Efficient implementation of code-based cryptography

D. J. Bernstein
University of Illinois at Chicago &
Technische Universiteit Eindhoven

Joint work with:

Tung Chou
Technische Universiteit Eindhoven

Peter Schwabe
Radboud University Nijmegen

Objectives

Set new speed records
for public-key cryptography.

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`mceliece` encrypt	73092
(2008 Biswas–Sendrier, $\approx 2^{80}$)	
`gls254` DH	76212
(binary elliptic curve; CHES 2013)	
`kummer` DH	88448
(hyperelliptic; Asiacrypt 2014)	
`curve25519` DH	182708
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`mceliece` **decrypt**	1130908
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Talk will focus on this case.

(Decryption is slightly slower:
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Or use Chien search: compute $c_i g^i$, $c_i g^{2i}$, $c_i g^{3i}$, etc. Cost per point: again 41 adds, 41 mults.

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Write f as $f_0(x^2) + x f_1(x^2)$.
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7714 for BM.
14794 for roots.
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Also $10\times$ speedup

More information:
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Gao and Mateer evaluate $f = c_0 + c_1 x + \cdots + c_{n-1} x^{n-1}$ on a size-n $\mathbf{F}_2$-linear space.

Their main idea: Write f as $f_0(x^2 + x) + x f_1(x^2 + x)$.

Big overlap between $f(\alpha) = f_0(\alpha^2 + \alpha) + \alpha f_1(\alpha^2 + \alpha)$ and $f(\alpha + 1) = f_0(\alpha^2 + \alpha) + (\alpha + 1) f_1(\alpha^2 + \alpha)$.

"Twist" to ensure $1 \in$ space. Then $\{\alpha^2 + \alpha\}$ is a size-$(n/2)$ $\mathbf{F}_2$-linear space. Apply same idea recursively.

Results

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What you find in p

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What you find in paper:

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