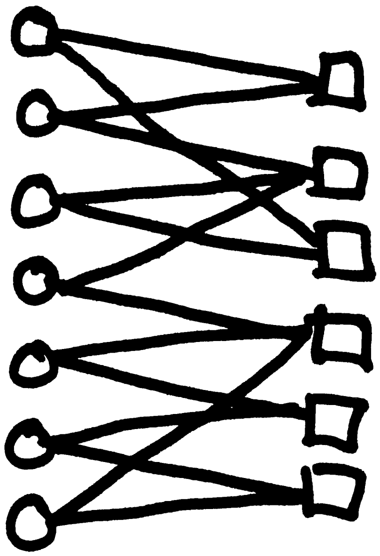
The Dumbbell Code

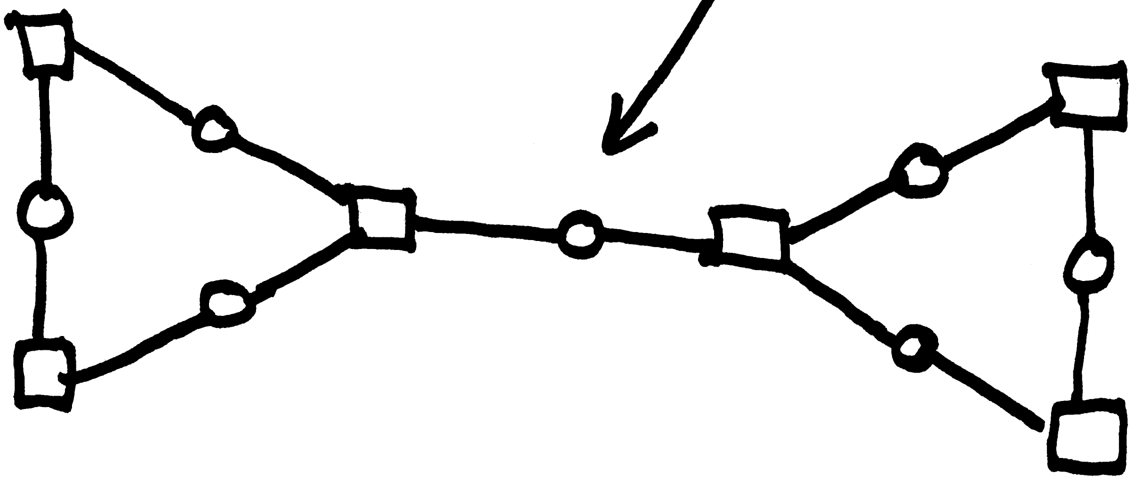
$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Codewords

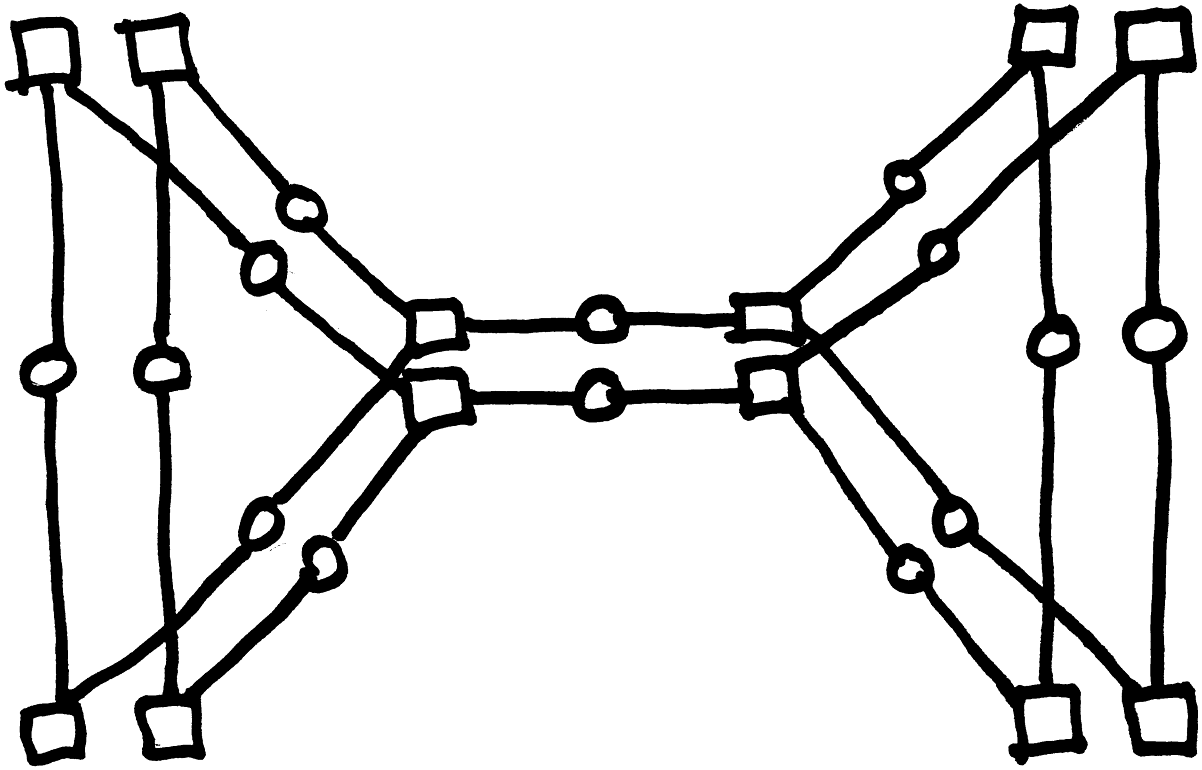
(0,0,0,0,0,0,0)
(1,1,1,0,0,0,0)
(0,0,0,0,1,1,1)
(1,1,1,0,1,1,1)



Tanner Graph



A 2-Cover



Fundamental Cone

$$\mathcal{K}(H) = \left\{ (a,a,a,b,c,c,c) \in \mathbb{R}^7 \;\middle|\; \begin{array}{c} a \geq 0,\, b \geq 0,\, c \geq 0 \\ 2a \geq b,\, 2c \geq b \end{array} \right\}$$

Fundamental Polytope

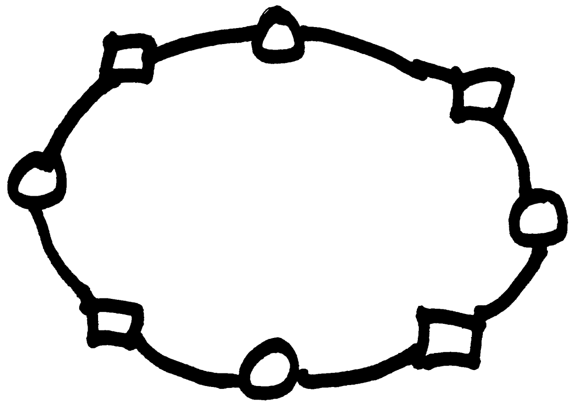
$\mathcal{P}(H)$ has 5 vertices:

$$\left.\begin{matrix}(0,0,0,0,0,0,0)\\(1,1,1,0,0,0,0)\\(0,0,0,0,1,1,1)\\(1,1,1,0,1,1,1)\end{matrix}\right\}\text{Codewords}$$

$$\left(\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},1,\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}\right)$$

The Circle [4,1,4] Repetition Code

(Kelley - Sridhara)

$T =$  $H = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

Fundamental Polytope $= \{(a,a,a,a) \in \mathbb{R}^4 \mid 0 \le a \le 1\}$

*If $a \notin \{0,1\}$, then (a,a,a,a) is not the normalized pseudocodeword of any codeword on any connected cover of T.

The O-Plus [4,1,4] Repetition Code

(Kelley - Sridhara)