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Fractional group operators and the classical special functions.

The classical special functions are all connected with the representation of Lie groups and appear as factors in multivariable functions $F_{\alpha,\dots}(w)$ on which the action of a classical Lie algebra is realized by linear differential operators $D(w, \partial_w)$. The actions of appropriate elements D of the Lie algebra lead to the standard differential recurrence relations for the functions, schematically of the form $DF_{\alpha,\dots} = cF_{\alpha\pm 1,\dots}$ where the α 's label the realization of the Lie algebra and D is a stepping operator. The action of group elements e^{-tD} can be interpreted in terms of generating functions dependent on the group parameter t . In the present work, we define fractional generalizations D^μ of the D 's in the context of Lie theory, determine their formal properties, and illustrate their usefulness in obtaining interesting relations among the functions. These include integral transformations, integral representations, and fractional integrals for the functions. Most of the specific results have been derived historically in other ways, but are unified in this approach in a group setting. I will present the theory of the fractional operators and representative examples of their use. (Received February 06, 2006)