What is
Variable-

e a prime makes"

p is

$+ 2^{96} - 1.$

specifies

re given

than p^2":

$, A_{11}, A_{10}, A_9,$

$_4, A_3, A_2, A_1, A_0),$

i.

$D_1; D_2; D_3; D_4$

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0);$
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0);$
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0);$
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8);$
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9);$
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11});$
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12});$
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13});$
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14}).$

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

What is "a few co

Variable-time loop

makes”

1.

, A_9,

A_1, A_0),

; D_4

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p “by adding or subtracting a few copies” of p.

What is “a few copies”?

Variable-time loop is unsafe.

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

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$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
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Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

What is "a few copies"?
Variable-time loop is unsafe.

Correct but quite slow:
conditionally add $4p$,
conditionally add $2p$,
conditionally add p,
conditionally sub $4p$,
conditionally sub $2p$,
conditionally sub p.

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

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Delay until end of computation?
Trouble: "A less than p^2".

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

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conditionally add p,
conditionally sub $4p$,
conditionally sub $2p$,
conditionally sub p.

Delay until end of computation?
Trouble: "A less than p^2".

Even worse: what about platforms where 2^{32} isn't best radix?

$A_5, A_4, A_3, A_2, A_1, A_0)$;
$_4, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$_4, 0, 0, 0, A_{10}, A_9, A_8)$;
$, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

e $T + 2S_1 + 2S_2 + S_3 +$
$- D_2 - D_3 - D_4$.

modulo p "by adding or
ing a few copies" of p.

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conditionally add $4p$,
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conditionally add p,
conditionally sub $4p$,
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There ar
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correct c

e.g. ECD
of scalar

e.g. ECD
addition
EdDSA

$_3, A_2, A_1, A_0)$;
$_2, A_{11}, 0, 0, 0)$;
$A_{12}, 0, 0, 0)$;
$A_{10}, A_9, A_8)$;
$A_{13}, A_{11}, A_{10}, A_9)$;
$_{13}, A_{12}, A_{11})$;
$A_{14}, A_{13}, A_{12})$;
$_8, A_{15}, A_{14}, A_{13})$;
$A_9, 0, A_{15}, A_{14})$.

$+ 2S_2 + S_3 +$
$D_3 - D_4$.

"by adding or
copies" of p.

What is "a few copies"?
Variable-time loop is unsafe.

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conditionally add $4p$,
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There are many m
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correct constant-ti

e.g. ECDSA needs
of scalars. EdDSA

e.g. ECDSA splits
additions into seve
EdDSA uses comp

A_0);
, 0);
);
);
A_{10}, A_9);
$_1$);
A_{12});
, A_{13});
A_{14}).

$S_3 +$

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p.

What is "a few copies"?
Variable-time loop is unsafe.

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conditionally add $2p$,
conditionally add p,
conditionally sub $4p$,
conditionally sub $2p$,
conditionally sub p.

Delay until end of computation?
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There are many more ways t
cryptographic design choices
affect difficulty of building fa
correct constant-time softwa

e.g. ECDSA needs divisions
of scalars. EdDSA doesn't.

e.g. ECDSA splits elliptic-cu
additions into several cases.
EdDSA uses complete formu

What is “a few copies”?
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There are many more ways that cryptographic design choices affect difficulty of building fast correct constant-time software.

e.g. ECDSA needs divisions of scalars. EdDSA doesn’t.

e.g. ECDSA splits elliptic-curve additions into several cases. EdDSA uses complete formulas.

What’s better use of time: implementing ECDSA, or upgrading protocol to EdDSA?