Efficient implementation of code-based cryptography

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. . . all of the above *at once*.

The track record

1978 McEliece proposed public-key code-based crypto.

Has held up well after extensive optimization of attack algorithms:
1962 Prange. 1981 Omura.
1988 Lee–Brickell. 1988 Leon.
1989 Krouk. 1989 Stern.
1989 Dumer.
1990 Coffey–Goodman.
1990 van Tilburg. 1991 Dumer.
1991 Coffey–Goodman–Farrell.
1993 Chabanne–Courteau.
1993 Chabaud.

1994 van Tilburg.

1994 Canteaut–Chabanne.

1998 Canteaut–Chabaud.

1998 Canteaut–Sendrier.

2008 Bernstein–Lange–Peters.

2009 Bernstein–Lange–Peters–van Tilborg.

2009 Bernstein (post-quantum).

2009 Finiasz–Sendrier.

2010 Bernstein–Lange–Peters.

2011 May–Meurer–Thomae.

2011 Becker–Coron–Joux.

2012 Becker–Joux–May–Meurer.

2013 Bernstein–Jeffery–Lange–Meurer (post-quantum).

Examples of the competition

Some cycle counts on `h9ivy`
(Intel Core i5-3210M, Ivy Bridge)
from `bench.cr.yp.to`:

`mceliece` encrypt	73092
(2008 Biswas–Sendrier, $\approx 2^{80}$)	
`gls254` DH	76212
(binary elliptic curve; CHES 2013)	
`kummer` DH	88448
(hyperelliptic; Asiacrypt 2014)	
`curve25519` DH	182708
(conservative elliptic curve)	
`mceliece` **decrypt**	1130908
`ronald1024` decrypt	1313324

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All load/store addresses and all branch conditions are public. Eliminates cache-timing attacks etc.

Similar improvements for CFS.

Constant-time fanaticism

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Handle all secret data
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"How can this be
competitive in speed?
Are you really simulating
field multiplication with
hundreds of bit operations
instead of simple log tables?"

Yes, we are.

Not as slow as it sounds!
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Low-end smartphone CPU:
128-bit XOR every cycle.

Ivy Bridge:
256-bit XOR every cycle,
or three 128-bit XORs.

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Typical decoding algorithms
have add, mult roughly balanced.

Coming next: how to save
many adds and *most* mults.
Nice synergy with bitslicing.

The additive FFT

Fix $n = 4096 = 2^{12}$, $t = 41$.

Big final decoding step
is to find all roots in $\mathbf{F}_{2^{12}}$
of $f = c_{41}x^{41} + \cdots + c_0 x^0$.

For each $\alpha \in \mathbf{F}_{2^{12}}$,
compute $f(\alpha)$ by Horner's rule:
41 adds, 41 mults.

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$c_i g^i$, $c_i g^{2i}$, $c_i g^{3i}$, etc. Cost per
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Our cost: **6.01** adds, **2.09** mults.

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Wait a minute.
Didn't we learn in school
that FFT evaluates
an n-coeff polynomial
at n points
using $n^{1+o(1)}$ operations?
Isn't this better than $n^2/\lg n$?

Standard radix-2 FFT:

Want to evaluate
$f = c_0 + c_1 x + \cdots + c_{n-1} x^{n-1}$
at all the nth roots of 1.

Write f as $f_0(x^2) + x f_1(x^2)$.
Observe big overlap between
$f(\alpha) = f_0(\alpha^2) + \alpha f_1(\alpha^2)$,
$f(-\alpha) = f_0(\alpha^2) - \alpha f_1(\alpha^2)$.

f_0 has $n/2$ coeffs;
evaluate at $(n/2)$nd roots of 1
by same idea recursively.
Similarly f_1.

Useless in char 2: $\alpha = -\alpha$.
Standard workarounds are painful.
FFT considered impractical.

1988 Wang–Zhu,
independently 1989 Cantor:
"additive FFT" in char 2.
Still quite expensive.

1996 von zur Gathen–Gerhard:
some improvements.

2010 Gao–Mateer:
much better additive FFT.

We use Gao–Mateer,
plus some new improvements.

Gao and Mateer evaluate
$f = c_0 + c_1 x + \cdots + c_{n-1} x^{n-1}$
on a size-n $\mathbf{F}_2$-linear space.

Their main idea: Write f as
$f_0(x^2 + x) + x f_1(x^2 + x)$.

Big overlap between $f(\alpha) =$
$f_0(\alpha^2 + \alpha) + \alpha f_1(\alpha^2 + \alpha)$
and $f(\alpha + 1) =$
$f_0(\alpha^2 + \alpha) + (\alpha + 1) f_1(\alpha^2 + \alpha)$.

"Twist" to ensure $1 \in$ space.
Then $\left\{\alpha^2 + \alpha\right\}$ is a
size-$(n/2)$ $\mathbf{F}_2$-linear space.
Apply same idea recursively.

Results

60493 Ivy Bridge cycles:

8622 for permutation.
20846 for syndrome.
7714 for BM.
14794 for roots.
8520 for permutation.

Code will be public domain.
We're still speeding it up.

Also $10\times$ speedup for CFS.

More information:
cr.yp.to/papers.html#mcbits

What you find in paper:

Cryptosystem specification.

Our speedups to additive FFT.
(We now have more speedups:
`cr.yp.to/papers.html#auth256`.)

Fast syndrome computation
without big precomputed matrix.
Important for lightweight!

Fast secret permutation
using bit operations:
sorting networks,
permutation networks.