

Non-uniform
cracks in the concrete:
the power of free precomputation

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2012.02.19 Koblitiz–Menezes

“Another look at HMAC”:

“... Third, we describe a fundamental flaw in Bellare’s 2006 security proof for HMAC, and show that with the flaw removed the proof gives a security guarantee that is of little value in practice.”

2012.03.02: *“Bellare contacted us and told us that he strongly objected to our language—especially the word ‘flaw’—...”*

Yehuda Lindell: *“This time they really outdid themselves since there is actually no error. Rather the proof of security is in the non-uniform model, which they appear to not be familiar with. . . . There is NO FLAW here whatsoever.”*

Jonathan Katz: *“Many researchers are justifiably concerned about the fact that Alfred Menezes will be giving an invited talk at Eurocrypt 2012 related to his line of papers criticizing provable security. I share this concern.”*

Bellare to Koblitz (according to 2012.10 Koblitz talk): *“It never occurred to me that a reader would not understand that when complexity is concrete, we have non-uniformity. . . . If you want . . . to gain respect among theoretical cryptographers, it would benefit from reflecting our feedback and being better informed about the basics of the field. . . . Uniform and non-uniform complexity are typically taught in a graduate course in computational complexity theory.”*

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[youtube?v=1560Rg5xXkk](https://www.youtube.com/watch?v=1560Rg5xXkk)

Understanding the dispute

What is the best chosen-plaintext
AES-128 key-recovery attack?

Attack input: a black box
that contains a secret key k
and computes $p \mapsto \text{AES}_k(p)$.

Attack output: k .

Standard definition of “best”:
minimize “time”.

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More generally, allow attacks with
<100% success probability;
analyze tradeoffs between
“time” and success probability.

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Standard conjecture:

For each $p \in [0, 1]$,
each AES key-recovery attack
with success probability $\geq p$
takes “time” $\geq 2^{128}p$.

See, e.g., 2005 Bellare–Rogaway.

Interlude regarding “time”

How much “time” does the following algorithm take?

```
def pidigit(n0,n1,n2):  
    if n0 == 0:  
        if n1 == 0:  
            if n2 == 0: return 3  
            return 1  
        if n2 == 0: return 4  
        return 1  
    if n1 == 0:  
        if n2 == 0: return 5  
        return 9  
    if n2 == 0: return 2  
    return 6
```

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Generalization: There exists an algorithm that, given $n < 2^k$, prints the n th digit of π using $k + 1$ “steps”.

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Variant: There exists a 256-“step” AES key-recovery attack (with 100% success probability).

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Variant: There exists a 256-“step” AES key-recovery attack (with 100% success probability). If “time” means “steps” then the standard conjecture is wrong.

2000 Bellare–Kilian–Rogaway:

“We fix some particular Random Access Machine (RAM) as a model of computation. . . . A’s running time [means] A’s actual execution time plus the length of A’s description . . . This convention eliminates pathologies caused [by] arbitrarily large lookup tables . . . Alternatively, the reader can think of circuits over some fixed basis of gates, like 2-input NAND gates . . . now time simply means the circuit size.”

Side comments:

1. Definition from Crypto 1994
Bellare–Kilian–Rogaway was
flawed: failed to add length.

Paper conjectured “useful” DES
security bounds; any reasonable
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2. Many more subtle issues defining RAM “time”: see 1990 van Emde Boas survey.

3. NAND definition is easier but breaks many theorems.

Reductions

Another standard conjecture:
Each AES-CBC-MAC q -block
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probability $\geq p + q(q - 1)/2^{129}$
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Why should users have any
confidence in this conjecture?

How many researchers have really
tried to break AES-CBC-MAC?
AES-CTR? AES-GCM? Other
AES-based protocols? Far less
attention than for key recovery.

Provable security to the rescue!

Prove: if there is
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Oops: This turns out to be hard.
But changing from key-recovery
attack to PRF distinguishing
attack allows a proof:
1994 Bellare–Kilian–Rogaway.

Similar pattern throughout the “provable security” literature.

Protocol designers (try to) prove that hardness of a problem P (e.g., AES PRF attacks) implies security of various protocols Q .

After extensive cryptanalysis of P , maybe gain confidence in hardness of P , and hence in security of Q .

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Why not directly cryptanalyze Q ?

Cryptanalysis is hard work: have to focus on *a few* problems P .

Proofs scale to *many* protocols Q .

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These conjectures are wrong.

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Good candidate for attack:

$\text{MD5}_0(7, \text{AES}_k(0), \text{AES}_k(1)) = 1$
with probability $\geq 1/2 + 2^{-64}$;

$\text{MD5}_0(7, F(0), F(1)) = 1$
with probability $\leq 1/2$.

Here $\text{MD5}_0(x) = \text{bit}_0(\text{MD5}(x))$.

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If this candidate doesn't work,
replace 7 with 8 or 9 or

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Example: There exists
an AES key-recovery attack
with success probability ≈ 1
taking “time” $\approx 2^{86}$.

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The conjectures are still wrong!

Example: There exists an AES key-recovery attack with success probability ≈ 1 taking “time” $\approx 2^{86}$.

The attack algorithm:

iterate $k \mapsto \text{AES}_k(0) \oplus 7$

2^{43} times, look up in

a size- 2^{43} Hellman table;

iterate $k \mapsto \text{AES}_k(0) \oplus 8$

2^{43} times, look up in

a size- 2^{43} Hellman table; etc.

How about NIST P-256?

ECDL input: points P, Q ,
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Standard conjecture:

For each $p \in [0, 1]$,

each P-256 ECDL algorithm

with success probability $\geq p$

takes “time” $\geq 2^{128} p^{1/2}$.

Cube-root ECDL algorithms

Assuming plausible heuristics,
overwhelmingly verified by
computer experiment:

There exists a P-256 ECDL
algorithm that takes “time” $\approx 2^{85}$
and has success probability ≈ 1 .

“Time” includes algorithm length.

Inescapable conclusion: **The
standard conjectures** (regarding
P-256 ECDL hardness, P-256
ECDSA security, etc.) **are false.**

Should P-256 ECDSA users
be worried about this
P-256 ECDL algorithm A ?

No!

We have a program B
that prints out A ,
but B takes “time” $\approx 2^{170}$.

We conjecture that
nobody will ever print out A .

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that prints out A ,
but B takes “time” $\approx 2^{170}$.

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But A *exists*, and the standard
conjecture doesn't see the 2^{170} .

Cryptanalysts *do* see the 2^{170} .

Common parlance: We have
a 2^{170} “precomputation”
(independent of Q) followed by
a 2^{85} “main computation”.

For cryptanalysts: This costs
 2^{170} , much worse than 2^{128} .

For the standard security
definitions and conjectures:
The main computation costs 2^{85} ,
much better than 2^{128} .

What the algorithm does

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1999 Escott–Sager–Selkirk–
Tsapakidis, also crediting
Silverman–Stapleton:

Computing (e.g.) $\log_P Q_1$,
 $\log_P Q_2$, $\log_P Q_3$, $\log_P Q_4$, and
 $\log_P Q_5$ costs only $2.49\times$ more
than computing $\log_P Q$.

The basic idea:

compute $\log_P Q_1$ with rho;
compute $\log_P Q_2$ with rho,
reusing distinguished points
produced by Q_1 ; etc.

2001 Kuhn–Struik analysis:

cost $\Theta(n^{1/2}\ell^{1/2})$

for n discrete logarithms

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2004 Hitchcock–

Montague–Carter–Dawson:

View computations of

$\log_P Q_1, \dots, \log_P Q_{n-1}$ as

precomputation for main

computation of $\log_P Q_n$.

Analyze tradeoffs between

main-computation time and

precomputation time.

2012 Bernstein–Lange:

- (1) Adapt to interval of length ℓ inside much larger group.
- (2) Analyze tradeoffs between main-computation time and precomputed table size.
- (3) Choose table entries more carefully to reduce main-computation time.
- (4) Also choose iteration function more carefully.
- (5) Reduce space required for each table entry.
- (6) Break $\ell^{1/4}$ barrier.

Applications:

- (7) Disprove the standard 2^{128} P-256 security conjectures.
- (8) Accelerate trapdoor DL etc.
- (9) Accelerate BGN etc.;
this needs (1).

Bonus:

- (10) Disprove the standard 2^{128} AES, DSA-3072, RSA-3072 security conjectures.

Credit to earlier Lee–Cheon–Hong paper for (2), (6), (8).

Standard walk function:

choose uniform random

$c_1, \dots, c_r \in \{1, 2, \dots, \ell - 1\};$

walk from R to $R + c_{H(R)}P$.

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Start some walks at yP

for random choices of y .

Build table of distinct

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(If this fails, rerandomize Q .)

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The following sketch
is not the state of the art —
but good enough to break
the 2^{128} assumption.

Let $g \in \mathbf{F}_p^*$ have order q , $h = g^k$.
Goal: Find k .

Precomputation:

Take $y = 2^{110}$,

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for every prime number $x \leq y$.

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Try to write h as

quotient h_1/h_2 in $\mathbf{F}_p^*$

with $h_2 \in \{1, 2, 3, \dots, 2^{1535}\}$,

$h_1 \in \{-2^{1535}, \dots, 0, 1, \dots, 2^{1535}\}$,

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into primes $\leq y$.

If this fails, try again

with hg, hg^2 , etc.

Analysis

About $y / \log y \approx 2^{103.75}$ primes $\leq y$
for a total of $2^{109.33}$ bytes
to store all small DLs.

Can write h as h_1/h_2 with
probability $\approx (6/\pi^2)2^{3071}/p$.

h_i is y -smooth with probability
very close to $u^{-u} \approx 2^{-53.06}$
where $u = 1535/110$.

Overall the attack requires
between $2^{107.85}$ and $2^{108.85}$
iterations; batch smoothness
detection is fast.

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(4) Add effectivity. Include cost for finding the algorithm.

(5) Add uniformity.

Clearly stops attacks

but breaks most theorems.

Abandons goal of defining concrete security of AES etc.